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MODEL PREDICTIVE CONTROL OF INTEGRATED QUANTITY AND QUALITY IN DRINKING WATER DISTRIBUTION SYSTEMS

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Abstract: The paper considers a feedback optimising control of drinking water distribution systems (DWDS). Although the optimised pump and valves scheduling and disinfectant injection control attracted considerable attention over last two decades most of the contributions were limited to an open-loop optimisation repetitively performed during the DWDS operation. An information feedback from the DWDS is needed in reality in order to compensate uncertainty in the water demand prediction and the modelling errors. Also, while a strong interaction between the water quantity and quality exists most of the proposals regards either quality or quantity control. In this paper a generic model predictive controller for the optimising control of an integrated quantity and quality in DWDS is derived and applied to the case – study system. The simulation results based on real data records are presented..

Keywords: predictive control, closed-loop control, integrated plant control, genetic algorithms, water quantity and quality.

1. INTRODUCTION AND PROBLEM STATEMENT

Drinking water distribution system (DWDS) delivers water to domestic users. Hence, main objective is to meet water demand of required quality every consumer (Brdys and Ulanicki, 1994) . For safe and efficient process operation the monitoring and control systems are needed. In the paper the monitoring system is assumed in place and the control system for DWDS is pursued. There are two major aspects in control of drinking water distribution systems (DWDS): quantity and quality. The quantity control deals with the pipe flows and pressures at the water network nodes producing optimised pump and valve control schedules so that water demand at the consumption nodes is met and the associated electrical energy cost due to the pumping is minimised (Brdys and Ulanicki, 1994; Boulos, *et al.*, 2004). Maintaining concentrations of water quality parameters within prescribed limits throughout the network is a major objective of the quality control. In the paper, only one quality parameter is considered that is chlorine. It is the most common disinfectant used in the DWDS. It is not expensive and effectively controls a number of disease-causing organisms. As the chlorine reactions with certain organic compounds produce disinfectant by-products that are health dangerous (Boccelli, *et al.*, 2003) the allowed chlorine residuals are bounded above. Hence, the objective of maintaining desired water quality is expressed by certain lower and upper limits on the chlorine residuals at the consumption nodes. The chlorine residuals are directly controlled within the treatment plants so that the water entering the DWDS has prescribed residual values. However, when travelling throughout the network the disinfectant reacts and consequently its major decay may occur so that a bacteriological safety of water may not be guaranteed particularly at remote consumption nodes. Therefore, post chlorination by means of using booster stations located at certain intermediate nodes is needed. The booster station allocation problem was presented in (Prasad, *et al.*, 2004; Ewald, *et al.*, 2006) and the solution methods based on multiobjective optimisation were provided. The chlorine residuals at the nodes representing outputs from the treatment plant and at the booster station nodes are the direct control variables for the quality control. Electricity charges due to pumping constitute main component of the operational cost to be minimised. As an interaction exists between the quality and quantity control problems due to the transportation delays when transferring the chlorine throughout the network a proposal to integrate these two control issues into one integrated optimising control problem was made in (Brdys, *et al.*, 1995) and a receding horizon model predictive control technique (MPC) was applied that assumed periodically varying and very similar demands over a number of subsequent days. Also the quality and quantity modelling errors were not addressed by means of feedback. These assumptions allowed for a simplified implementation of the MPC were the feedback was taken once per day. Several solvers of the MPC optimisation task were proposed applying the genetic search (Ostfeld, *et al.*, 2002), mixed integer linear (MIL) algorithm (Brdys, *et al.*, 1995), sequential quantity-quality hybrid search and genetic-MIL approach (Trawicki, *et al.*, 2003) and nonlinear programming approach (Sakarya and Mays, 2000). Due to different time scales in the hydraulic variations (slow) and internal chlorine decay dynamics (fast) the integrated optimisation task complexity did not allow applying the integrated control to many realistic size DWDS. With the hydraulic time step typically one hour, quality time step for example five minutes and the prediction horizon due to tank capacities typically 24 hrs the problem dimension largely increases even for small size systems. A suboptimal two layer hierarchical control structure was proposed in (Brdys, *et al.*, 2000). The *optimising controller* at the *upper control layer* operates according to a simplified receding horizon strategy. At the beginning of a 24 hours time period the DWDS quantity and quality states are measured or estimated and sent to the integrated quantity and quality optimiser. The consumer demand prediction is also sent to the optimiser. The quality model assumes the same time step as the quantity dynamics model. Hence, the problem dimension is vastly reduced but the quality modelling error is significantly increased. Hence, solving the integrated quantity-quality optimisation problem produces the optimised chlorine injection schedules at the booster and treatment plant output nodes of poor quality and good suboptimal optimised pump and valve schedules over next 24 hours. The pump and valve schedules are applied to the DWDS and maintained during so called control time horizon e.g., 2 hours. The quality controls need to be improved and this is performed at the lower correction layer by the fast *feedback quality controller* that samples the chlorine residual concentrations as it is required by its decay dynamics e.g., with one minute sampling interval. The robustly feasible correction controller that employs the on line determined safety zones in order to guarantee feasibility of the quality controls under uncertain models and inputs was proposed in (Duzinkiewicz, *et al.*, 2005). An adaptive indirect modelled reference controller was proposed in (Polycarpou, *et al.*, 2001) for a prescribed chlorine level tracking. In this paper the optimising MPC is considered with full information feedback under different daily demands and uncertain demand predictions and modelling errors. The MPC genetic optimisation task solver is an enhanced genetic algorithm. The operational requirements on the tank volumes at the ends of the prediction horizons are suitably incorporated into the MPC optimisation tasks. The paper is organised as follows. In section 2 the MPC optimisation problem is formulated. The optimising MPC is derived in section 3 and applied to the case study DWDS in Gdynia, northern Poland in section 4. The conclusions in section 4 complete the paper.

2. FORMULATION OF MPC OPTIMISATION PROBLEM

2.1 Performance index

An electrical energy cost due to pumping constitutes the main component of an overall DWDS operational cost (Brdyś and Ulanicki, 1994). The control actions are piecewise constant and as the controller is meant to operate at the upper layer of the hierarchical control structure presented above the water quantity and quality sampling intervals are assumed the same and equal to T. Hence, for any variable x we shall denote $x(k)=x(kT)$ where k is integer valued variable denoting control and prediction time steps of the length T. Also it represents a discrete time variable. A prediction horizon is composed of H_p time steps. A prediction of value of variable z at instant lT that is the value $z(lT)$ performed at instant kT is denoted by $x(l|k)$. The control input sequence over the prediction horizon that is produced by solving MPC optimisation problem at instant $t_k = kT$ is denoted by $\{u(k+i\,|\,k)\}_{i\in\overline{0:H_p-1}}$. The piecewise control inputs that are applied to the system over $t\in(kT,(k+H_p)T]$, where t denotes a continuous time variable, are $u(t)=u(k+i\,|\,k), t\in((k+i)T,(k+i+1)T],\ i\in\overline{0:H_p-1}$.

The predicted operational cost at time instant kT over the prediction horizon can then be expressed as

$$F(k)=\sum_{p\in P}\ \sum_{i\in 1:H_p}\eta(k+i)E_{p,j}(k+i\,|\,k) \qquad (1)$$

where P – pump index set; $E_{p,j}(k+i|k)$ – predicted at time instant k electrical energy usage of by pump

j over time step $k+i$; $\eta(k+i)$ – unit energy cost over time step $k+i$.

The energy consumed by pump over time step depends on the head drop across the pump and flow through the pump. The pump control variables are: pump status *on* or *off* described by binary variable $x_i \in \{0,1\}$ and speed $s_i, i \in P$, in the case of variable speed pump. The pipe flows are also controlled by valves varying pipe resistances. The valve control variables are denoted $v_i, i \in V$ and $v_i = 1$ for fully open valve (the resistance unchanged) and $v_i = 0$ for a closed valve (the resistance is infinite and there is no flow through the pipe). Hence, the quantity control sequences generated at *k* over H_p are:

$$\left\{u^{qn}(k+i\,|\,k)\right\}_{i\in\overline{0:H_p-1}} = \{x_1(k+i\,|\,k), s_1(k+i\,|\,k), .., x_p(k+i\,|\,k), s_p(k+i\,|\,k), v_1(k+i\,|\,k), ..., v_v(k+i\,|\,k)\}_{i\in\overline{0:H_p-1}} \quad (2)$$

Applying these controls to DWDS at time kT over H_p forces the system to respond in terms of its outputs that are: pump flows $q_p = \{q_{p,i}\}_{i\in P}$, pipe flows $q = \{q_i\}_{i\in PP}$ and nodal heads (pressure + nodal elevation) including tank/reservoir nodes $\{h_i\}_{i\in TN\cup JN}$, where PP, TN, JN denote the pipe, tank/reservoir node and pipe junction node index sets, respectively. Clearly, the responding flows and heads depend on the tank levels at the instant kT and the water demands at the consumption nodes over the time step. The demands are assumed constant over the time steps so that the demand vector over time step *n* is $d(n) = \{d_i(n)\}_{i\in DN}$, where DN is demand node index set. As at $t = kT$ the system demand over the prediction horizon $\{d(k+i)\}_{i\in\overline{0:H_p-1}}$ is not exactly known its prediction $\{d(k+i\,|\,k)\}_{i\in\overline{0:H_p-1}}$ is used to determine under known from measurements tank heads $h_i(k)_{i\in TN}$ the predicted DWDS flow and head responses $\{q_p(k+i\,|\,k)\}_{i\in\overline{0:H_p-1}}$, $\{q(k+i\,|\,k)\}_{i\in\overline{0:H_p-1}}$, $\{\{h_j(k+i\,|\,k)\}_{j\in TN\cup JN}\}_{i\in\overline{1:H_p}}$ and to calculate the predicted energy costs $\{E_{p,j}(k+i\,|\,k)\}_{i\in\overline{0:H_p-1}}, j \in P$ in (1). Determining the predicted flows and heads from the control inputs and predicted disturbances (demands) is done by employing a hydraulic input – output model (Brdys and Ulanicki, 1994; Boulos *et al.*, 2004) and solving the model equations. This involves advanced methods for integration of ordinary differential equations mixed with a set of nonlinear algebraic equations. The EPANET simulation package (USEPA, 2001) is used in the paper.

2.2 Constraints

Quality output constraints. Main quality control objective is to maintain the chlorine concentrations c_i at the monitored nodes $i \in QMN$ that include the demand nodes and certain, critical for the quality, nodes over the network within the bounds specified for the nodes that is:

$$c_i^{min} \le c_i \le c_i^{max}, i \in QMN \quad (3)$$

The upper bounds reflect the consumer preferences and creation of dangerous for health products. The direct quality control inputs are free chlorine concentrations $c_i, i \in QCN$ at the quality control nodes where the prescribed chlorine concentration levels u_i^{dqu} are forced and maintained by simple PI local control loops injecting the required amount of chlorine into the nodes. We shall further neglect the PI loop dynamics and assume that $u_i^{dqu} = c_i$, $i \in QCN$. The injected chlorine travels then throughout the DWDS network to reach the monitoring nodes so that the required chlorine concentrations at these nodes remain within the prescribed bounds. The resulting transportation delays are time varying and depend on the flows and this is the one way interaction between the quality and control problems. Heavy pumping and storing water in the tanks during low electricity tariff period and delivering the stored water to the demand nodes by gravity transport when the energy cost is high is a principle of the optimising quality control. However, slow flows may produce large delays so that it may not be possible to meet the quality limits (3). Thus, the quantity control inputs are also indirect quality control inputs and there is a need for an integrated control.

The travelling chlorine is also subject to reactions. A complete chlorine concentration $c_{p,i}(l,t)$ dynamics at a distance $l \in [0, L_i]$ from the pipe $i \in PP$ input node at time instant t can be modelled for a turbulent flow as (Al-Omari and Chaudhry, 2001; Males, *et al.*, 1988; Rossman, *et al.*, 1994):

$$\frac{\partial c_{p,i}(l,t)}{\partial t} + v(l,t)\cdot\frac{\partial c_{p,i}(l,t)}{\partial l} - k_{p,i}\cdot c_{p,i}(l,t) = 0 \quad (4)$$

where $\upsilon_i(l,t)$ - linear pipe flow velocity and $k_{p,i}$ - bulk reaction rate coefficient,
under the prescribed initial and boundary conditions

$$c_{p,i}(l,0) = c_{p,i}^{l,0}(l),\; c_{p,i}(0,t) = c_{p,i}^{0,t}(t) \quad (5)$$

The boundary conditions in the interconnected network are calculated from chlorine mass balance at the pipe input node being a junction of several incoming pipes (Rossman, *et al.*, 1994). The chlorine dynamics in a tank/reservoir is modelled by applying the mass balance principle and the reaction kinetics as in (4) but it is lumped. Notice that the quality control inputs enter the model through the boundary conditions in (4) and the concentrations $c_{p,i}(L_i, 0)$ in the pipes delivering water to the tanks.

The above equations constitute the quality model linking the direct quality inputs and quantity control inputs (indirect quality control inputs)

u^{qn}, $u_i^{dqu} = c_i$, $i \in QCN$, respectively and disturbance inputs to the chlorine concentrations at the monitored nodes, the quality outputs. Applying the control inputs sequences, $\{\{u_i^{dqu}(k+i|k)\}_{i \in QCN}\}_{i \in \overline{0:H_p-1}}$, $\{u^{qn}(k+i\,|\,k)\}_{i \in \overline{0:H_p-1}}$ at time instant kT to DWDS under demand $\{d(k+i)\}_{i \in \overline{0:H_p-1}}$ and under the initial tank levels $h_i(k)_{i \in TN}$, initial chlorine concentrations in the tanks $c_i(k), i \in TN$, initial chlorine concentrations along pipes $c_{p,i}(l,k)$, $i \in PP$ and chlorine concentration at the input nodes of pipes $c_{p,i}(0,t)$, $t \in [kT,(k+H_p)T]$, $i \in PIN$ being the input pipes into the DWDS (DWDS quality boundary conditions) forces the system quality outputs to move over the period $t \in [kT,(k+H_p)T]$ along trajectories $c_i(t), i \in QMN$. As previously, at the time instant kT only the predicted quality output trajectories $c_i(t\,|\,kT), i \in QMN$ can be evaluated by employing an integrated model of the hydraulics and quality. In order to formulate the output constraints for the MPC optimisation problem we shall consider the output values only at the sampling instants. Hence, the MPC quality output constraints read (see (3)):

$$c_j^{min} \le c_j(k+i\,|\,k) \le c_j^{max};\ j \in QMN, i \in \overline{1:H_p} \tag{6}$$

The Epanet simulation package is very suitable to calculate the predicted outputs in (6).

Quantity output and control input constraint. The quantity output constraints are formulated based on the predicted output values at sampling time instants. The constraints are in the form of lower and upper bounds on certain flows and junction heads and on all tank heads in order to meet the tank capacity constraints. All the continuously valued control inputs are bounded above and bellow. The pump structure constraints are naturally bounded. As certain pump operating sequences are not allowed there is also a logic type of constraints.

Tank initial – final volume constraints. A standard operational requirement at DWDS is that the initial leading volume tank heads, thus volumes, are approximately restored at the end of the prediction horizon. This is incorporated into the MPC optimiser as the constraints:

$$h_i(k+H_p\,|\,k) = h_i(k) + \varepsilon, i \in LTN \tag{7}$$

where ε is a small number to be selected bearing in mind the tank level reachability and risk of not meeting the demand or violating tank hard capacity constraints.

The MPC optimisation problem. Given the demand and DWDS boundary quality predictions $\{d(k+i\,|\,k)\}_{i \in \overline{0:H_p-1}}$, $c_{p,i}(0,t\,|\,kT)$, $t \in [kT,(k+H_p)T]$, $i \in PIN$ respectively, and the initial quantity and quality conditions:

$h_i(k)$; $c_i(k), i \in TN$; $c_{p,i}(l,k), l \in [0,L_i]$, $i \in PP$.

The MPC optimiser solves at $t = kT$ the following optimisation problem:

$$Minimise\ \ F(k)$$

$$\textit{with respect to}: \{u^{qn}(k+i\,|\,k)\}_{i \in \overline{0:H_p-1}} \tag{8}$$

$$\{\{u_j^{dqu}(k+i|k)\}_{j \in QCN}\}_{i \in \overline{0:H_p-1}}$$

subject to: DWDS control input and predicted output constraints, initial-final volume constraint (7).

3. OPTIMISING MODEL PREDICTIVE CONTROLLER

The chlorine distributions along pipes, tank heads and chlorine concentrations in tanks are DWDS state variables. Let us denote the state vector at time instant t as

$$X(t) \triangleq \{h_i(t), c_i(t), i \in TN; c_{p,i}(l,t), l \in [0,L_i], i \in PP\}$$

As presented before, knowing $X(kT)$ and the DWDS inputs/input predictions over $t \in [kT,(k+H_p)T]$ the DWDS output responses will uniquely be forced and their predictions over the prediction horizon would be uniquely calculated by employing the DWDS simulator. For the optimising control purposes the prediction horizon needs to be properly selected in order to capture the tank and chlorine dynamics. The latter is much slower than the former one. The optimising MPC generates at the sampling time instant kT the control input values $u(k)$ that are then applied to DWDS and maintained till next sampling instant $(k+1)T$. Its operation at kT is as follows:

Step 1: The DWDS state $X(kT)$ is measured or estimated and the demand and DWDS quality boundary conditions are predicted.
Step 2: The MPC optimisation problem (8) is solved.
Step 3: Only the first optimised control action is used and applied to DWDS that is: $u(k) = u^{opt}(k\,|\,k)$.
Step 4: Set $k := k+1$ and return to ***Step 1***.

The MPC feedback from DWDS consists in updating the predictions and replacing model state values by the real measured or estimated from the measurements in the DWDS. This is to overcome the model-reality differences. Due to very practical reasons the functions $c_{p,i}(l,t), l \in [0,L_i]$ are estimated by measuring/estimating the chlorine concentrations at the pipe input an output nodes and then applying linear approximation along the pipe. The recently developed fast estimator of chlorine concentration at certain point along the pipe (Langowski and Brdys, 2006) can be used in order to improve the approximation accuracy. The optimisation problem is very complex as it is nonlinear, nonconvex in continuous variables and mixed integer. An advanced genetic type of search coupled with EPANET simulator of DWDS was applied in this paper with the specialised for the problem at hand genetic operators. The GA details are not reported here due to space limitations.

4. APPLICATION TO GDYNIA DWDS CASE STUDY

The skeleton of DWDS in Gdynia is shown in Fig. 1. There are 3 underground water sources, 4 tanks, 3 reservoirs of a total capacity of $12 \cdot 10^3 m^3$, 10 variable speed pumps, 4 valves, 5 quality booster stations, 148 pipes and 134 junction nodes (87 demand nodes) in the system. The prediction horizon and sampling interval are $T = 0.5$ hour and $H_p = 24$, respectively. The demand prediction is provided with accuracy $\pm 5\%$ during the first five hours that decreases up to 20% over subsequent hours of the prediction horizon. During 6am-12am and 3pm-9pm, $\eta = 0.12$ \$/kWh and $\eta = 0.06$ \$/kWh during 10pm-5am. There are 5 quality control nodes with the booster stations and 129 monitored nodes. The MPC controller was simulated over 72 hrs time period starting at 12pm. The relative speed of the pumps Sieradzka, Kolibki and Reda generated by the controller are illustrated in Fig. 2 and compared against the speeds seen in the system. The MPC controller takes full advantage of the existing reservoir capacities in order to exploit different electricity tariff periods. As the result the pump speed trajectories vary more than it is seen during the current system operation. Better usage of the reservoir capacity by the MPC can be also noticed by inspecting the results illustrated in Fig. 3. Indeed, the controller is more aggressive in entering the lower reservoir levels. Ability to maintain feasibility of the reservoir operation over long control period due to the condition (7) is crucial in achieving such effect. The energy cost due to pumping over the simulation period of 72hrs was about 13% smaller than the current cost. The disinfectant concentration at the node 1 is shown in Fig. 4. It can be seen that the MPC controller has managed to keep the concentration low within the allowed limits.

5. CONCLUSIONS AND FUTURE WORK

The paper has derived the optimising model predictive controller for a feedback control of an integrated quantity and quality in DWDS. The controller has been validated by simulation based on real data records from a case study DWDS in Gdynia to produce promising results. For the first time the realistic simulation results of the feedback controller but not only the open loop optimiser have been produced.

Fig.1. The skeleton of DWDS in Gdynia

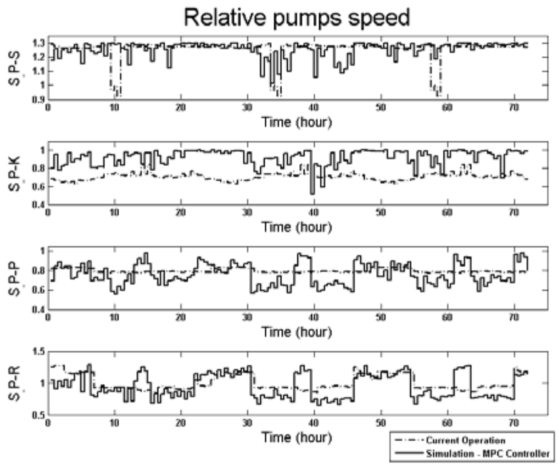

Fig.2. Relative speeds of the pumps Sieradzka, Kolibki, P1 and Reda

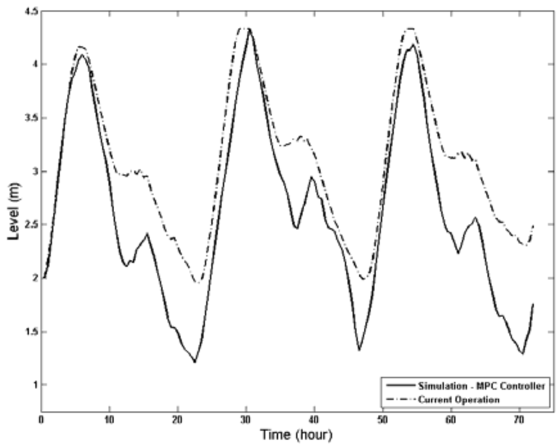

Fig.3. Level trajectory of Witomino reservoir

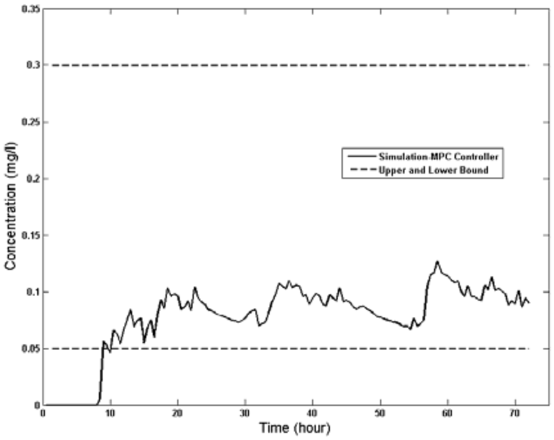

Fig.4. Chlorine concentration at node 1

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