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STABILITY OF SUPERVISED ADAPTIVE CONTROL

Eduardo J. Dozal-Mejorada and B. Erik Ydstie

Chemical Engineering Department
Carnegie Mellon University
Pittsburgh, PA USA 15213
{*edozal,ydstie*}*@andrew.cmu.edu*

Abstract: This paper analyzes a new dual model supervised adaptive control algorithm for stability and robustness. The algorithm addresses infinite drift and parameter bursting by using one model as a supervisor and one as the controller. The main contribution of this paper is the demonstration that the dual model supervised adaptive control algorithm is robust with respect to small model/plant mismatch and bounded disturbances. The theoretical results are based on the Switching Lemma, Lyapunov stability, variance estimation and data normalization.

Keywords: stability; robustness; infinite drift; parameter bursting; adaptive control; self tuning control; supervised control; switching lemma

1. INTRODUCTION

Un-modeled dynamics along with unmeasured disturbances typically cause two main types of instabilities in adaptive control. In the first case, the estimated parameters de-stabilize the closed-loop system dynamics. This de-stabilization results in excitation in the input and output signals in the form of *bursting*. The observed excitation improves the signal-to-noise ratio which leads to a readjustment of the parameter estimates. The closed-loop stability is regained after the readjustment. However, once the closed loop has been stabilized the drifting resumes and the unstable cycle restarts as can be seen in Figure (1). In the second case, the estimated parameters *drift* without foreseeable convergence. Unfortunately, this phenomenon does not completely de-stabilize the system signals. The end result is that the signals are not excited and thus no estimated parameter improvement occurs. Different approaches have been proposed to address infinite drift and bursting. References (Åström and Wittenmark, 1995),

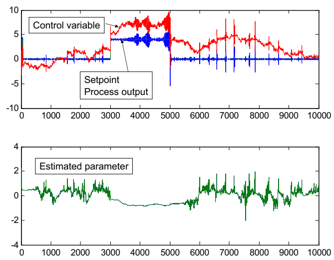

Fig. 1. Typical bursting behavior of a simple system.

(Ioannou and Sun, 1996), (Mareels and Polderman, 1996) and (Middleton *et al.*, 1988) present some of the proposed approaches focusing on their respective strengths and weaknesses. We briefly review three of the more common methods.

Parameter *projection* onto a convex set was introduced in (Egardt, 1979) to address infinite parameter drift. One strength of this approach is that it requires very limited a-priori system information. However, parameter estimation never stops, therefore bursting may persist. Parameter *leakage* was introduced to drive the estimates towards a specific family of reference values. Utilizing strong leakage leads to successful parameter drift elimination, however the parameters become biased towards the chosen references (Ioannou and Kokotovic, 1983). Furthermore, adequate reference values must be chosen to accurately describe the observed input-output plant behavior (Hovd and Bitmead, 2006). Therefore leakage typically degrades performance. The *deadzone* approach works by stopping the parameter estimation once the prediction error gets below a certain threshold (Peterson and Narendra, 1982) and (Hill and Ydstie, 2004). The main advantage of this method is that the parameters are forced to converge thereby eliminating infinite drift. However, the performance of this technique depends on correctly choosing the deadzone which may not be easy or intuitive in practical applications.

In this paper we present stability and robustness results of a proposed new approach (Dozal-Mejorada *et al.*, 2006). The method uses a dual model approach to determine when to update the control model parameters. In this respect the new approach is related to the deadzone in that it only allows estimation when new valuable data becomes available.

The rest of the paper is organized as follows. In Section 2 we present the problem statement. The supervised adaptive control algorithm with dual models is reviewed in Section 3. We present stability and robustness results in Section 4. Lastly, the Appendix contains detailed proofs of the technical results.

2. PROBLEM DEFINITION

Consider the self tuning of the single-input single-output (SISO) discrete-time system given by

$$A(q^{-1})y(t) = B(q^{-1})u(t) + \gamma(t) \tag{1}$$

The measured output and manipulated input are given by the signals $\{y(t), u(t)\}$ respectively. The effects of un-modeled dynamics and unknown disturbances are captured in the signal $\gamma(t)$. $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator q^{-1}

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \tag{2}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m} \tag{3}$$

Parameters of the $A(q^{-1})$ and $B(q^{-1})$ polynomials are aligned into the parameter vector

$$\theta^\star = (a_1, \ldots, a_n, b_0, \ldots, b_m)^T \tag{4}$$

In certainty equivalence adaptive control the unknown parameters of system (1) are replaced with estimates

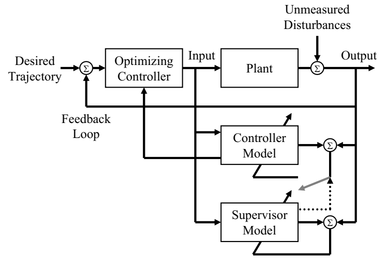

Fig. 2. Block diagram showing the supervised dual-model algorithm approach.

$$\hat{\theta}(t) = (\hat{a}_1(t), \ldots, \hat{a}_n(t), \hat{b}_0(t), \ldots, \hat{b}_m(t))^T \tag{5}$$

The estimated parameters are then used to design a stabilizing feedback control law as if they were the true ones. The main challenge consists of designing parameter estimation and control laws so that the closed loop adaptive system is stable and its performance over time approaches that of the controller based on the true system parameters. In this paper we address the stability problem and outline very briefly how we will address performance related issues.

3. SUPERVISED ADAPTIVE CONTROL

A new method for supervised adaptive control has been proposed (Dozal-Mejorada *et al.*, 2006). The algorithm is based on the idea of *supervising* the closed-up behavior of the estimated control model. Figure 2 shows the dual-model architecture under consideration. The first model is used by the controller to achieve the control objectives while the second model is used to supervise the performance of the control model and decide when to update the control model estimates.

In order to present the supervisory approach and its properties, consider the recursive least squares (RLS) algorithm with variable forgetting factor applied to the control and supervisor models (Goodwin and Sin, 1984)

$$\begin{aligned}\hat{\theta}_i(t) &= F_{\Theta^\star}\left\{\hat{\theta}_i(t-1)\right\}\\ &+ F_{\Theta^\star}\left\{\frac{P(t-1)_i\varphi(t-1)e_i(t)}{\lambda_i(t)r_i(t) + \varphi(t-1)^T P_i(t-1)\varphi(t-1)}\right\}\end{aligned} \tag{6}$$

$$\begin{aligned}P_i(t) &= C_i(t) + P_i(t-1)\lambda_i(t)^{-1}\\ &- \left\{\frac{P_i(t-1)\varphi(t-1)\varphi(t-1)^T P_i(t-1)}{\lambda_i(t)r_i(t) + \varphi(t-1)^T P_i(t-1)\varphi(t-1)}\right\}\lambda_i(t)^{-1}\end{aligned} \tag{7}$$

where $i \in [C, S]$ represent the Control model and Supervisor model respectively. $\lambda(t)$ is the forgetting factor. $F_{\Theta^\star}$ is an operator which projects the estimates onto the convex set $\Theta^\star$. We assume that

$\theta^\star \in \Theta^\star$ (Ydstie, 1989). $C_i(t)$ are matrices chosen to bound the covariance matrices so that

$$P_{i,min} \leq P_i(t) \leq P_{i,max} \tag{8}$$

This approach is called condition number monitoring. Lastly, the regression vector $\varphi(t)$ is composed of past output and input signals

$$\begin{aligned}\varphi(t)^T = [&-y(t), \dots, -y(t-n+1),\\ &u(t), \dots, u(t-m+1)]\end{aligned} \tag{9}$$

The prediction errors for each model are given by

$$e_i(t) = y(t) - \varphi(t-1)^T \hat{\theta}_i(t-1) \tag{10}$$

Define the signals $r_i(t)$ to be estimates of the variance of the disturbance sequences

$$r_i(t) = \sigma^2 r_i(t-1) + (1-\sigma^2) e_i(t)^2 \tag{11}$$

The time constant for the variance estimate is

$$\sigma = 1 - M_0^{-1} \tag{12}$$

with $M_0 > 1$ being the estimation memory length. The normalization sequences are

$$n_i(t) = \lambda_i(t) r_i(t) + \varphi(t-1)^T P_i(t-1) \varphi(t-1) \tag{13}$$

The switch condition is the last component of the algorithm. The switch determines when to update the control model. Model performance evaluation is based on the magnitudes of the corresponding model prediction errors.

$$Switch = \begin{cases} 1(ON) & \text{if } ||e_C(t)||^2 \geq \varepsilon ||e_S(t)||^2 \\ 0(OFF) & \text{else} \end{cases} \tag{14}$$

The parameter $\varepsilon > 0$ determines when the control model is updated. Typically the tolerance is set so that $\varepsilon \in [2,5]$. The smaller number leads to more frequent updates. The performance of the algorithm is adjusted by the parameter ε and with leakage. Bursting can be avoided by choosing $\varepsilon \geq 2$ and choosing some appropriate leakage. Algorithm performance degrades with increasing ε. The **Supervised Adaptive Control** algorithm, some performance related issues and the leakage is further discussed in the companion paper (Dozal-Mejorada *et al.*, 2006).

4. STABILITY AND TECHNICAL RESULTS

In this section we present stability and robustness results for the dual model supervised adaptive control algorithm. First we characterize the class of model mismatch and control design laws that are handled by the approach. We demonstrate convergence and boundedness of all signals by constructing a comparison signal $x(t)$ which measures all signals in the adaptive control problem. Let $x(t)$ be a comparison signal defined in the following way

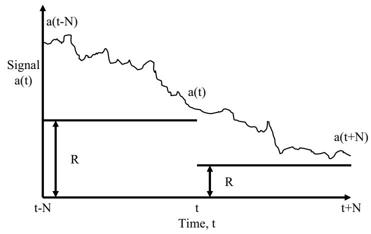

Fig. 3. Nonnegative comparison signal $a(t)$.

$$x(t) = \sigma^2 x(t-1) + (1-\sigma^2) e_C(t)^2 \tag{15}$$

with $x(0) < \infty$. Concerning the control design and un-modeled dynamics we make the following assumptions.

Assumption A1: Constants K_γ, K_φ, k_γ, and k_φ exist so that

$$||\varphi(t-1)||^2 \leq K_\varphi x(t-1) + k_\varphi \tag{16}$$

$$\gamma(t)^2 \leq K_\gamma x(t-1) + k_\gamma \tag{17}$$

The first inequality is satisfied provided we use a controller which stabilizes the estimated model at each sampling time. The second inequality states that the external perturbations are bounded and the un-modeled dynamics have bounded H_∞ norm. The number K_γ can be related to this norm (Kelly and Ydstie, 1997) and (Ydstie, 1992).

Main Result:

The supervised adaptive control algorithm introduced above is stable in the sense that $\limsup y(t)^2$ and $\limsup u(t)^2$ are bounded provided the the parameter K_γ is sufficiently small.

Proof: The proof consists of four independent steps. In the first step we introduce and review Ydstie's Switching Lemma (Ydstie, 1992). In the second step we relate the comparison and normalization sequences $x(t)$ and $r_C(t)$ to each other. The last two steps involve presenting Lyapunov stability results for the single model and dual model adaptive control cases.

4.1 The Switching Lemma

Introduce the indicator function $A(t) \in [0,1]$ and consider the nonnegative signal $a(t)$ updated so that

$$\begin{aligned}a(t+1) = &A(t)[g_1 a(t) + K_1]\\ &+[1-A(t)][g_2 a(t) + K_2]\end{aligned} \tag{18}$$

where

$$0 < g_2 < 1 < g_1 < \infty \quad \text{and} \quad K_1, K_2 \geq 0 \tag{19}$$

Figure 4 shows the signal architecture and Figure 3 depicts typical signal behavior for the given architecture. Define the auxiliary variables

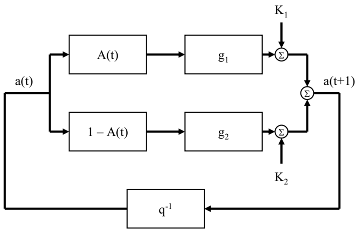

Fig. 4. Signal architecture for the comparison sequence.

$$\begin{aligned} u_1 &= \ln\left\{g_1 + K_1 R^{-1}\right\} \\ u_2 &= -\ln\left\{g_2 + K_2 R^{-1}\right\} \end{aligned} \quad (20)$$

Assume constants R and N exist so that

$$a(t-i) \geq R \quad \forall i = 0, 1, \ldots, N \quad (21)$$

implies that

$$\frac{1}{N}\sum_{i=t-N}^{t} A(i) = U < \frac{u_2}{u_1 + u_2} \quad (22)$$

Therefore, *Lemma 1:*

(1) $a(t+1) \leq \max\left\{R, e^{-\delta(N+t)}a(0)\right\} g_2^{-N}$ where $\delta > 0$
(2) $\frac{k_2}{1-g_2} \liminf a(t) \leq R$ and $\limsup a(t) \leq g_2^{-N} R$

Proof: Given in (Ydstie, 1992).

4.2 Relationship of the Comparison Signals

Here we present two results. The first one relates the normalization sequence $r_C(t)$ to $x(t)$. The second one relates the comparison sequence $x(t)$ and the normalization sequence $n_C(t)$ to the model prediction error $e_C(t)$ and the Switching Lemma.

Lemma 2:

$$K_{rxc,min} \leq \frac{r_C(t)}{x(t)} \leq K_{rxc,max} \quad (23)$$

Proof: From the algorithm we recall expression (11). Expand the definition of the normalization sequence r_C in time

$$r_C(t) = \sigma^{2t} r_C(0) + (1-\sigma^2)\sum_{i=1}^{t}\left\{\sigma^{2(t-i)} e_C(i)^2\right\} \quad (24)$$

Examining (24) and (15) we see that

$$x(t) = \sigma^{2t} x(0) + (1-\sigma^2)\sum_{i=1}^{t}\left\{\sigma^{2(t-i)} e_C(i)^2\right\} \quad (25)$$

Then

$$\frac{r_C(t)}{x(t)} \leq 1 + \frac{r_C(0)}{x(0)} \quad (26)$$

Q.E.D.

Let $A(t)$ be an indicator function so that

$$A(t) = 1 \text{ if } x(t) \geq \sigma x(t-1)$$

and $A(t) = 0$ zero otherwise.

Lemma 3:

$$\frac{1}{1+\sigma} A(t) \leq \frac{e_C(t)^2}{n_C(t)} \frac{n_C(t)}{x(t)} \quad (27)$$

Proof: Multiplying the $A(t)$ indicator function through expression (15)

$$A(t)\left\{x(t) = \sigma^2 x(t-1) + (1-\sigma^2) e_C(t)^2\right\} \quad (28)$$

Rearranging and multiplying top and bottom of the RHS by $n_C(t)$

$$A(t)\left[1 - \frac{\sigma^2 x(t-1)}{x(t)}\right] = A(t)(1-\sigma^2)\frac{e_C(t)^2}{n_C(t)} \frac{n_C(t)}{x(t)} \quad (29)$$

$A(t)$ was defined so that $x(t) \geq \sigma x(t-1)$ if $A(t) = 1$ we therefore get

$$\frac{(1-\sigma)}{(1-\sigma)(1+\sigma)} A(t) \leq \frac{e_C(t)^2}{n_C(t)} \frac{n_C(t)}{x(t)}$$

Q.E.D.

4.3 Single Model Lyapunov Stability

We now review some results for the single model case. Define the parameter error

$$\tilde{\theta}_C(t) = \theta_C^{\star} - \hat{\theta}_C(t) \quad (30)$$

and propose the Lyapunov candidate function

$$V_C(t) = \tilde{\theta}_C(t)^T P_C(t)^{-1} \tilde{\theta}_C(t) \quad (31)$$

Then, *Lemma 4:*

$$V_C(t) \leq \lambda_C(0) V_C(0) + \sum_{i=1}^{t} \lambda_C(i)\left\{\frac{\gamma(i)^2}{r_C(t)} - \frac{e_C(t)^2}{n_C(t)}\right\} \quad (32)$$

Proof: See Appendix.

Lemma 5: There exist constants so that the Switching Lemma applies to the supervisory adaptive control approach and

$$\frac{1}{1+\sigma}\sum_{t-N}^{t} A(i) \leq U \quad (33)$$

bounding $x(t)$ and effectively bounding all system signals.

Proof: See Appendix.

4.4 Dual Model Supervised Lyapunov Stability

The last step in the proof is to extend the single model results to the dual model approach of (Dozal-Mejorada *et al.*, 2006).

Lemma 6: Following Lemma 5, an extension for the dual model states that there exist some constants so that the Switching Lemma bounds $x(t)$.

$$\frac{1}{1+\sigma}\sum_{t-N}^{t} A(i) \le U_{dual} \qquad (34)$$

Hence, effectively bounding all system signals.

Proof: See Appendix.

5. CONCLUSIONS

Un-modeled dynamics and unmeasured disturbances may cause adaptive controllers to experience instabilities such as infinite drift or bursting. In this paper we prove stability of an algorithm which has been shown to address these instabilities using estimator supervision. The supervisor decides when to update the controller by examining the model prediction errors. An update is performed when there is new, valuable information present. In this paper we have shown that the algorithm tolerates small model/plant mismatches and bounded disturbances. In a companion paper we have presented simulation and experimental results that show that bursting and drift is effectively prevented.

6. APPENDIX

6.1 Proof of Lemma 4

Rewrite the parameter update law using the parameter error definition

$$\tilde{\theta}_C(t) = \tilde{\theta}_C(t-1) - P_C(t-1)\varphi(t-1)e_C(t)n_C(t)^{-1}$$

The Matrix Inversion Lemma (MIL) is given by

$$P_C(t)^{-1} = \lambda_C(t)P_C(t-1)^{-1} + \varphi(t-1)\varphi(t-1)^T r_C(t)^{-1}$$

Applying the MIL to the covariance matrix update and combining with the parameter update law

$$\tilde{\theta}_C(t) = \tilde{\theta}_C(t-1) - P_C(t)\varphi(t-1)e_C(t)r_C(t)^{-1}$$

We now substitute this expression twice onto the proposed Lyapunov function

$$V_C(t) = \left[\tilde{\theta}_C(t-1) - P_C(t)\varphi(t-1)e_C(t)r_C(t)^{-1}\right]^T \cdot P_C(t)^{-1}\left[\tilde{\theta}_C(t-1) - P_C(t)\varphi(t-1)e_C(t)r_C(t)^{-1}\right]$$

Expanding

$$\begin{aligned} V_C(t) = {} & \lambda_C(t)V_C(t-1) + (\tilde{\theta}_C(t-1)^T\varphi(t-1))^2 \\ & -2\varphi(t-1)^T\tilde{\theta}_C(t-1)e_C(t)r_C(t)^{-1} \\ & +\varphi(t-1)^T P_C(t)\varphi(t-1)e_C(t)^2 r_C(t)^{-2} \end{aligned}$$

Add and subtract $e_C(t)^2 r_C(t)^{-1}$ and combine like terms

$$\begin{aligned} V_C(t) = {} & \lambda_C(t)V_C(t-1) + (\tilde{\theta}_C(t-1)^T\varphi(t-1) \\ & -e_C(t))^2 r_C(t)^{-1} \\ & -\left[1 - \frac{\varphi(t-1)^T P_C(t)\varphi(t-1)}{n_C(t)}\right]\frac{e_C(t)^2}{r_C(t)} \end{aligned}$$

Then

$$V_C(t) \le \lambda_C(0)V_C(0) + \sum_{i=1}^{t}\lambda_C(i)\left\{\frac{\gamma(i)^2}{r_C(t)} - \frac{e_C(t)^2}{n_C(t)}\right\}$$

valid for $\lambda_{C,min} \le \lambda_C(t) \le 1$. Furthermore, since the covariance matrix update is really given by $P_C(t) + C_C(t)$ as long as $C_C(t) \ge 0$ the result holds. □

6.2 Proof of Lemma 5

Recall the relationship between the Switching Lemma, $e_C(t)$ and $x(t)$

$$\frac{1}{1+\sigma}A(t) \le \frac{e_C(t)^2}{n_C(t)}\frac{n_C(t)}{x(t)}$$

Examine the last term of the expression and substitute the definition of $n_C(t)$

$$\frac{n_C(t)}{x(t)} = \lambda_C(t)\frac{r_C(t)}{x(t)} + \frac{\varphi(t-1)^T P_C(t-1)\varphi(t-1)}{x(t)}$$

Using Lemma 2, Assumption A1, and the facts that $\lambda_{C,min} \le \lambda_C(t) \le 1$ and $P_{C,min} \le P_C(t) \le P_{C,max}$

$$\frac{n_C(t)}{x(t)} \le K_{rxc,max} + K_\varphi\sigma^{-1}P_{C,max} + \frac{k_\varphi P_{C,max}}{x(t)}$$

Since we assume $\exists(N,R)$ so that $x(t) \ge R\forall t \in [t-N,t]$ then

$$\frac{n_C(t)}{x(t)} \le C_1$$

with

$$C_1 = K_{rxc,max} + K_\varphi\sigma^{-1}P_{C,max} + k_\varphi P_{C,max}R^{-1}$$

The Switching condition becomes

$$\frac{1}{1+\sigma}A(t) \le C_1\frac{e_C(t)^2}{n_C(t)}$$

Multiplying top and bottom by $\lambda_C(t)$

$$\frac{1}{1+\sigma}A(t) \le \frac{C_1}{\lambda_{C,min}}\lambda_C(t)\frac{e_C(t)^2}{n_C(t)}$$

Substituting the result of Lemma 4

$$\frac{1}{1+\sigma}A(t) \le \frac{C_1}{\lambda_{C,min}}\left[\lambda_C(t)V_C(t-1) - V_C(t) + \frac{\gamma(t)^2}{r_C(t)}\right]$$

Using Assumption A2 the last term can be rewritten so that

$$\frac{\gamma(t)^2}{r_C(t)} \le \frac{K_\gamma x(t-1) + k_\gamma}{r_C(t)}$$

Using Lemma 2 and $x(t) \geq R$

$$\frac{\gamma(t)^2}{r_C(t)} \leq K_{rxc,min}K_\gamma + k_\gamma(K_{rxc,min}R)^{-1}$$

Then the overall expression becomes

$$\frac{1}{1+\sigma}A(t) \leq \frac{C_1}{\lambda_{C,min}}\left[\,\lambda_C(t)V_C(t-1) - V_C(t) + K_{rxc,min}K_\gamma + k_\gamma(K_{rxc,min}R)^{-1}\,\right]$$

Summing from $t-N$ to t

$$\frac{1}{1+\sigma}\sum_{i=t-N}^{t} A(i) \leq \frac{C_1}{\lambda_{C,min}}\left[V_C(t-N) + K_{rxc,min}K_\gamma + k_\gamma(K_{rxc,min}R)^{-1}\,\right]$$

We arrive at our final result

$$\frac{1}{1+\sigma}\sum_{i=t-N}^{t} A(i) \leq U$$

where

$$U = \frac{C_1}{\lambda_{C,min}}\left[\,V_C(t-N) + K_{rxc,min}K_\gamma + k_\gamma(K_{rxc,min}R)^{-1}\,\right]$$

which bounds the comparison signal $x(t)$ as long as Assumption A1 is satisfied. □

6.3 Proof of Lemma 6

The stability proof follows that of the single model case with some very specific additions. Here we introduce these modifications and provide an outline for the proof. First define a new $[0,1]$ indicator function so that

$$\Delta(t) = \begin{cases} 1 & e_C(t)^2 \geq \varepsilon e_S(t)^2 \\ 0 & e_C(t)^2 < \varepsilon e_S(t)^2 \end{cases}$$

The next step is to define a new comparison signal which incorporates both prediction errors. Add and subtract $\Delta(t)e_C(t)^2$ to the original comparison signal

$$x(t) = \sigma^2 x(t-1) + (1-\sigma^2)[\Delta(t)e_C(t)^2 + (1-\Delta(t))e_C(t)^2]$$

Since $\Delta(t) = 1$ only if $e_C(t)^2 \geq \varepsilon e_S(t)^2$ then $x(t) = \sigma^2 x(t-1) + (1-\sigma^2)[\Delta(t)e_C(t)^2 + (1-\Delta(t))e_C(t)^2]$

$$x(t) \leq \sigma^2 x(t-1) + (1-\sigma^2)[\Delta(t)e_C(t)^2 + (1-\Delta(t))\varepsilon e_S(t)^2]$$

The next step is to modify the Lyapunov candidate function to reflect the sporadic nature of the update. Let

$$V_C(t) = [1-\Delta(t)]V_C(t-1) + \Delta(t)\left[\lambda_C(t)V_C(t-1) - \lambda_C(t)\frac{e_C(t)^2}{n_C(t)} + \frac{\gamma(t)^2}{r_C(t)}\,\right]$$

The rest of the proof follows exactly the single model with the implementation of the appropriate aforementioned definitions and invoking the Switching Lemma

$$\frac{1}{1+\sigma}\sum_{i=t-N}^{t} A(i) \leq \Delta(t)\frac{e_C(t)^2}{x(t)} + (1-\Delta(t))\varepsilon\frac{e_S(t)^2}{x(t)}$$

□

REFERENCES

Dozal-Mejorada, E.J., P. Thakker and B.E. Ydstie (2006). Supervised adaptive predictive control using dual models. *DYCOPS 2006-Submitted.*

Egardt, B. (1979). *Stability of Adaptive Controllers.* Springer-Verlag. New York.

Goodwin, G.C. and K.S. Sin (1984). *Adaptive Filtering, Prediction and Control.* Prentice Hall. Englewood Cliffs, New Jersey.

Hill, J.H. and B.E. Ydstie (2004). Adaptive control with selective memory. *Int. J. Adapt. Control Signal Process.* **18**, 571–587.

Hovd, M. and R.R. Bitmead (2006). Directional leakage and parameter drift. *Int. J. Adapt. Control Signal Process.* **20**, 27–39.

Ioannou, P.A. and J. Sun (1996). *Robust Adaptive Control.* Prentice Hall.

Ioannou, P.A. and P.V. Kokotovic (1983). *Adaptive Systems with Reduced Models.* Springer. Berlin, Germany.

Kelly, J.H. and B.E. Ydstie (1997). Adaptive h_∞ control with application to systems with structural flexibility. *IEEE Transactions on Automatic Control* **42**(10), 1358–1369.

Mareels, I. and J.W. Polderman (1996). *Adaptive Systems, An Introduction.* Birkhauser. Boston.

Åström, K.J. and B. Wittenmark (1995). *Adaptive Control.* first edition ed.. Addison-Wesley Publishing Company, Inc. Reading, Massachusetts.

Middleton, R.H., G.C. Goodwin, D.J. Hill and D.Q. Mayne (1988). Design isues in adaptive control. *IEEE Transactions on Automatic Control* **33**(1), 50–58.

Peterson, B.B. and K.S. Narendra (1982). Bounded error adaptive control. *IEEE Transactions on Automatic Control* **27**(6), 1161–1168.

Ydstie, B.E. (1989). Stability of discrete model reference adaptive control-revisited. *Systems and Control Letters* pp. 429–438.

Ydstie, B.E. (1992). Transient performance and robustnes of direct adaptive control. *IEEE Transactions on Automatic Control* **37**(8), 1091–1105.