A NEW PERFORMANCE EVALUATION STRATEGY FOR DECENTRALIZED MULTIVARIABLE PID CONTROL SYSTEMS

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Abstract: This paper addresses the problem of performance evaluation for decentralized multivariable PID controllers, by comparing the actual performance with that of a benchmark multivariable state-feedback LQR controller. The procedure can be used at a design stage and for closed-loop performance monitoring of industrial plants. An overall measure of controller suboptimality is proposed along with a matrix that quantifies the closed-loop interactions among control loops. The minimum interactions evaluated under the benchmark controller can be regarded as an intrinsic process property. Finally, a retuning procedure for the decentralized controller is proposed with the aim of improving the overall closed-loop performance. Several extensions of the benchmark controller are discussed, such as how to identify a process model from data and how to consider output feedback. The features of the proposed methodology are explained by application to the Shell Control problem.

Keywords: Decentralized control systems, PID control, LQR, subspace identification, control loop performance monitoring

1. INTRODUCTION

Control Loop Performance Monitoring (CLPM) is widely recognized as a mean of large importance to improve product quality and then the overall economy of industrial plants. The complete job of a CLPM system includes a reliable performance evaluation, a prompt detection of low performing loops, a detection of the causes and suggestion of actions to take. Tools for automatic monitoring control loops recently proposed by software houses (e.g. AspenWatch/PIDWatch by Aspentech, LoopScout by HoneyWell) aim at this scope by different ways. Despite of this fact, several issues are still open, not only in implementation (e.g. on-line versus off-line architectures, interaction with the operator) but also with regard to theoretical aspects. In the case of multivariable plants, the very basic issue of defining appropriate performance indices is not yet completely solved (Schäfer and Cinar, 2004) and also in the tracking of root cause of a perturbation there is still much work to do (Tornhill and Horch, 2006).

This paper focuses on the evaluation of loop performance in the particular case of multi-input multi-output (MIMO) plants under decentralized control, an aspect of interest both for a continuous evaluation/improvement of SISO loops and to decide from a "una tantum" analysis to change to a MIMO control system. Traditionally, the problem of evaluating the suitability of a decentralized control to cope with a MIMO process has been faced starting from the analysis of plant interactions (e.g. with RGA technique), seen as an intrinsic property of the process (to be minimize with an appropriate choice of couplings between manipulated and controlled variables).

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However, it is evident that a perturbation transmitted from one loop to another depends both on process characteristics and on controller tuning/design; thus the evaluated performance index is necessarily affected by both features. Following this track in previous work two indexes were proposed: a Time Domain Interaction and a Controller Performance Index (Rossi *et al.*, 2006). Going one step forward, in the present work a minimum degree of interaction, depending on the optimal MIMO controller (benchmark) which can be designed for that plant, is defined. With these premises, a global approach to the problem has been undertaken trough a strategy based on the following steps: i) definition of a performance index for the actual controller, ii) performance evaluation by comparison with a benchmark MIMO controller, iii) retuning procedure to obtain the "best" achievable performance.

2. METHOD: BASIC IDEAS

2.1 Preliminary definitions

In this work we consider linear multivariable discrete time-invariant systems in the form:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + e_k \ , \end{aligned} \tag{1}$$

in which $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^m$ is the output, $e \in \mathbb{R}^m$ is white noise, (A, B, C) have appropriate dimensions, the pair (A, C) is observable and the pair (A, B) is controllable, and the following condition is satisfied:

$$\text{rank} \begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix} = n + m \ . \tag{2}$$

The process (1) is assumed to be controlled by a decentralized multivariable proportional-integral-derivative (PID) control systems in the form:

$$u_k = K_P \epsilon_k + K_I \sum_{j=0}^{k} \epsilon_j + K_D(\epsilon_k - \epsilon_{k-1}) \ , \tag{3}$$

in which K_P, K_I, K_D are square diagonal matrices of dimension $m \times m$ and $\epsilon \in \mathbb{R}^m$ is the tracking error:

$$\epsilon_k = r - y_k \ , \tag{4}$$

where r is the output setpoint.

It is assumed that outputs are appropriately scaled so that an equal tracking error of different outputs can be regarded as equivalent in terms of performance loss. Similarly, it is assumed that inputs are appropriately scaled so that an equal variation of different inputs can be regarded as equivalent in terms of control effort. Given a sequence of input and output data, collected in closed loop from time $k = 0$ up to time $k = N-1$, the following "cost function" is defined for each control loop $i = 1, 2, \ldots, m$:

$$\Phi_i = \sum_{k=0}^{N-1} (\mathbf{1}_i' \epsilon_k)^2 + \lambda(\mathbf{1}_i' \Delta u_k)^2 \ , \tag{5}$$

in which $\mathbf{1}_i \in \mathbb{R}^m$ is $i-$th column of the identity matrix, $\Delta u_k = u_k - u_{k-1}$ is the input variation, and λ is a positive scalar, defined by the user to weigh the relative importance between output tracking error and input usage. Consequently the "overall" cost function is defined as:

$$\Phi = \sum_{i=1}^{m} \Phi_i = \sum_{k=0}^{N-1} \epsilon_k' \epsilon_k + \lambda \Delta u_k' \Delta u_k \ . \tag{6}$$

2.2 Closed-Loop Interaction Array (CLIA)

In order to quantify the effect of "perturbations" transmitted from one loop to the other ones, we propose to collect closed-loop data during setpoint changes superimposed on each loop, separately, i.e. with unchanged setpoints of all other loops. This is a commonly accepted industrial practice, used for instance in preliminary identification tests for MPC projects. A typical pattern requires the setpoint of the $i-$th loop to be changed from 0 (i.e. the reference value) to a positive value a, then from the corresponding negative value $-a$, and finally back to 0 (while the setpoints of all other loops are kept equal to 0). Let Φ_j^i be the cost function defined in (5) for the $j-$th loop during the time period involving setpoints changes in the $i-$th loop, and let Φ^i be the corresponding overall cost function defined in (6). With these definitions we can now introduce the *Closed-Loop Interaction Array* (CLIA) as follows:

$$\Gamma = \{\gamma_{ji}\} = \left\{ \frac{\Phi_j^i}{\Phi^i} \right\} \ . \tag{7}$$

It is straightforward to see that the matrix $\Gamma \in \mathbb{R}^{m \times m}$ satisfies the following conditions:

$$\gamma_{ji} \geq 0, \qquad \sum_{j=1}^{m} \gamma_{ji} = 1 \ . \tag{8}$$

It is clear from its definition that the ideal situation from an interaction point of view is when Γ is equal (or close) to the identity matrix. In practice, instead, the matrix Γ may be significantly different from the identity matrix, thus showing relevant interactions among the control loops, especially when a decentralized controller is adopted.

2.3 Benchmark controller definition

In order to evaluate how far the current controller's performance is from the least interacting situation, we propose to define a "benchmark" controller the solution to the following optimization problem:

$$\min_{u_0, u_1, \ldots} \sum_{k=0}^{\infty} \epsilon_k' \epsilon_k + \lambda \Delta u_k' \Delta u_k \tag{9a}$$

subject to:

$$x_0 = 0, \quad u_{-1} = 0 \tag{9b}$$
$$x_{k+1} = Ax_k + Bu_k \tag{9c}$$
$$\epsilon_k = r - Cx_k \ . \tag{9d}$$

In order to solve problem (9) we preliminary make two assumptions, which will be removed later on, namely: (i) the full state of the system x is measurable, (ii) the system matrices (A, B, C) are known. First we evaluate the steady-state targets for state and input as the solution of following linear square system:

$$\begin{bmatrix} I - A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = \begin{bmatrix} 0 \\ r \end{bmatrix} , \tag{10}$$

to obtain $\begin{bmatrix} \bar{x} \\ \bar{u} \end{bmatrix} = Mr$ (with M trivially defined), and we introduce the following augmented state and decision variable:

$$w_k = \begin{bmatrix} x_k - \bar{x} \\ u_{k-1} - \bar{u} \end{bmatrix}, \qquad v_k = \Delta u_k \ . \tag{11}$$

Next, we rewrite the problem (9) in a standard Linear Quadratic Regulation (LQR) form:

$$\min_{v_0, v_1, \dots} \sum_{k=0}^{\infty} w_k' Q w_k + v_k' R v_k \qquad \text{s.t.} \tag{12a}$$
$$w_{k+1} = \hat{A} w_k + \hat{B} v_k \ , \tag{12b}$$

where the matrices $Q, R, \hat{A}, \hat{B}$ are defined as follows:

$$Q = \begin{bmatrix} C'C & 0 \\ 0 & 0 \end{bmatrix}, \quad R = \lambda I,$$
$$\hat{A} = \begin{bmatrix} A & B \\ 0 & I \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\ I \end{bmatrix} \ . \tag{13}$$

Hence, the solution of (12) is [see e.g. (Kwakernaak and Sivan, 1972)]:

$$v_k = Kw_k = -(R + \hat{B}'\Pi\hat{B})^{-1}\hat{B}'\Pi\hat{A}w_k \ , \tag{14}$$

in which Π is the unique stabilizing solution of the following Riccati equation:

$$\Pi = Q + \hat{A}'\Pi\hat{A} - \hat{A}'\Pi\hat{B}(R + \hat{B}'\Pi\hat{B})^{-1}\hat{B}'\Pi\hat{A} \ , \tag{15}$$

and, therefore, the optimal *state feedback* control law solution to (9) is given by:

$$u_k = u_{k-1} + Kw_k = u_{k-1} + K_1(x_k - \bar{x}) + K_2(u_{k-1} - \bar{u}) \ , \tag{16}$$

where $K_1 \in \mathbb{R}^{m \times n}$ and $K_2 \in \mathbb{R}^{m \times m}$ are trivially defined.

By simulating the same pattern of setpoint changes, we compute the cost associated to each "loop" during setpoint changes on the $i-$th loop, denoted with $\bar{\Phi}^i_j$, an overall cost, denoted with $\bar{\Phi}^i$, then we build the "optimal" CLIA, denoted as $\bar{\Gamma} = \{\gamma_{ji}\}$. Notice that, in general, the optimal cost function for a setpoint change r can be computed analytically as $\bar{\Phi} = r'M'\Pi Mr$.

2.4 Decentralized controller performance evaluation

The performance of the decentralized controller can be now assessed in terms of global (i.e. multivariable) cost function during the pattern of setpoint changes by defining the *Controller Suboptimality Index* (CSI):

$$\sigma = \frac{1}{m} \left(\sum_{i=1}^{m} \frac{\Phi^i}{\bar{\Phi}^i} \right) - 1 \ . \tag{17}$$

We propose the following "grades" for evaluation of the overall performance:

- if $\sigma \leq 0.25$, suboptimality is *irrelevant*;
- if $0.25 < \sigma \leq 1.0$, suboptimality is *moderate*;
- if $1.0 < \sigma$, suboptimality is *significant*.

Since CSI does not explain directly which closed-loop interactions are more relevant, we can compute the difference between the off-diagonal elements of CLIA and the corresponding elements of the "optimal" CLIA, i.e.

$$\delta_{ji} = \gamma_{ji} - \bar{\gamma}_{ji}, \qquad j \neq i \ , \tag{18}$$

and *propose* the following "grades" of interaction toward the $j-$th loop during a setpoint change in the $i-$th loop:

- if $\delta_{ji} < 0.05$, interaction is *irrelevant*;
- if $0.05 \leq \delta_{ji} < 0.20$, interaction is *moderate*;
- if $0.20 \leq \delta_{ji} \leq 1$, interaction is *significant*.

It is clear that the proposed grades, although derived from extensive simulations, are somewhat arbitrary and one may consider different grades specifically suited for each application.

2.5 Decentralized controller retuning

If the performance of the multivariable decentralized controller is regarded as unsatisfactory, we can try to modify the controller's tuning parameters. We propose to compute the new tuning parameters from the solution of an optimization problem in which the overall cost function is minimized, as detailed. Let $\theta \in \mathbb{R}^{3m}$ be a vector containing the diagonal elements of the controller matrices K_P, K_I, K_D. Next, let Φ be overall closed-loop cost function defined over an infinite horizon, i.e. $\Phi = \sum_{k=0}^{\infty} \epsilon_k'\epsilon_k + \lambda\Delta u_k'\Delta u_k$, associated to a generic setpoint change vector r, when the controller parameters are given in θ, and let $\bar{\Phi}$ be the corresponding "optimal" cost function achieved by the benchmark controller. With these definitions, the "optimal" decentralized controller tuning parameters are obtained from the following optimization problem:

$$\theta^* = \operatorname*{argmin}_{\theta} \frac{\Phi}{\bar{\Phi}} - 1, \qquad \text{s.t. (9b)–(9d) and} \tag{19a}$$
$$\theta_{\min} \leq \theta \leq \theta_{\max} \ , \tag{19b}$$

in which the bounds $\theta_{\min}$, $\theta_{\max}$ are defined by the user. The closed-loop evolution of the process model controlled by the multivariable decentralized controller can be written by introducing the augmented state and output: $\mathcal{X}_k = \begin{bmatrix} x_k' & I_{k-1}' & \epsilon_{k-1}' & \epsilon_{k-2}' \end{bmatrix}'$, $\mathcal{Y}_k = \begin{bmatrix} \epsilon_k' & \Delta u_k' \end{bmatrix}'$, as follows:

$$\begin{aligned}\mathcal{X}_{k+1} &= \mathcal{A}\mathcal{X}_k + \mathcal{B}r \\ \mathcal{Y}_k &= \mathcal{C}\mathcal{X}_k + \mathcal{D}r\ ,\end{aligned} \tag{20}$$

in which

$$\mathcal{A} = \begin{bmatrix} A - BK_TC & BK_I & -BK_D & 0 \\ -C & I & 0 & 0 \\ -C & 0 & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \quad \mathcal{B} = \begin{bmatrix} BK_T \\ I \\ I \\ 0 \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} -C & 0 & 0 & 0 \\ -K_TC & 0 & -K_P - 2K_D & K_D \end{bmatrix} \quad \mathcal{D} = \begin{bmatrix} I \\ K_T \end{bmatrix}$$

and $K_T = K_P + K_I + K_D$. We can now write

$$\Phi = \sum_{k=0}^{\infty} \mathcal{Y}_k' \mathcal{Q} \mathcal{Y}_k\ , \tag{21}$$

where $\mathcal{Q} = \begin{bmatrix} I & 0 \\ 0 & \lambda I \end{bmatrix}$. After some algebraic manipulations (omitted in the sake of space) it we can see that:

$$\Phi = r'\mathcal{M}'\mathcal{P}\mathcal{M}r \tag{22}$$

in which $\mathcal{M} = (I - \mathcal{A})^{-1}\mathcal{B}$ and $\mathcal{P}$ is solution to the Lyapunov equation:

$$\mathcal{P} = \mathcal{C}'\mathcal{Q}\mathcal{C} + \mathcal{A}'\mathcal{P}\mathcal{A}\ . \tag{23}$$

It is important to point out that the solution to (19) depends on the setpoint vector r, because the considered controller is decentralized and with fixed (PID) structure, while the optimal multivariable controller defined as a benchmark is independent of the setpoint vector. Clearly, one can compute the optimal controller parameters for the most frequent "directions" of the setpoint, or simply for the m unitary directions, $\mathbf{1}_i$ $(i = 1, \ldots, m)$. According to this latter choice we rewrite (19) as (recall that $\bar{\Phi} = r'M'\Pi Mr$):

$$\theta^* = \operatorname*{argmin}_{\theta} \sum_{i=1}^{m} \frac{\{\mathcal{M}'\mathcal{P}\mathcal{M}\}_{ii}}{\{M'\Pi M\}_{ii}}, \quad \text{s.t. (19b)}\ , \tag{24}$$

which can be solved by means of constrained nonlinear optimization routines.

3. METHOD: COMPLEMENTS

The procedure outlined in the previous section can be used in a straightforward way at a design stage, when a process model is known, to evaluate the suitability of decentralized PID control systems against a multivariable alternative. In the framework of CLPM, instead, when the process model is not known or changes with the operating conditions and only input/output data are available, the two preliminary assumptions considered in the benchmark controller definition can be removed, as discussed in the sequel.

3.1 Output feedback and noise

Since the decentralized control system is an output feedback controller, a fairer comparison can be made against an output feedback controller that solves (9). Moreover, during the simulation of the benchmark controller response it may be appropriate to add normally distributed noise to the process output.

As it is standard in LQ control (Kwakernaak and Sivan, 1972, Ch.5,Sec.6.6) we make use of an observer to obtain an estimate of the state vector from measurements of the output vector and then apply the optimal state feedback control law (16) to this estimate. More in detail, let $\hat{x}_k$ denote an estimate of x_k computed at time $k-1$, and let y_k be the measured output. We now define the control input at time k as:

$$u_k = u_{k-1} + K_1(\hat{x}_k - \bar{x}) + K_2(u_{k-1} - \bar{u})\ , \tag{25}$$

and then compute the estimate of the state for the next sampling time as:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L(y_k - C\hat{x}_k)\ , \tag{26}$$

in which $L \in \mathbb{R}^{n\times m}$ is a constant matrix chosen such that $A - LC$ has all eigenvalues inside the unit disk.

3.2 Subspace model identification

Since in general the process model is not known we propose to use a Subspace IDentification (SID) algorithm to obtain a model necessary to build the benchmark controller and evaluate the "optimal" performance. The identification algorithm is a variant of the closed-loop "projection" method proposed in (Huang *et al.*, 2005), and is applied to closed-loop data collected to evaluate the performance of the decentralized controller.

3.2.1. Data projection. Given a positive integer ℓ, assumed to satisfy $\ell > n$, let the vector of "future" outputs $\bar{y}_k = \begin{bmatrix} y_k' & y_{k+1}' & \cdots & y_{k+\ell-1}' \end{bmatrix}'$ and similar straightforward definitions are given for vectors of future inputs and noise, denoted with $\bar{u}_k$ and $\bar{e}_k$. From the model (1), we can obtain:

$$\bar{y}_k = O_\ell x_k + H_\ell \bar{u}_k + \bar{e}_k\ , \tag{27}$$

in which O_ℓ is the extended observability matrix, H_ℓ is a lower block-triangular Toeplitz matrix (omitted in the sake of space). Next, we write (27) for $k = \ell, \ldots, \ell + M - 1$:

$$Y_f = O_\ell X + H_\ell U_f + E_f\ , \tag{28}$$

where Y_f, X, U_f, E_f are constructed just placing side by side vectors $\bar{y}_k$, x_k, $\bar{u}_k$ and $\bar{e}_k$ respectively, for M sampling times. Now, post-multiplying both sides of (28) by an appropriate matrix, W', such that $\lim_{M\to\infty} \frac{1}{M} E_f W' = 0$, we can rewrite (28) as follows:

$$\begin{bmatrix} I & -H_\ell \end{bmatrix} Z_f W' = O_\ell X W'\ , \tag{29}$$

in which $Z_f = [Y_f'\ U_f']'$. Since the data are collected in closed loop, a possible choice to avoid problems of correlation of W' with the future noise matrix is to choose: $Z_{CL} = \begin{bmatrix} R_f' & Z_p' \end{bmatrix}'$, in which R_f is the matrix of future setpoints, defined similarly to Y_f, and Z_p is defined as Z_f but with data shifted ℓ times in the past.

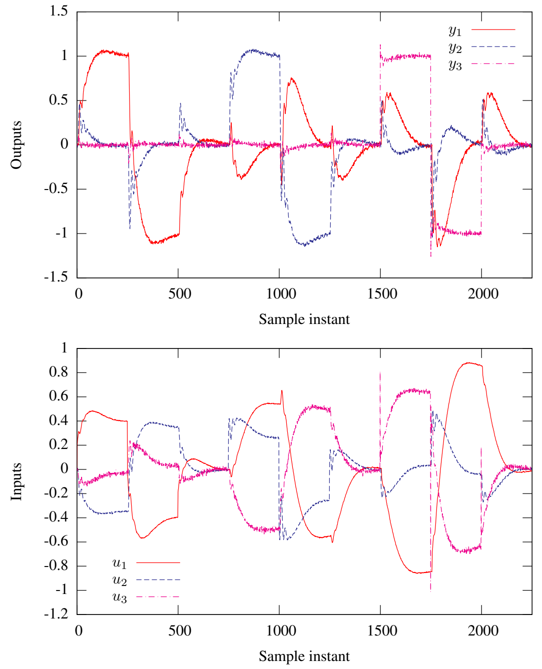

Fig. 1. Closed-loop simulation results with decentralized controller (original tuning)

3.2.2. Recovering model matrices. In order to obtain O_ℓ, we can pre-multiply (29) by a matrix $O_\ell^\perp$ orthogonal to O_ℓ, i.e. such that $O_\ell^\perp O_\ell = 0$, and obtain

$$O_\ell^\perp \left[I \ -H_\ell\right] Z_f W' = 0 \,. \tag{30}$$

Let $Z = Z_f W'$, it is clear that $O_\ell^\perp \left[I \ -H_\ell\right] = Z^\perp$ and hence it is necessary to compute $Z^\perp$. This can be done performing an SVD of Z:

$$Z = \left[U_1 \ U_2\right] \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} , \tag{31}$$

in which the dimension of the square diagonal matrix S_1, i.e. the rank of Z, should be $m\ell + n$ (Wang and Qin, 2002, Lemma 1). In practice, however, the rank of Z may be different from $m\ell + n$ because of the noise. Hence, the dimension of S_1, and consequently the system's order n, should be obtained from the singular values, using e.g. an Akaike Information Criterion as in (Wang and Qin, 2002) or a heuristic PCA approach as described in (Micchi and Pannocchia, 2006). Next, from (31) it can be seen that (30) is satisfied if:

$$O_\ell^\perp \left[I \ -H_\ell\right] = T U_2' \,, \tag{32}$$

where T is a nonsingular transformation matrix of dimension $m\ell - n$. Finally, by partitioning:

$$T U_2' = \left[P_1' \ P_2'\right] \,, \tag{33}$$

in which $P_1 \in \mathbb{R}^{m\ell \times (m\ell - n)}$, it is easy to see that:

$$P_1' O_\ell = 0, \qquad -P_1' H_\ell = P_2' \,, \tag{34}$$

which can be readily solved to find estimates of O_ℓ and H_ℓ. Once the estimate of O_ℓ is computed, it is possible to recover an estimate of A and C simply observing that (in a MATLAB notation)

$$C = O_\ell(1:m,:) \tag{35a}$$
$$O_\ell(1:(\ell-1)m,:)A = O_\ell(m+1:m\ell,:) \,, \tag{35b}$$

and solving equation (35b) for A in least square sense. The original method (Wang and Qin, 2002; Huang *et al.*, 2005) computes B from H_ℓ, which is also obtained from (29). Here, instead, we propose to solve (34) directly in terms of B enforcing causality. In particular we exploit the structure of H_ℓ (linear in B) and rewrite the second equation in (34) as:

$$M_1 B = M_2 \,, \tag{36}$$

with M_1 and M_2 suitably defined. Clearly (36) is solved for B in a least-square sense.

4. ILLUSTRATIVE EXAMPLE

4.1 Process and controller

As an example we consider the Shell Control Problem, whose process transfer function is given below:

$$G(s) = \begin{bmatrix} \dfrac{4.5e^{-27s}}{50s+1} & \dfrac{1.77e^{-28s}}{60s+1} & \dfrac{5.88e^{-27s}}{50s+1} \\ \dfrac{5.39e^{-18s}}{50s+1} & \dfrac{5.62e^{-14s}}{60s+1} & \dfrac{6.9e^{-15s}}{50s+1} \\ \dfrac{4.38e^{-20s}}{33s+1} & \dfrac{4.42e^{-22s}}{44s+1} & \dfrac{7.2}{19s+1} \end{bmatrix} , \tag{37}$$

and the sampling time is $T_s = 4$. A normally distributed output noise with standard deviation of 0.01 is added to each output. A decentralized control system of PI type is considered ($K_D = 0$) in which the elements of K_P, K_I are obtained from the SIMC rules (Skogestad, 2003) applied to the corresponding diagonal elements of $G(s)$ (including $T_s/2$ as additional delay due too the sampling).

4.2 Performance evaluation and controller retuning

We consider a pattern of setpoint changes on each loop (with setpoints of the other loops kept at 0): $r_i = 1$ for 250 sampling times, followed by $r_i = -1$ for 250 sampling times, and finally $r_i = 0$ for 250 sampling times. Closed-loop outputs and inputs are reported in Figure 1. Next, by means of the SID algorithm discussed in the previous section we obtain a process model (in state-space form, with $n = 28$). By choosing a steady-state Kalman filter as observer gain, tuned with state and output noise covariance of 10^{-4}, we build an output feedback LQ controller (using $\lambda = 1.0$). Thus, for the current decentralized controller the following CSI and CLIA are evaluated:

$$\sigma = 7.19, \quad \Gamma = \begin{bmatrix} 0.834 & 0.509 & 0.808 \\ 0.160 & 0.480 & 0.108 \\ 0.006 & 0.011 & 0.084 \end{bmatrix} , \tag{38}$$

while for the "optimal" controller ($\sigma = 0$), CLIA is:

$$\bar{\Gamma} = \begin{bmatrix} 0.958 & 0.026 & 0.054 \\ 0.032 & 0.965 & 0.047 \\ 0.010 & 0.009 & 0.899 \end{bmatrix} , \tag{39}$$

Table 1. Decentralized controller tuning parameters (original and optimized)

Case	Gain			Integral time		
	# 1	# 2	# 3	# 1	# 2	# 3
Original	0.192	0.334	0.660	50.0	60.0	16.0
Optimized	0.176	0.524	0.514	10.0	89.4	25.9

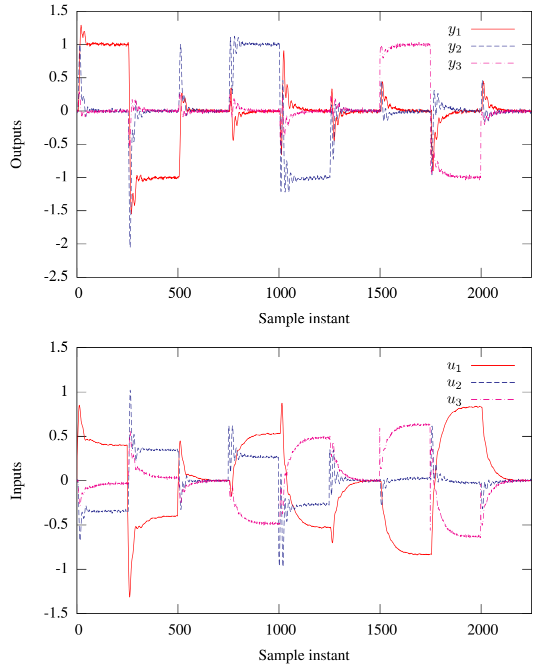

Fig. 2. Closed-loop simulation results with decentralized controller (optimized tuning)

Finally, we apply the retuning procedure outlined in the previous section, and the obtained tuning parameters are reported in Table 1 along with the original ones. Closed-loop results obtained with the retuned decentralized controller are reported in Figure 2. The corresponding performance measures are:

$$\sigma = 1.45, \quad \Gamma = \begin{bmatrix} 0.528 & 0.189 & 0.389 \\ 0.439 & 0.749 & 0.274 \\ 0.033 & 0.062 & 0.337 \end{bmatrix}. \qquad (40)$$

4.3 Discussion

The results presented show that the plant controlled by the original decentralized controller suffer from significant interactions especially towards the first control loop. In particular when a setpoint change occurs in the third loop, relevant perturbations are shown in the first loop ($\delta_{12} = \gamma_{12} - \bar{\gamma}_{12} = 0.483$, $\delta_{13} = 0.754$). As a result, the overall performance is significantly suboptimal ($\sigma = 7.2$). On the other hand, irrelevant interactions are shown in the third loop when setpoint changes occur in other loops ($\gamma_{31} = 0.006$, $\gamma_{32} = 0.011$). Notice that results of the optimal controller clarify that almost "perfect" decoupling among the loops is achievable if an appropriate multivariable controller is adopted ($\bar{\gamma}_{ii}$ are at least 0.9). The optimal "picture" shown by $\bar{\Gamma}$ is measure of the "true" degree of interactions, to be regarded as an intrinsic process property. While it is correct to claim that the actual performance depends on both the process and the controlled used, in general it is not appropriate to discuss whether a process is interacting or not without specifying the controller used. Finally, the proposed retuning procedure is able to improve relevantly the overall performance of the decentralized controller ($\sigma = 1.45$), although the performance is still far from that of the multivariable benchmark controller.

5. CONCLUSIONS

In the present paper we proposed a new methodology for performance evaluation of decentralized multivariable PID control systems, to be used at a design stage or for closed-loop monitoring. The main point is to define a benchmark controller against which the performance of the current controller is compared. This allows us to quantify an optimal performance and a minimum degree of interaction that should be regarded as an intrinsic process property. An overall performance index, the Controller Suboptimality Index, is proposed whose computation is based on closed-loop data collected during setpoint changes superimposed on each loop, separately. Furthermore, a Closed-Loop Interaction Matrix is computed to quantify clearly the interactions among loops. Finally, we proposed a retuning procedure aimed at finding the tuning parameters that guarantee the "best" overall performance achievable with a decentralized PID controller. When the process model required for the definition of the benchmark controller is not known, we proposed using a multivariable subspace identification procedure, based on the closed-loop data collected.

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