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GENERALIZED PREDICTIVE CONTROL BASED IN MULTIVARIABLE BILINEAR MULTI-MODEL

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Abstract: This paper shows a new approach of Model Predictive Control (MPC). A multivariable bilinear multi-model is presented. A set of local bilinear models is identified and the proposed algorithm implements the timestep quasilinearization in trajectory. A metric based in norms is presented to measure the distance from the current operation point to a designed controller in other operation point. Application results are showed in a case study.

Keywords: Predictive control; Nonlinear systems; MIMO; Model-based control; Metrics.

1. INTRODUCTION

Linear controllers (like MPC based in linear models) can exhibit some performance problems when applied in processes with strong nonlinearities, since linear models can not represent the process behavior in whole operating range as showed in (Aslan, *et al.*, 2004).

Several algorithms and techniques have been proposed by researches in order to improve control systems with strong nonlinearities. This approach considers one trajectory with many equilibrium points. For each equilibrium point, a bilinear model is obtained and a quasilinear GPC (Generalized Predictive Controller) is designed.

The multi-model idea has been proposed for many researches in order to solve control system problems for strongly nonlinear plants operating in a large range. In (Foss, *et al.*, 1995), a set of nonlinear state-space models is obtained, and an interpolation function is defined to build a global nonlinear model. In that case, the optimization problem is solved by a nonlinear programming algorithm. (Foss, *et al.*, 1995) applied this described method in a batch fermentation process, that is kind of process that exhibit large variations in the operating conditions during a batch.

In (Azimzadeh, *et al.*, 1998) a multi-model based approach is used with approximated local linear state-space state. In that approach, a local model validity function based in statistical datas from the process (means and standard deviations) is presented.

A similar method based in gap metrics is presented in (Aslan, *et al.*, 2004) where authors apply a closed loop gap metric to measure the distance from the current operation point to a tabled operation point (where a PI controller is designed).

The basic idea of multivariable multi-model based control is to identify local models and to build a global controller with these models (by using a proposed metric).

In this work, for each local model, a 2-norm is calculated like weighting factor to generate the suitable control signal in that operation point.

2. THE METRIC

In this paper, a different metric is proposed in order to measure the distance from a designed multivariable quasilinear predictive controller in a known operation point to the current operation point. In multivariable case, in a process with p-inputs and q-outputs, the output is $y \in R^q$ and the input is $u \in R^p$. In a known trajectory of process output, the distance from the first operation point to the last operation point is given by:

$$d_{1,NOP} = \left\| y_{NOP} - y_1 \right\|_2 \tag{1}$$

where NOP is the number of operation points. For each operation point, a controller is designed.

To measure the distance from the current operation point to the operation point of i^{th} designed controller, we can use the expression:

$$\delta_i = \frac{d_{1,NOP}}{\left\| y_{COP} - y_i \right\|_2}; \quad i = 1, \cdots, NOP \tag{2}$$

where COP is the current operation point in trajectory. The weighting factor for the i^{th} designed controller is given by:

$$w_i = \frac{\delta_i}{\sum_{j=1}^{NOP} \delta_j}; \quad i = 1, \cdots, NOP \tag{3}$$

This metric has the property $\sum_{i=1}^{NOP} w_i = 1$ defined.

In the metric proposed by (Aslan, *et al*., 2004) the metric is based in H_∞ norm. For this reason, although this metric guarantee the closed loop stability, its calculation is much more complex than this proposed metric and in each instant, a transfer function for that operating regime must be available. In the metric proposed by (Azimzadeh, *et al.*, 1998) the metric is based in Gaussian local model validity function. No comparison has been done between this approach in relation of other metrics.

3. QUASILINEAR MULTIVARIABLE GENERALIZED PREDICTIVE CONTROL

The designed controllers are based in quasilinear generalized predictive control (QGPC). Theses controllers are based in multivariable bilinear NARIMAX (Non Linear, Auto-Regressive, Moving Average, with exogenous input) models.

The basic idea of QGPC algorithm is calculate a control effort sequence, based in the minimization of a multi-step objective function, in a defined prediction horizon.

3.1 Multivariable Model

The multivariable bilinear NARIMAX model with p-inputs and q-outputs is given by:

$$\begin{aligned} &A(q^{-1})\Delta_q(q^{-1})y(k) = B(q^{-1})\Delta_p(q^{-1})u(k-1) + \\ &D_e(q^{-1})D[u(k-1)]D_d(q^{-1})\Delta_q(q^{-1})y(k-1) + \\ &C(q^{-1})e(k) \end{aligned} \tag{4}$$

where $y(k) \in R^q$ is the process output vector, $u(k) \in R^p$ is the process input vector and $e(k) \in R^q$ is the gaussian white noise with zero mean and covariance $diag(\sigma^2)$. The matrices $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are polynomials matrices in shift operator q^{-1} and are defined by:

$$A(q^{-1}) = I_{q\times q} + A_1 q^{-1} + \cdots + A_{na} q^{-na} \tag{5}$$

$$B(q^{-1}) = B_0 + B_1 q^{-1} + \cdots + B_{nb} q^{-nb} \tag{6}$$

$$C(q^{-1}) = I_{p\times p} + C_1 q^{-1} + \cdots + C_{nc} q^{-nc} \tag{7}$$

$$D_d(q^{-1}) = D_{d,0} + D_{d,1} q^{-1} + \cdots + D_{d,nd_d} q^{-nd_d} \tag{8}$$

$$D_e(q^{-1}) = D_{e,0} + D_{e,1} q^{-1} + \cdots + D_{e,nd_e} q^{-nd_e} \tag{9}$$

where $A(q^{-1}) \in R^{q\times q}$, $B(q^{-1}) \in R^{q\times p}$, $C(q^{-1}) \in R^{q\times q}$, $D_e(q^{-1}) \in R^{q\times p}$ and $D_d(q^{-1}) \in R^{p\times q}$ and the matrix $D[u(k-1)]$ is defined as:

$$D[u(k-1)] = \begin{bmatrix} u_1(k-1) & 0 & \cdots & 0 \\ 0 & u_2(k-1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & u_p(k-1) \end{bmatrix} \tag{10}$$

This nonlinear model is quasilinearized to be used in QGPC. The multivariable quasilinear model must be obtained by rewriting the expression (4) of the following form:

$$\begin{aligned} &A(q^{-1},u)\Delta_q(q^{-1})y(k) = B(q^{-1})\Delta_p(q^{-1})u(k-1) + \\ &+ C(q^{-1})e(k) \end{aligned} \tag{11}$$

where:

$$A(q^{-1},u) = A(q^{-1}) + D_e(q^{-1})D[u(k-1)]D_d(q^{-1}) \tag{12}$$

The polynomial matrix $A(q^{-1},u)$ is calculated considering $D[u(k-1)]$ as constant in prediction horizon. The polynomial matrix $A(q^{-1},u)$ is considered diagonal in this paper.

3.2 The Predictor

The output prediction *i-step* ahead may be calculated making:

$$\begin{aligned}&\widetilde{A}(q^{-1},u)y(k+i)=\\&B(q^{-1})\Delta_p(q^{-1})u(k+i-1)+C(q^{-1})e(k+i)\end{aligned} \tag{13}$$

where $\widetilde{A}(q^{-1},u)=A(q^{-1},u)\Delta_q(q^{-1})$.

In this case, the polynomial matrix $C(q^{-1})=I_{p\times p}$ is uncorrelated (white noise). Consider the following Diophantine equation:

$$I_{p\times p}=E_i(q^{-1},u)\widetilde{A}(q^{-1},u)+q^{-i}F_i(q^{-1},u) \tag{14}$$

where:

$$E_i(q^{-1},u)=E_{i,o}(u)+E_{i,1}(u)q^{-1}+\cdots+E_{i,i-1}(u)q^{-(i-1)} \tag{15}$$

$$F_i(q^{-1},u)=F_{i,o}(u)+F_{i,1}(u)q^{-1}+\cdots+F_{i,na}(u)q^{-na} \tag{16}$$

Pre-multiplying (13), with $C(q^{-1})=I_{p\times p}$, for $E_i(q^{-1},u)$ we obtain:

$$\begin{aligned}&E_i(q^{-1},u)\widetilde{A}(q^{-1},u)y(k+i)=\\&E_i(q^{-1},u)B(q^{-1})\Delta_p(q^{-1})u(k+i-1)+\\&E_i(q^{-1},u)C(q^{-1})e(k+i)\end{aligned} \tag{17}$$

Rewriting (14) of the following form:

$$E_i(q^{-1},u)\widetilde{A}(q^{-1},u)=I_{p\times p}-q^{-i}F_i(q^{-1},u) \tag{18}$$

Substituting (18) in (17) we obtain:

$$\begin{aligned}&y(k+i)=F_i(q^{-1},u)y(k)+\\&E_i(q^{-1},u)B(q^{-1})\Delta_p(q^{-1})u(k+i-1)+\\&E_i(q^{-1},u)C(q^{-1})e(k+i)\end{aligned} \tag{19}$$

As the degree of $E_i(q^{-1},u)$ is $j-1$, then the sub-optimal prediction of $y(k+i)$ is:

$$\begin{aligned}&\hat{y}(k+i)=F_i(q^{-1},u)y(k)+\\&E_i(q^{-1},u)B(q^{-1})\Delta_p(q^{-1})u(k+i-1)\end{aligned} \tag{20}$$

Make:

$$E_i(q^{-1},u)B(q^{-1})=H_i(q^{-1},u)+q^{-i}H_{ipa}(q^{-1},u) \tag{21}$$

As the degree of $H_i(q^{-1},u)$ is less than $j-1$, the predictor may be written as:

$$\begin{aligned}&\hat{y}(k+i)=F_i(q^{-1},u)y(k)+\\&H_{ipa}(q^{-1},u)\Delta_p(q^{-1})u(k-1)+\\&H_i(q^{-1},u)\Delta_p(q^{-1})u(k+i-1)\end{aligned} \tag{22}$$

The last term of (22) considers the future inputs (forced response) and the two first terms consider only past inputs (free response).

Make:

$$\hat{y}(k+i)=H_i(q^{-1},u)\Delta_p(q^{-1})u(k+i-1)+Y_{li} \tag{23}$$

where:

$$Y_{li}=F_i(q^{-1},u)y(k)+H_{ipa}(q^{-1},u)\Delta_p(q^{-1})u(k-1) \tag{24}$$

3.3 The Objective Function

The objective function is given by:

$$J=\sum_{i=N_1}^{NY}\left\|r(k+i)-\hat{y}(k+i)\right\|_R^2+\sum_{i=1}^{NU}\left\|\Delta u(k+i-1)\right\|_Q^2 \tag{25}$$

Where N_1 is minimum prediction horizon, N_2 is prediction horizon, NU is the control horizon, R and Q are weighting matrices of error signal and control effort, respectively, $\hat{y}(k+i)$ is the sub-optimum i-step ahead predicted output, $r(k+i)$ is the future reference trajectory.

3.4 The Control Law

The control effort is obtained, without constraints, by the minimization of the objective function. This minimization is obtained by the calculation of its gradient (making it equals zero), of the following form:

$$\frac{\partial J}{\partial u}=0 \tag{26}$$

Consider the predictions set:

$$y_{N_{1y}}=H_{N_{1yu}}u_{NU}+y_{lN_{1y}} \tag{27}$$

where:

$$y_{N_{1y}}=\begin{bmatrix}\hat{y}(k+N_1) & \hat{y}(k+N_1+1) & \cdots & \hat{y}(k+NY)\end{bmatrix}^T \tag{28}$$

$$H_{N_{1yu}}=\begin{bmatrix}H_{N_1-1} & H_{N_1-2} & \cdots & H_{N_1-NU}\\ H_{N_1} & H_{N_1-1} & \cdots & H_{N_1+1-NU}\\ \vdots & \vdots & \ddots & \vdots\\ H_{NY-1} & H_{NY-2} & \cdots & H_{NY-NU}\end{bmatrix} \tag{29}$$

$$u_{NU} = \begin{bmatrix} \Delta_p(q^{-1})u(k) \\ \Delta_p(q^{-1})u(k+1) \\ \vdots \\ \Delta_p(q^{-1})u(k+NU-1) \end{bmatrix} \quad (30)$$

$$y_{IN_{1y}} = \begin{bmatrix} Y_{IN_1} \\ Y_{IN_1+1} \\ \vdots \\ Y_{INY} \end{bmatrix} \quad (31)$$

The objective function (25) may be rewritten of the following form:

$$J = (H_{N_{1yu}}u_{NU} + y_{IN_{1y}})^T \overline{R}(H_{N_{1yu}}u_{NU} + y_{IN_{1y}}) + u_{NU}^T \overline{Q} u_{NU} \quad (32)$$

Where $\overline{R} = diag[R, \cdots, R]$ and $\overline{Q} = diag[Q, \cdots, Q]$.

The minimization of (32) produces the following control law:

$$u = (H_{N_{1yu}}^T H_{N_{1yu}} + \overline{Q})^{-1} H_{N_{1yu}}^T \overline{R}(r - y_{IN_{1y}}) \quad (33)$$

Because of the receding control horizon, only the first p rows of (33) are computed.

3.5 The Control Law Considering the Presented Metric

For each operation point, a quasilinear controller is designed. So, there is $p \times NOP$ control efforts computed. The control effort sent to the process is a weighting combination of control efforts calculated for each operation point:

$$u_i(k) = \sum_{j=1}^{NOP} w_j u_{i,j}(k) \quad (34)$$

where $i = 1, \cdots, p$.

4. CASE STUDY: DEBUTANIZER DISTILLATION COLUMN

4.1 Description of Distillation Column

Debutaziner distillation column is usually used to remove the light components from the gasoline stream to produce Liquefied Petroleum Gas (LPG) as showed in (Fontes, *et al.*, 2006).

The most common control strategy is to manipulate the reflux flow rate and the temperature in column's bottom and, to control the concentrations of any product in *butanes* stream and in C5+ stream as showed in (Almeida, *et al.*, 2000). The chosen process variables are: concentration of i-pentane in butanes stream (y_1) and concentration of i-butene in C5+ stream (y_2). The studied column is simulated in Hysys software and is showed in Figure 1.

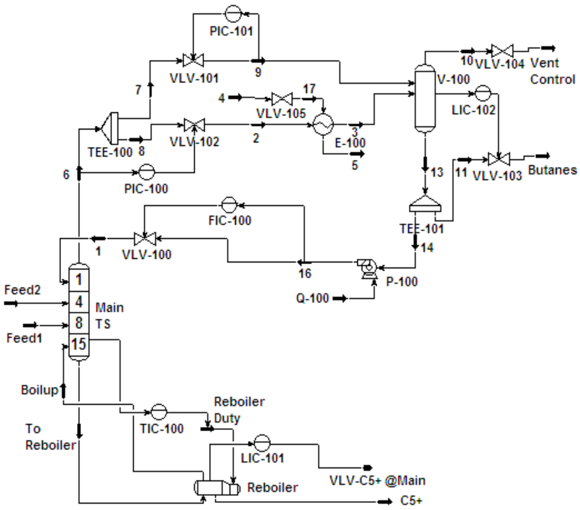

Fig. 1. Distillation Column simulated in Hysys Software.

The reflux flow rate (u_1) is manipulated through the FIC-100 controller and the temperature of column's bottom (u_2) is manipulated through the TIC-100 controller. The reflux flow rate is measured in m^3/h and the temperature of column's bottom is measured in ^{o}C.

4.2 Chosen Operation Points

In this case study, three operation points were chosen, as showed in Table 1. The identified bilinear models were obtained using the multivariable recursive least squares algorithm and the model's structure has been chosen by using the Akaike criterion. In all points, the chosen sample rate is 4 minutes.

For this article, only monotonic trajectories are being considered. The trajectory of y_1 is monotonically increasing and the trajectory of y_2 is monotonically decreasing.

Table 1 Three operarion points chosen in distillation column.

Operation Point	Input	Output (Mass Fractions)
1	u_1 = 40 m^3/h u_2 = 147 ^{o}C	y_1 = 0.014413 y_2 = 0.001339
2	u_1 = 37 m^3/h u_2 = 147.5 ^{o}C	y_1 = 0.017581 y_2 = 0.001161
3	u_1 = 34 m^3/h u_2 = 148 ^{o}C	y_1 = 0.021994 y_2 = 0.001004

This distillation column has nonlinear behaviour (like anti-symmetrical behavior) in chosen trajectory. A single-model based controller presents a poor performance when applied in all trajectory. The next subsection will show the results comparing the single-model and multi-model approach. The subsection 4.4 shows the comparison results, by using some indices in order to quantitatively assess the performance of the systems.

4.3 Results

The proposed quasilinear multi-model is compared with quasilinear single-model (using the 3rd bilinear model). Figures 2 and 3 show the output comparison.

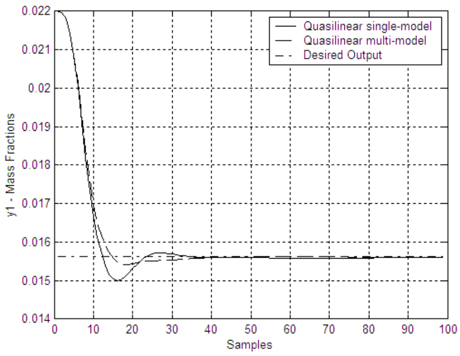

Fig. 2. Concentration of i-pentane (in mass fraction) in butanes stream.

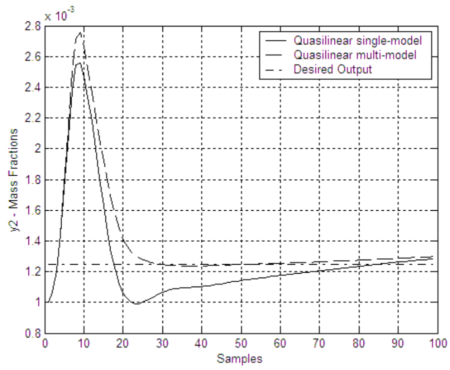

Fig. 3. Concentration of i-butene (in mass fraction) in C5+ stream.

The process, showed in Figures 3 and 4, is in 3rd operation point and the controllers will lead the process until close to the 1st operation point. The comparisons of control efforts are showed in Figures 4 and 5.

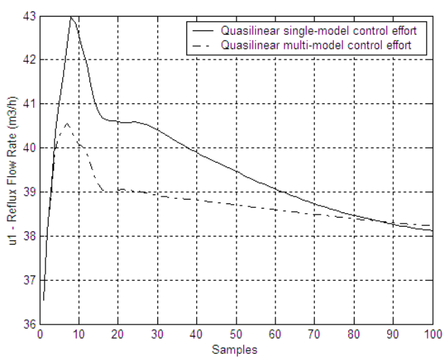

Fig. 4. Control effort (reflux flow rate in m^3/h).

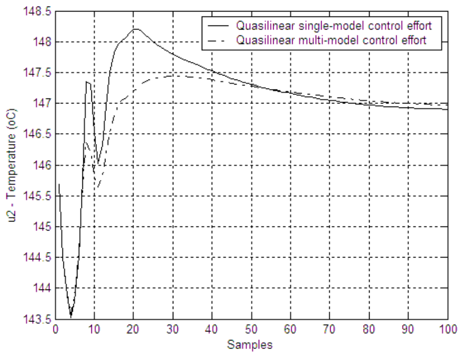

Fig. 5. Control effort (temperature in column's bottom in ^{o}C).

Figure 6 shows the behaviour of weighting factors in time. In beginning of simulation, the 3rd weighting factor is close to 1 and will be decreased (because the process is not close to this point anymore). The first and the second weighting factor are close to zero in beginning (because the process is close to the third operation point).

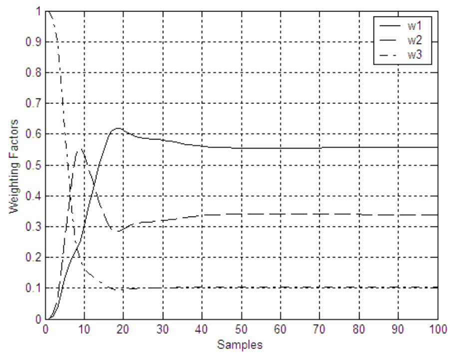

Fig. 6. Behaviour of weighting factors in time and in trajectory consequently.

4.4 Comparison of Results

Observing the Figures 2, 3, 4 and 5, we can observe that quasilinear multi-model presents better performance in relation to the product quality and better performance in control effort (less control effort). In order to quantitatively assess the performance of multi-model quasilinear GPC, some indices like showed in (Goodhart, *et al.*, 1994) are calculated. Theses indices may be extended to multivariable case, of the following form:

$$\varepsilon_{1,i} = \sum \left| u_i(k) \right| / N \tag{35}$$

where $i = 1, \cdots, p$ and N is the amount of control effort applied in the process to achieve the desired response. The index showed in (35) is the account of total control effort to achieve a given response. The variance of controlled actuators is:

$$\varepsilon_{2,i} = \sum (u_i(k) - \varepsilon_{1,i})^2 / N \tag{36}$$

The deviation of the process of integral of absolute error (IAE) is:

$$\varepsilon_{3,j} = \sum \left| r_j(k) - y_j \right| / N \tag{37}$$

where $j = 1, \cdots, q$.

The overall measure of effectiveness is defined as:

$$\varepsilon_j = \sum_{i=1}^{p} (\alpha_i \varepsilon_{1,i} + \beta_i \varepsilon_{2,i}) + \rho_j \varepsilon_{3,j} \tag{38}$$

where $j = 1, \cdots, q$. The factors α_i, β_i and ρ_j are weightings chosen to reflect the actual financial cost of energy usage, actuator wear and product quality, respectively. In this case, we consider $\alpha_i = 0.1$, $\beta_i = 0.15$ and $\rho_j = 0.5$ because we have established as priority the product quality. The values of Equation (37) are multiplied by 10^6 in order to keep the same order of magnitude of (36) and (35).
The Table 2 shows the comparison between quasilinear single-model and quasiliner multi-model.

Table 2 Comparison of Performance indices between Quasilinear single-model and Quasilinear multi-model

I/O	**Model**	ε_1	ε_2	ε_3	ε
1	Single	38.41	1.41	251.61	144.64
2	Multi	38.38	0.32	248.41	142.83
1	Single	147.01	0.37	117.16	77.42
2	Multi	146.94	0.29	103.48	70.36

Table 2 shows the performance of quasilinear multi-model approach in terms of less energy usage, less actuator wear and better product quality in relation to quasilinear single-model performance.

5. CONCLUSIONS

This paper has presented a new approach of quasilinear predictive control by using a defined discrete quasilinear multi-model. The case with constraints treatment was not analyzed. Simulation results have shown the best performance (quantitative and qualitative) of quasilinear multi-model approach in relation of quasilinear single-model. The analysis robust stability was not studied. The next step of this research consists of to analyze the robustness and stability of this approach.

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