8th International IFAC Symposium on
Dynamics and Control of Process Systems
Preprints Vol.3, June 6-8, 2007, Cancún, Mexico

OPTIMAL SENSOR NETWORK DESIGN FOR MULTIRATE SYSTEMS

Sachin C. Kadu[*#], Mani Bhushan[*1], Ravindra Gudi[*]

[*] *Department of Chemical Engineering,IIT,Powai,Mumbai 400076,India*
[#] *Reactor Projects Division, B.A.R.C.,Mumbai 400085,India*

Abstract: A methodology for determining optimal sensor network design for multirate systems is presented. This methodology is based on generating trade-off (Pareto optimal) solutions between the quality of state estimation and the total measurement cost associated with the sensor network. Variances associated with the states estimated using a Kalman filter in a multirate setting are used to calculate the state estimation quality. The resulting multiobjective optimization problem is solved using the well known non-dominated sorting genetic algorithm-II. The resulting solutions can be then further analyzed by the process designer for determining the optimal sensor network. The methodology is demonstrated using simulations on the benchmark quadruple tank set up (Johansson, 2000).

Keywords: Multiobjective optimization, Genetic algorithms, Multirate systems, Sensor, Network, Kalman filter

1. INTRODUCTION

In the process industry, measurements of different process variables are often obtained at different sampling rates. Such multirate measurement scenarios are encountered frequently in chemical and bio-chemical processes, for e.g. secondary variables (such as temperature, pressure) are measured at a higher frequency while some quality variables (such as molecular weight) are measured relatively slowly. In such situations, for monitoring and control, it is possible to generate estimates of the quality variables and secondary variables at frequent rates using (inferential) observer-based schemes that rely on both fast and slow measurements, as well as the process model. The quality of estimates (in terms of their variance) generally increases with the number of sensors and their sampling frequencies, but this also increases the associated measurement cost. Hence, there is a trade-off between the quality of estimates that can be obtained and the measurement cost. In sensor network design literature this trade-off is analyzed through the formulation of a Pareto optimization problem.

State space models are widely used in process control and estimation literature for representing process dynamics. Kalman filtering techniques are preferred techniques to infer process variables (states) using incomplete or partial measurements and a first-principles based state space model. The performance, as characterized by the uncertainty in the estimated states in Kalman filtering strategy, can be improved by formally incorporating the infrequently available measurements. The uncertainty in the estimated states depends on the total number of sensors (and their noise characteristics), their location (in terms of which variables they sense) and their sampling frequencies.

In literature, recent research in the area of sensor network design based on state space models includes the work of Muske and Georgakis (2003) and Musulin et al. (2005). Earlier approaches for optimal sensor location and their selection are reviewed briefly by Muske and Georgakis (2003). The trade-off between measurement cost and the process information in the optimal determination of measurement systems for chemical processes is considered by Muske and Georgakis (2003). The inverse norm of a weighted steady state prediction error covariance matrix (SPECM) was used in their work as a scalar measure of the process information associated with a given measurement system. The corresponding measurement cost is determined by summation of the installation and operating cost for

[1] Author for correspondence Email:mbhushan@iitb.ac.in; Tel #: +91-22-2576-7214; Fax #: +91-22- 2572-6895; *A fuller version of this manuscript is under preparation for submission to J. of Process Control, April 2007.*

each sensor. The same idea is extended by considering time varying state estimation error covariance matrix (SEECM) to compute scalar measure of process information by Musulin et al. (2005).

All of the above work deals with sensor network design for single rate processes. However in a typical chemical process, variables are measured at different rates. There are several incentives to include the slowly sampled and/or irregularly available measurements, in the overall estimation scheme (Gudi et al. 1995). While these measurements have been shown to enhance observability of the system as well as reduce variance of the estimates, they can also influence the overall sampling requirements of the frequently measured variables. Considering that even the frequency of the regular measurements is influenced by cost considerations, it is important to include this frequency as a decision variable in a generic multirate framework that focuses on cost as well as accuracy of the estimates. Such a multirate treatment of the sensor network design problem is quite relevant in the context of chemical and biochemical systems and has not received any attention in the literature.

In this work, the problem of sensor network design has been formulated in a generic, multirate framework that considers both location of the sensors and frequencies of sampling as decision variables to assess the trade-off between quality of the estimates and the overall cost of the measurement system. As in existing literature reported earlier, we analyze the trade-off by representing it as a Pareto optimization problem, which is solved using the popular non-dominated sorting genetic algorithm (NSGA-II) (Deb et al. 2002). We demonstrate the suitability of the proposed approach by implementing it on the benchmark multivariable quadruple tank problem (Johansson, 2000).

The organization of this article is as follows. A brief description of the sensor network design approach followed in this work is presented in section 2. The case study results are discussed in section 3 and concluding remarks are made in section 4.

2. DESIGN APPROACH

In present work, the problem of sensor network design includes two objective functions viz. maximizing the quality of estimates and minimizing the measurement cost subject to a constraint of system detectability. The objective function values characterizing the quality of estimates are based on a Kalman filter implemented for different rates of sampling. We next briefly review the basic Kalman filter and present its extension to the multirate estimation scenario.

2.1. *Kalman filtering algorithm*

A Kalman filter is an optimal recursive data processing algorithm. Consider the problem related to the optimal estimation of states of a linear discrete time invariant (LTI) system represented by:

$$x_{k+1} = \varphi x_k + \Gamma u_k + w_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where k represents the sampling instant, $x_k \in R^n$ represents the vector of states of interest for measurement and control, $u_k \in R^m$ is the vector of manipulated input variables, $y_k \in R^r$ is the vector of measured variables, $\varphi \in R^{nxn}$ is the state transition matrix, $\Gamma \in R^{nxm}$ is the control gain matrix and $C \in R^{rxn}$ is the measurement matrix. The process noise w_k represents the unmodelled process dynamics in equation (1) and is assumed to be Gaussian with zero mean and covariance matrix Q. The measurements are corrupted by noise v_k which is also assumed to be Gaussian with zero mean and covariance matrix R. The error covariance matrices associated with predicted state vector $\hat{x}_k(-)$ and estimated state vector $\hat{x}_k(+)$ are given by (Gelb, 1988):

$$P_k(-) = \varphi P_{k-1}(+)\varphi^T + Q \tag{3}$$

$$P_k(+) = [\, I - K_k C \,] P_k(-) \tag{4}$$

where I is the identity matrix of suitable size, $P_k(-)$ and $P_k(+)$ are called as SPECM and SEECM respectively. The (–) and (+) refers to the time just before and immediately after measurement respectively. The Kalman gain matrix K is given by

$$K_k = P_k(-)C^T [CP_k(-)C^T + R\,]^{-1} \tag{5}$$

Equation (4) can be interpreted as a step towards minimization of the objective function of estimation error suitably weighed by the Kalman gain. The weights in Kalman gain matrix are generated by use of model, measurements and measurements noise. To implement the algorithm, the following initial conditions should be provided:

$$E\left[x_0\right] = \hat{x}_0 \;;\; E\left[\left(x_0 - \hat{x}_0\right)\left(x_0 - \hat{x}_0\right)^T\right] = P_0 \tag{6}$$

However, the Kalman filtering algorithm presented above assumes that measurements are available at every sampling instant and as such it can be used only for single rate systems. But in actual practice, a multirate scenario could be possible since all measurements may not be available at all time instants. Further, as mentioned earlier, there appear to be sufficient incentives to actually explore such a scenario of multirate measurements from the perspective of trade-off between cost and quality of estimates. To account for different measurements at different time instants we propose to suitably modify the above Kalman filter algorithm. Since at each time instant the size of r (number of measurements available) changes, the sizes of matrices C and R vary. With the following two modifications, above Kalman filtering algorithm can be used for multirate scenario:

Generation of appropriate C & R matrices

Let s sensors, viz. $s_1, s_2,\ldots, s_s$ be available for 1 to s measurements respectively and further let us assume that i^{th} sensor can be used for measurements with a sampling time $f^i_t \in D_i$, where $D_i=\{ f^i_1, f^i_2,\ldots f^i_{Ni}\}$ is the set of allowed sampling times for sensor i. Let k=1 to T be the time interval used for Kalman filter to generate time varying SEECM. Under this situation, if the remainder of $k/f^i_t=0$, then measurement from i^{th} sensor is available at the k^{th} instant. Hence the corresponding i^{th} row in C and R matrices are retained. On other hand, if remainder of $k/ f^i_t \neq 0$, then i^{th} sensor measurement is not available at k^{th} instant and the i^{th} row in C and R matrices are deleted. Once this is done for all sensors, the resulting C and R matrices are used in Kalman filtering calculations. Hence the Kalman filtering algorithm discussed earlier can now be used for multirate processes with appropriate C and R matrices at different sampling instants to calculate SEECM at each sampling instant. It is to be noted that in this multirate Kalman filtering procedure, use of the updated values of C and R at each time instant provides a link between sensor availability, accuracy and state estimation quality through equations (4) and (5).

Time interval for simulation (T)

For the multirate sampling system, due to the periodic availability of measurements, a periodic output relation is obtained (Lennartson,1988) with period z where z is the least common multiple (LCM) of the different sampling times, i.e. $z=LCM\{f^1_t, f^2_t,\ldots f^s_t\}$. A sensor network with large sampling time for each sensor will have large value of z. The simulation interval T for multirate Kalman filtering algorithm should be larger than z to allow the SEECM to attain steady state.

2.2. *Scalar measure for quality of estimate*

The model based multirate Kalman filtering strategy as described in section 2.1 is used to generate optimally filtered estimates of the states from the measurements which arrive at different sampling times. For a particular sensor network, the multirate Kalman filter provides the knowledge of SEECM, which varies with each time instant. The trace of SEECM gives the sum of variances of all the estimated states, which in turn gives an idea about the quality of estimation. As in earlier work (Musulin et al. 2005), the sum of trace of SEECM over the selected simulation time interval T is considered in our work as scalar measure of process information i.e. quality of estimation.

2.3. *Scalar measure of cost*

In general, a scalar measure of cost can be computed as the sum of installation and operating cost. Generally, the cost of sensor varies inversely with its measurement accuracy. Hence, accurate sensors measured frequently will lead to good quality estimates but will incur high costs, and vice-versa low cost sensors may not lead to good quality estimates. Optimization problem to generate this tradeoff between cost and estimation quality is discussed next.

2.4. *Sensor network optimization problem*

The sensor network optimization problem is multiobjective in nature. It can be formulated as follows:

$$\begin{aligned} &\min\{F_1(q),F_2(q)\} \qquad (7)\\ &\text{s.t. system detectability,}\\ &\quad q\in D \end{aligned}$$

where q is the decision variable vector i.e. time intervals at which different measurements can be taken. D is feasible space as characterized by set of $\{D_1,D_2,\ldots D_s\}$. It is to be noted that since D_i is a set of integers, the optimization problem (7) is non-convex. Further D_i can also be a union of disjoint subsets, for e.g. $D_i = \{[0,1,\ldots,5]\cup[15,16,\ldots,20]\}$sec for a given i. It is also to be noted that if the sampling time is zero for a particular sensor, then that sensor is not part of the measurement scheme. Hence, in our proposed formulation for sensor network design for multirate systems, there is no need to introduce artificial variables to indicate absence or presence of a sensor. The objective functions $F_1(q)$ and $F_2(q)$ corresponding to quality of estimation and cost respectively, are computed as:

$$F_1(q)=\sum_{k=1}^{T}\left[\text{trace}\{P_k(+)\}\right] \qquad (8)$$

$$F_2(q)=\sum_{i=1}^{s}Fc_i+\sum_{k=1}^{T}\sum_{i}^{s}\delta_{ik}c_i \qquad (9)$$

where $F_1(q)$ is sum of trace of SEECM over simulation time T. In equation (9) the first term represents total installation cost and the second term represents measurement or operating cost. The variables Fc_i and c_i represent the installation cost and operating cost for sensor i respectively. The variable δ_{ik} can take a value of either 0 or 1 and is defined as:

$$\delta_{ik}=\begin{cases}1, \text{ if remainder}\{k/f^i_t\}=0\\ 0, \text{ otherwise}\end{cases} \qquad (10)$$

The value of $\delta_{ik}=1$ indicates presence of i^{th} sensor at k^{th} time instant.

Problem (7) is a multiobjective optimization problem since in absence of any weighting criteria for combining the objectives $F_1(q)$ and $F_2(q)$ into a single objective, this problem may have several solutions corresponding to trade-offs between these two objectives. Generation of these trade-off or Pareto solutions is the aim in this article. The Pareto solutions are the set of non-dominated feasible solutions. Solution q_i is said to be non-dominated if it is feasible and there is no other feasible solution q_j which has better (lower) values of both objectives F_1

and F_2 compared to the values obtained with q_i (Deb, 2003). The concept of Pareto optimal solutions is illustrated in Figure 1. In this Figure, solutions A, B and C are Pareto optimal since they are not dominated by any other solution, but solution D is not Pareto optimal since it is dominated by solution B. The set of Pareto optimal points is also labelled as the Pareto optimal front.

Another feature of formulation (7) is that it is not in standard explicit optimization form since calculation of $F_1(q)$ given the decision variables q, requires the solution of multirate Kalman filtering equations.

Due to these features (non-convexity, implicit nature, and multiple objectives) traditional optimization solution strategies are not suited for solving this problem. Hence we use genetic algorithms for solving problem (7). This solution strategy is discussed next.

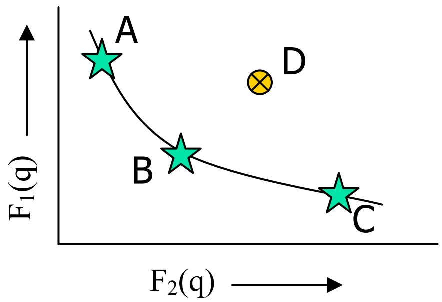

Figure1: Concept of Pareto optimal solutions

2.5. *Solution approach*

In this work we propose the use of genetic algorithm since it does not require the problem to be convex or be presented in an explicit optimization form. Further, it is suited for generating multiple solutions since it works with a population and not with a single solution. The well-known NSGA-II as introduced by Deb et al., (2002) is used to solve above Pareto optimization problem where the decision variables are sampling times for the available sensors. In NSGA-II algorithm, the population needs to be sorted according to ascending levels of non-domination. It uses crowding distance technique for maintaining diversity in the population. Further, the non-dominating sorting is done for combined population (parent and offspring) to preserve elitism. The algorithm is terminated when number of generations reaches a predefined maximum value or when not much improvement in population is obtained in successive generations.

3. CASE STUDY AND RESULTS

The issues related to trade-off in the sensor design objectives as discussed in this article, have been further analyzed by application on the quadruple tank setup of Johannsen (2000) which has been used as a benchmark problem in the process control literature.

3.1. *Case study : Quadruple Tank set-up*

The quadruple tank process shown in Figure 2, consists of four interconnected water tanks and two pumps. The inputs are the voltages to the two pumps and the outputs are the levels in the tanks as measured by voltages from level measurement devices. A non-linear mathematical model for the quadruple tank process based on first principles is given as (Johansson, 2000)

$$\frac{dh_1}{dt} = -\frac{ac_1}{Ac_1}\sqrt{2gh_1} + \frac{ac_3}{Ac_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{Ac_1}\nu_1 \qquad (11)$$

$$\frac{dh_2}{dt} = -\frac{ac_2}{Ac_2}\sqrt{2gh_2} + \frac{ac_4}{Ac_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{Ac_2}\nu_2 \qquad (12)$$

$$\frac{dh_3}{dt} = -\frac{ac_3}{Ac_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{Ac_3}\nu_2 \qquad (13)$$

$$\frac{dh_4}{dt} = -\frac{ac_4}{Ac_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{Ac_4}\nu_1 \qquad (14)$$

where, the voltage applied to Pump p is ν_p and the corresponding flow is $k_p\nu_p$. The parameters $\gamma_1, \gamma_2 \in (0,1)$ are determined from setting of valve positions. The flow to tank 1 is $\gamma_1 k_1 \nu_1$ and flow to tank 4 is $(1-\gamma_1)k_1\nu_1$ and similarly for tanks 2 and 3. Variables h_1 to h_4 are the heights in the tanks 1 to 4 respectively. The measurement level signal from tank j is $k_c h_j$ where k_c is a constant. Detailed description of this model and the values of various parameters have been obtained from Johansson (2000). In this work, we restrict our attention to linear state space models. The continuous time, linear state space model for the above presented non-linear equations (Equations 11-14) is obtained using a first order Taylor series expansion around the steady state values for these equations. The discretization of the linearized continuous model to obtain φ and Γ matrices (Equation 1) is performed using the matlab command "c2d" with discretization time=1 sec.

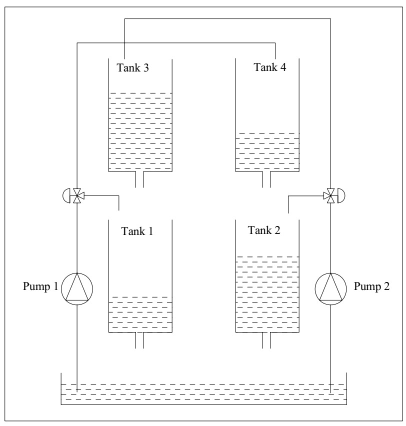

Figure 2: Schematic diagram of quadruple tank process

3.2. *Results and discussion*

The water level in each tank is considered as a state. There are four states and two input variables. We have considered summation of trace of recursive SEECM over the time interval T=1000 sec, as a scalar measure of process information. The inverse of variance of a sensor noise is considered to be its measurement cost when the corresponding state is measured. The total cost is obtained by summation of total measurement cost for all sensors over given time interval. The installation cost is considered to be zero. The parameters used for the genetic algorithm and multirate Kalman filter are given in Table 1.

The decision variables in this problem are the sampling times for measuring the different height signals. For each of the four signals, the allowed set of sampling times D_i={0,1,2…….31}sec ∀ i=1,2,3,4. Hence there are a total of 32^4=1048576 (≈10^6) measurement combinations possible. For the present work, population size of 1000 and 100 generations in the GA strategy are used. Hence a total of 10^5 combinations of sensors networks (less than 10% of all possible combinations) are enumerated by GA. To check the quality of results obtained from GA we additionally also enumerate all feasible solutions. The final Pareto optimal front obtained by GA and that obtained by complete enumeration are shown in Figure 3. It can be seen from this Figure that the Pareto optimal front found by GA is very close to true front obtained by complete enumeration. The optimal front is divided into 3 regions viz. regions-A, B and C. These regions are broadly highlighted in Figure 3. Regions A and C are sensor networks corresponding to low cost-poor estimation quality and high cost-good estimation quality respectively. On other hand region-B sensor networks have moderate cost and quality of estimates. Now the process designer can select a particular sensor network depending on his/her constraint on available cost and need for accuracy in the state estimates. The results reported (in terms of optimal networks) did not change when the simulation time interval T for the multirate Kalman filtering calculations was increased to 10000 sec. The representative solutions for above three groups are shown in Table 2.

It can be noticed from this table that height of the liquid in Tank 4 is measured at each instant (maximum sampling rate) for each of the three solutions due to its low measurement cost compared to rest of the measurements. The variations in variance of state estimation error (VSEE) for different states corresponding to three representative measurement schedules are given in Figures 4-6. In this case study φ is stable; hence any measurement strategy ensures system detectability. For the detectable case, single rate system states converge to single steady state values, while multirate system states exhibit a periodic steady state pattern. Figure 4 shows the variation in VSEE corresponding to measurement schedule [0 31 27 1], which indicates that state h1 is not measured at any time while states h2,h3 and h4 are measured at every 31, 27 and 1sec intervals respectively. It can be seen from Figure 4 that the VSEE curve shows an increasing trend for state h1 since it is never measured. Even though no measurement is available for state h1, some reduction in its VSEE still occurs at time period of 27sec. This is due to the inferential correction of state h1 by the Kalman filter based on measurement of state h3 and its relationship with state h1 as captured by the state space model. Since system is detectable the VSEE will also settle for this unmeasured state h1 (Nicolao, 1992). The actual simulation is carried out for 1000 sec but the changes of VSEE in Figures 4-6 are shown only upto 100 sec for clarity. Even though Figure 4 shows continuous increment in state h1, it reaches its periodic steady state value after 700 sec. The state h4 is measured at every time instant and its VSEE quickly settles to a steady state value. Figure 4 shows a periodic pattern for states h2 and h3, since these states are measured infrequently and their VSEE value increases when the corresponding measurements are not available. However when measurements for these states are available the VSEE is reduced to a lower value due to the multirate Kalman filter correction. The same pattern is repeated over successive sampling instants and the VSEE finally attains a periodic steady state A similar description holds for Figures 5 and 6 depending on sampling time for each state

Table 1 GA and Kalman filter related parameters

Parameter		Value
Length of chromosomes	:	20 bits
Population size	:	1000
Number of generations	:	100
Crossover frequency	:	0.9
Mutation parameter	:	0.01
T for Kalamn filter	:	1000 sec
Q	:	Identity matrix of appropriate size
The variances of sensor noise (cm^2) for:		
i. state h1	:	0.01
ii. state h2	:	0.1
iii. state h3	:	1
iv. state h4	:	10

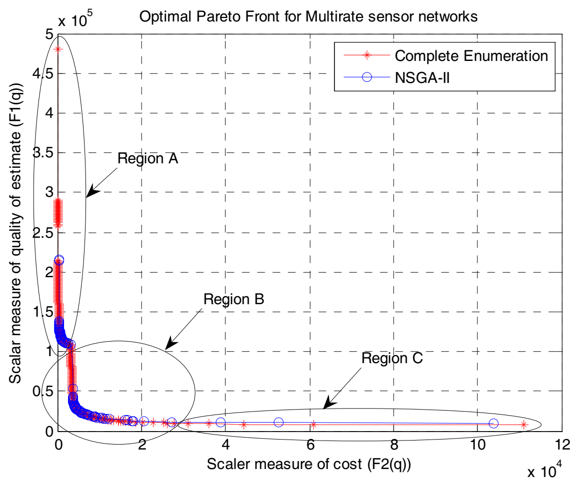

Figure 3: Optimal Pareto Front for Multirate sensor networks.

Table 2: Representative solutions for optimal measurement time

Region	Measurement time interval (sec) [h1 h2 h3 h4]	Measure of Cost x 10^{-5}	Measure of quality of estimate x 10^{-5}
A	0 31 27 1	0.0046	1.3748
B	31 14 3 1	0.0434	0.2893
C	1 3 3 1	1.0376	0.1008

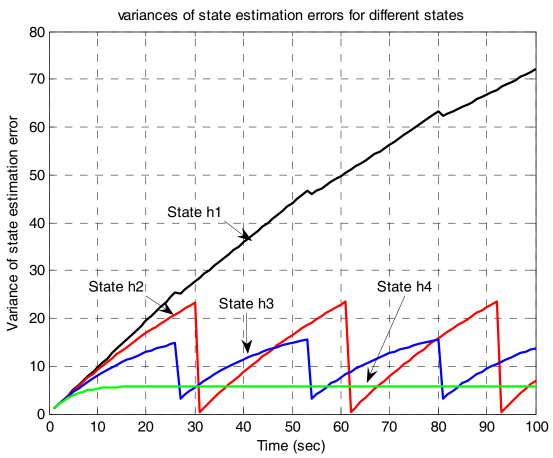

Figure 4: Variation in VSEE for sensor network [0 31 27 1]

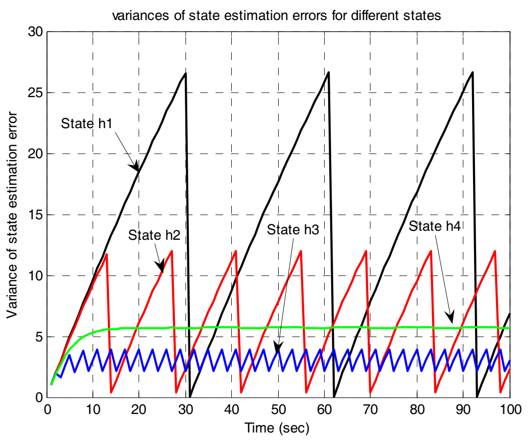

Figure 5: Variation in VSEE for sensor network [31 14 3 1]

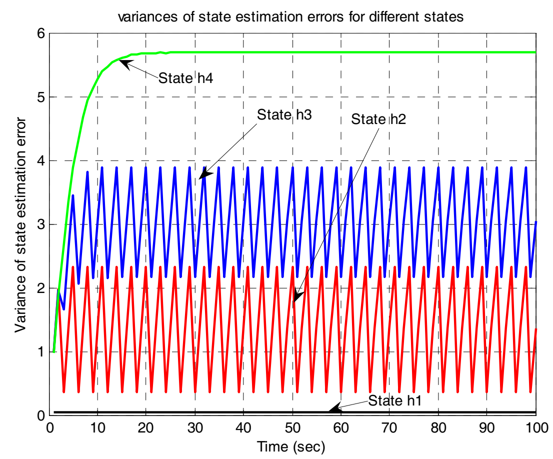

Figure 6: Variation in VSEE for sensor network [1 3 3 1]

4. CONCLUSIONS

The methodology for determining optimal sensor network design for multirate systems presented here is relevant to chemical and bio-chemical processes since they are multirate in nature. Further, the methodology is also applicable for the problem of determining measurement time intervals for regular systems, where measurement procedure/availability is not a constraint. The model based Kalman filtering strategy in the multirate framework is proposed to generate optimally filtered estimates of the states which in turn are used for determining the scalar measure of quality of estimation. The resulting Pareto optimization problem is solved by using NSGA-II. The optimal solutions provide several choices to process system designer to select a suitable sensor network depending upon his/her choice for measurement cost and quality of estimates. The methodology is demonstrated using simulations involving the benchmark quadruple tank set up. The extension of this methodology for dealing with non-linearity, non-Gaussian noise, and measurement delays is currently under study.

REFERENCES

Deb, K (2003). *Multi-Objective Optimization using Evolutionary Algorithms*, chapter 1. John Wiley & Sons (Asia) Pte. Ltd., Singapore.

Deb, K., A. Pratap, S. Agarwal and T. Meyarivan (2002). A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, **6(2)**, 182-197.

Gelb, A (1988). *Applied Optimal Estimation*, pp102-155.The M.I.T. press, England.

Gudi, R.D., S.L. Shah and M.R. Gray (1995). Adaptive Multirate State and Parameter Estimation Strategies with Application to a Bioreactor. *AIChE Journal,* **41(11),** 2451-2464.

Johansson, K.H (2000). The Quadruple-Tank Process: A Multivariable Laboratory Process with an Adjustable Zero. *IEEE Transactions on Control Systems Technology*, **8(3),** 456-465.

Lennartson, B (1988). Periodic solutions of Riccati equations applied to multirate sampling. *Int. J. Control* , **48(3),** 1025-1042.

Muske, K.R. and C. Georgakis (2003). Optimal Measurement System Design for Chemical Processes. *AIChE Journal,* **49(6)**, 1488-1494.

Musulin, E., C. Benqlilou, M.J. Bagajewicz and L. Puigjaner (2005). Instrumentation design based on optimal Kalman filtering. *Journal of Process Control*, **15**, 629-638.

Nicolao, G.D (1992). Cyclomonotonicity and Stabilizability Properties of Solutions of the Difference Periodic Riccati Equation. *IEEE Transactions on Automatic Control*, **37(9),** 1405-1410.