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PASSIVITY BASED CONTROL OF PROCESS NETWORKS

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Abstract: In this paper, an approach to control design for large process system networks is developed using the passivity theory. The process network considered includes processes which obey the laws of thermodynamics and have dynamics constrained by balance equations for mass, energy, charge and components. The flow amongst the different processes in the network depend on potential differences. Sufficient conditions under which a process network is passive so that it can be controlled by passive controllers.

Keywords: process networks; process systems; passivity

1. INTRODUCTION

Control problems with dynamics distributed in space and time can often be modeled using networks of interconnected, simpler processes. Examples include bio-chemical processes (Hatzimanikatis et al. (1996)), reaction networks (Fishtik et al. (2004)) and plant-wide control (Kumar and Daoutidis (2002)). In each of these examples the integrated system consists of sub-processes which interact through information and material flow and the dynamics the total process exhibits can be far more complex than that of the individual sub-components.

Network descriptions have several advantages (Oster et al. (1971); Gilles (1998)). Networks provide a graphical representation which makes it easy to visualize interconnections and build models. The topology often defines the character of the network and graphical representations can often provide physical insights that are difficult to extract from algebraic representations. One drawback is that the size and complexity of control design increases rapidly as the size of the network increases. There is therefore a need to develop scalable approaches for stability analysis and control system design.

In this paper, based on Jillson and Ydstie (2005) we develop an approach to control system design of networks which addresses scalability by using ideas from the passivity theory of nonlinear control. Passivity, like Lyapunov stability theory, uses a storage function which can be interpreted as the energy within the system. However, unlike Lyapunov theory, it provides a direct way to analyze input-output stability of interconnected systems and methods for decomposition and decentralization. Passivity was originally developed to study stability of complex electrical circuits (Desoer and Kuh (1969)), and has recently been extended to modeling and control of chemical processes and networks, e.g., Ydstie (2002); Bao (1998); Jillson and Ydstie (2005); Hangos et al. (1999).

The main contribution of the current paper is that we develop a systematic method to design passivity based controllers for large scale process networks. A process network is a collection of processes which obey the laws of thermodynamics and have dynamics constrained by balance equations for mass, energy, charge and components. The flows amongst the different processes in the network depend on potential differences. We show the sufficient conditions under which a process network is passive and stable so that it can be controlled by passive controllers. The application of the theory is illustrated in a simulation of a network of reaction diffusion equations.

2. PROCESS NETWORKS

In this paper, we consider the network of process systems. The topology of the network is represented by the graph $G = (P, F, T)$. The set of n_p vertices or nodes (P) specifies the locations of the elementary processes. The set of n_f edges (F) specifies connections amongst the vertices. And the set of n_t terminals (T) specifies how the network is connected with the environment or other independent networks.

At each node, the state, z is described by a set of extensive variables. For example, for a thermodynamic system with n_c components, the state consists of the internal energy, volume, and component masses:

$$z = [U, V, M_1, \ldots, M_{n_c}]^T \tag{1}$$

There is also a corresponding potential at each node, which is related to the state and a scalar entropy function, S by the following partial derivative:

$$\frac{\partial S}{\partial z} = w^T \tag{2}$$

For the states given in (1), the corresponding potentials are related to the temperature, pressure, and chemical potential of the node:

$$w = \left[\frac{1}{T}, \frac{P}{T}, \frac{\mu_1}{T}, \ldots \frac{\mu_{n_c}}{T}\right]^T \tag{3}$$

The potential difference, $X_{ij} = w_j - w_i$, between connected nodes acts as a driving force for flow, and is continuous around any closed loop.

The states must be conserved at each node, and the following differential equation holds:

$$\frac{dz}{dt} = p + \phi \tag{4}$$

where p is the net production (e.g. due to chemical reaction) and ϕ is the net flow of material, energy, etc. into the node. The production term p is a function of the states of the node and can be a complex non-linear expression.

We are concerned with two types of flow: convective and diffusive flows. Convective flow results from a bulk flow between nodes, and has the same composition as the node it exits, i.e. the convective flow from node i to node j:

$$f_{ij}^c = \hat{z}_i \dot{m}_{ij} \tag{5}$$

where $\hat{z}_i$ is a vector of state variables per total mass (specific quantity), and $\dot{m}_{ij}$ is the bulk mass flow rate between node i and node j. Diffusive flow is the flow of material, energy, etc. due to gradients in potentials (e.g. heat conduction between two units with different temperatures) as given by the following expression for linear diffusion:

$$f_{ij}^d = LX_{ij} \tag{6}$$

where L is a constant negative definite matrix.

A process system network can be represented by a collection of models which describes the dynamics of each node in the network:

$$\frac{dz_i}{dt} = p(z_i) - \sum_{j=1}^{m_i} f_{ij}^d - \sum_{j=1}^{n_i} f_{ij}^c \tag{7}$$

$i = 1, \ldots, n_p$, where m_i and n_i are number of diffusive and convective flows from node i respectively. It is assumed a physical outlet flow is positive and inlet flow is negative.

3. PASSIVITY OF PROCESS SYSTEM NETWORK

A process system with input u, output y and state z is said to be strictly state passive if there exists a positive semi-definite storage function $A(z)$ such that

$$A(z(t)) - A(z(0)) \leq \int_0^t y^T u ds - \int_0^t V(z) ds \tag{8}$$

for all u in the input space, and z in the state space, where $V(z)$ is a positive definite function. A nice property of a strictly state passive system is that it can be stabilized by *any* passive controllers, including the commonly used PID controllers. In this section, we study under what conditions the entire network is passive so that a passive control system can be implemented.

Define the storage function A_i for the i-th node as in Jillson and Ydstie (2005):

$$\begin{aligned} A_i(z_i) &= w_i^{*T} z_i - S_i(z_i) \\ &= (w_i^* - w_i)^T z_i \end{aligned} \tag{9}$$

where w_i^* is the reference of w_i and S_i is the entropy. Due to the concavity of S_i, we have $A_i > 0\ \forall w_i \neq w_i^*$ and $A_i = 0$ if $w_i = w_i^*$. The storage function A of the entire process network is:

$$A(z) = \sum_{i=1}^{n_p} A_i(z_i) \tag{10}$$

where $z = \left[z_1^T, z_2^T, \ldots z_{n_p}^T\right]^T$ is the vector of all states of the network. We have,

$$\begin{aligned}\frac{dA(z)}{dt} &= \sum_{i=1}^{n_p} \frac{dA_i(z_i)}{dt} \\ &= \sum_{i=1}^{n_p} \left[w_i^{*T}\frac{dz_i}{dt} - \frac{\partial S_i(z_i)}{\partial z_i}\frac{dz_i}{dt}\right] \\ &= \sum_{i=1}^{n_p} \left[w_i^{*T}\frac{dz_i}{dt} - w_i^T\frac{dz_i}{dt}\right] \qquad (11)\end{aligned}$$

Define deviation variables $\bar{w}_i = w_i - w_i^*$, $\bar{z}_i = z_i - z_i^*$, $\bar{f}_{ij} = f_{ij} - f_{ij}^*$ and $\bar{p}_i(z_i) = p_i(z_i) - p_i(z_i^*)$, where w_i^*, z_i^* and f_{ij}^* are of the reference point at steady state. Thus:

$$\begin{aligned}\frac{dA}{dt} &= \sum_{i=1}^{n_p}\left[-\bar{w}_i^T\frac{d\bar{z}_i}{dt}\right] \\ &= \sum_{i=1}^{n_p}\Bigg[-\bar{w}_i^T\bar{p}_i(z_i) + \bar{w}_i^T\sum_{j=1,j\neq i}^{n_p}\bar{f}_{ij}^d \\ &\quad + \bar{w}_i^T\sum_{j=1}^{n_p}\bar{f}_{ij}^c + \bar{w}_i^T\sum_{j=1}^{n_t}\bar{f}_{ij}^t\Bigg] \\ &= -\sum_{i=1}^{n_p}\bar{w}_i^T\bar{p}(z_i) + \sum_{i=1}^{n_p}\bar{w}_i^T\left(\sum_{j=1,j\neq i}^{n_p}L_{ij}(\bar{w}_i - \bar{w}_j)\right) \\ &\quad + \sum_{i=1}^{n_p}\bar{w}_i^T\sum_{j=1}^{n_p}\bar{f}_{ij}^c + \sum_{i=1}^{n_p}\bar{w}_i^T\sum_{j=1}^{n_t}\bar{f}_{ij}^t \qquad (12)\end{aligned}$$

where $\bar{f}_{ij}^t$ is the j-th terminal flow from i-th node. Denote n_f^d as the total number of diffusive flows and the k-th diffusive flow is from node i to j. Then $\bar{X}_k = \bar{w}_i - \bar{w}_j$. Define N_O as the set of nodes which a convective flow exits, N_I as the set of nodes which a convective flow enters. We have:

$$\begin{aligned}\frac{dA}{dt} &= -\sum_{i=1}^{n_p}\bar{w}_i^T\bar{p}(z_i) + \sum_{k=1}^{n_f^d}\bar{X}_k^T L_k\bar{X}_k \\ &\quad + \sum_{i\in N_O}\sum_{j\in N_I}(\bar{w}_i - \bar{w}_j)^T\bar{f}_{ij}^c + \sum_{k=1}^{n_t}\bar{w}_k^T\bar{f}_k^{tO} \\ &\quad - \sum_{k=1}^{n_t}\sum_{j=1}^{n_{I,k}^t}\bar{w}_k^T\bar{f}_{kj}^{tI} \\ &= -\sum_{i=1}^{n_p}\bar{w}_i^T\bar{p}(z_i) + \sum_{k=1}^{n_f^d}\bar{X}_k^T L_k\bar{X}_k \\ &\quad + \sum_{i\in N_O}\sum_{j\in N_I}(\bar{w}_i - \bar{w}_j)^T\left(\frac{z_i}{M_i}\dot{m}_{ij} - \frac{z_i^*}{M_i^*}\dot{m}_{ij}^*\right) \\ &\quad + \sum_{k=1}^{n_t}\bar{w}_k^T\left(\frac{z_k}{M_k}\dot{m}_k^{tO} - \frac{z_k^*}{M_k^*}\dot{m}_k^{*tO}\right) \\ &\quad - \sum_{k=1}^{n_t}\sum_{j=1}^{n_{I,k}^t}\bar{w}_k^T c_{kj}\overline{\dot{m}}_{kj}^{tI} \qquad (13)\end{aligned}$$

where $n_{I,k}^t$ denotes the number of inlet flows from the environment to k-th node, vectors $\bar{f}_k^{tO}$ and $\bar{f}_{kj}^{tI}$ denote the outlet and inlet component flows of node k to and from the environment, $\dot{m}_k^{tO}$ denotes the bulk outlet flow rate including all components from k-th node to environment (a terminal flow), $\dot{m}_{kj}^{tI}$ denotes the j-th inlet (bulk) flow to the k-th node from the environment, $\overline{\dot{m}}_{kj}^{tI} = \dot{m}_{kj}^{tI} - \dot{m}_{kj}^{tI*}$ is the deviation variable, and c_{kj} is a constant vector of quantity of components per unit mass or volume in the j-th flow that enters k-th node. Here we assume that *the total inventory of each node is controlled* by manipulating its total outlet flow. For node i:

$$\dot{m}_i = \sum_j \dot{m}_{ij} = k_i M_i \qquad (14)$$

where $k_i > 0$ is the constant gain of the proportional only controller applied to i-th node. Denote α_{ij} as the ratio of outlet flow rate from node i to node j to the total outlet flow rate from node i. Obviously $\alpha_{ij} \geq 0$ and $\sum_j \alpha_{ij} = 1$. Therefore the actual and reference flow rates from node i to node j under the inventory control are:

$$\dot{m}_{ij} = \alpha_{ij}k_i M_i,\ \dot{m}_{ij}^* = \alpha_{ij}k_i M_i^* \qquad (15)$$

Therefore,

$$\begin{aligned}\frac{dA}{dt} &= \sum_{i=1}^{n_p}\bar{w}_i^T\bar{p}(z_i) + \sum_{j=1}^{n_f^d}\bar{X}_j^T L_j\bar{X}_j + \\ &\quad \sum_{i\in N_O}\sum_{j\in N_I}(\bar{w}_i - \bar{w}_j)^T\bar{z}_i\alpha_{ij}k_i \\ &\quad + \sum_{k=1}^{n_t}\bar{w}_k^T\left(\frac{z_k}{M_k}\dot{m}_k^{tO} - \frac{z_k^*}{M_k^*}\dot{m}_k^{*tO}\right) \\ &\quad - \sum_{k=1}^{n_t}\sum_{j=1}^{n_{I,k}^t}\bar{w}_k^T c_{kj}\overline{\dot{m}}_{kj}^{tI} \qquad (16)\end{aligned}$$

Since $\sum_j \alpha_{ij} = 1$, we have,

$$\begin{aligned}&\sum_{i\in N_O}\sum_{j\in N_I}(\bar{w}_i - \bar{w}_j)^T\bar{z}_i\alpha_{ij}k_i \\ &= \sum_{i\in N_O}\bar{w}_i^T\bar{z}_i k_i - \sum_{i\in N_O}\sum_{j\in N_I}\bar{w}_j^T\bar{z}_i\alpha_{ij}k_i \qquad (17)\end{aligned}$$

The relationship between the intensive variable w_i and the extensive variable z_i is:

$$w_i = Q_i z_i \qquad (18)$$

where Q_i may not be constant, but is always negative definite:

$$Q_i < 0 \qquad (19)$$

Therefore,

$$\frac{dA}{dt} = \underbrace{\sum_{i=1}^{n_p} \bar{w}_i^T \bar{p}_i(z_i)}_{\text{net production}} + \underbrace{\sum_{j=1}^{n_f^d} \bar{X}_j^T L_j \bar{X}_j}_{\text{diffusive flows}}$$

$$+ \underbrace{\sum_{i \in n_O} k_i \bar{z}_i^T Q_i^T \bar{z}_i - \sum_{i \in n_O} \sum_{j \in n_I} \bar{w}_j^T \bar{z}_i \alpha_{ij} k_i}_{\text{convective flows}}$$

$$+ \underbrace{\sum_{k=1}^{n_t} \bar{w}_k^T \left(\frac{z_k}{M_k} \dot{m}_k^{tO} - \frac{z_k^*}{M_k^*} \dot{m}_k^{*tO} \right)}_{\text{terminal outlet flow}}$$

$$- \underbrace{\sum_{k=1}^{n_t} \sum_{j=1}^{n_{I,k}^t} \bar{w}_k^T c_{kj} \overline{\dot{m}}_{kj}^{tI}}_{\text{terminal inlet flow}} \quad (20)$$

For given manipulated variables $u(t)$ and controlled variables $y(t)$, the strict state passivity condition requires that the right hand side of the above equation to be less than $y^T(t) u(t) - V(z)$, where $V(z) > 0$. Now let us study each term in the right hand side of the above equation:

- **Net production**: If

$$-\bar{w}_i^T \bar{p}_i(z_i) < 0, \ \forall i = 1, \ldots, n_p \quad (21)$$

then we can choose

$$V(z) = \sum_{\iota=1}^{n_p} \bar{w}_i^T \bar{p}_i(z_i) > 0 \quad (22)$$

In this case, the production term is said to be dissipative. If $p_i(z_i)$ is linear, e.g., $p_i(z_i) = P_i z_i$, where P_i is a constant matrix. Then we have $\bar{p}_i(z_i) = P_i \bar{z}_i$ and

$$-\bar{w}_i^T \bar{p}_i(z_i) = -\bar{z}_i^T Q_i P_i \bar{z}_i \quad (23)$$

As $Q_i < 0$, (21) is equivalent to $\mathrm{Re}[\lambda(P_i)] < 0$ for all nodes. That is, each matrix P_i is stable.

- **Diffusive flows**:
Clearly,

$$\sum_{j=1}^{n_f^d} \bar{X}_j^T L_j \bar{X}_j < 0 \quad (24)$$

for any $\bar{X}_j \neq 0$.

- **Convective flows**:
Define $\tilde{z} = [\bar{z}_1^T, \ldots, \bar{z}_{n_p}^T]^T$

$$\sum_{i \in n_O} k_i \bar{z}_i^T Q_i^T \bar{z}_i - \sum_{i \in n_O} \sum_{j \in n_I} \alpha_{ij} \bar{z}_j^T Q_j^T \bar{z}_i k_i$$

$$= \tilde{z}^T \begin{bmatrix} Q_1 k_1 & \cdots & -Q_1 \alpha_{n_p,1} k_{n_p} \\ -Q_2 \alpha_{12} k_1 & & -Q_2 \alpha_{n_p,2} k_{n_p} \\ \vdots & \ddots & \vdots \\ -Q_{n_p} \alpha_{1,n_p} k_1 & \cdots & Q_{n_p} \alpha_{n_p,n_p} k_{n_p} \end{bmatrix} \tilde{z}$$

$$= \tilde{z}^T \Omega \Theta \tilde{z} \quad (25)$$

Where

$$\Omega = \begin{bmatrix} -Q_1 k_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & -Q_{n_p} k_{n_p} \end{bmatrix} \quad (26)$$

and

$$\Theta = \begin{bmatrix} -I & \cdots & \alpha_{n_p,1} I \\ \vdots & \ddots & \vdots \\ \alpha_{1,n_p} I & \cdots & -I \end{bmatrix} \quad (27)$$

Note that matrix Θ is column conservation matrix (i.e., $\sum_{j,j \neq i} |\alpha_{ij}| \leq |\alpha_{ii}|$ and $\alpha_{ii} < 0$ and $\alpha_{ij} > 0$ for $i \neq j$) (as in Hangos and Peni (2003)). Therefore $\Theta \leq 0$ and all its eigenvalues are real. It can be proved that

$$\Omega \Theta \leq 0 \quad (28)$$

for any $Q_i < 0$. Therefore,

$$\sum_{i \in n_O} k_i \bar{z}_i^T Q_i^T \bar{z}_i - \sum_{i \in n_O} \sum_{j \in n_I} \alpha_{ij} \bar{w}_j^T \bar{z}_i k_i \leq 0 \quad (29)$$

- **Terminal outlet flow:**
Assume $\dot{m}_k^{tO} = \alpha_{k0} M_k, \dot{m}_k^{*tO} = \alpha_{k0}^* M_k^*$, where α_{k0} is the ratio of the outlet flowrate of k-th node to the environment (often the product flow) to the total inventory in k-th node and α_{k0}^* is the value of α_{k0} at a reference point. α_{k0} can be understood as a relative outlet flow rate. We have:

$$\begin{aligned}
&\sum_{k=1}^{n_t} \bar{w}_k^T \bar{f}_{Tk}^c \\
&= \sum_{k=1}^{n_t} \bar{w}_k^T \left(\frac{z_k}{M_k} \dot{m}_k^{tO} - \frac{z_k^*}{M_k^*} \dot{m}_k^{*tO} \right) \\
&= \sum_{k=1}^{n_t} \bar{w}_k^T (z_k \alpha_{k0} - z_k^* \alpha_{k0}^*) \\
&= \sum_{k=1}^{n_t} \left[\bar{w}_k^T ((\bar{z}_k + z_k^*)(\bar{\alpha}_{k0} + \alpha_{k0}^*) - z_k^* \alpha_{k0}^*) \right] \\
&= \sum_{k=1}^{n_t} \left[\bar{w}_k^T (\bar{z}_k \bar{\alpha}_{k0} + \bar{z}_k \alpha_{k0}^* + z_k^* \bar{\alpha}_{k0}) \right] \\
&= \sum_{k=1}^{n_t} \bar{w}_k^T z_k \bar{\alpha}_{k0} + \sum_{k=1}^{n_t} \bar{w}_k^T \bar{z}_k \alpha_{k0}^* \\
&= \sum_{k=1}^{n_t} \bar{w}_k^T Q_k^{-1} \bar{w}_k \bar{\alpha}_{k0} + \sum_{k=1}^{n_t} \bar{z}_k^T Q_k \bar{z}_k \alpha_{k0}^*
\end{aligned} \quad (30)$$

Because $\alpha_{k0}^* > 0$, we have

$$\sum_{k=1}^{n_t} \bar{z}_k^T Q_k \bar{z}_k \alpha_{k0}^* < 0. \quad (31)$$

From the above results, we can derive the following sufficient condition for passivity of the entire network modeled in (7):

Theorem 1. The process system network as given in (7) under total inventory control of each node is strictly state passive if the controlled variables are

$$\left\{\bar{w}_k^T Q_k^{-1}\bar{w}_k, \bar{w}_k^T c_{kj}\right\},\ k=1\ldots n_t,\quad j=1\ldots n_{I,k}^t$$

and the manipulated variables are

$$\left\{\bar{\alpha}_{k0}, \overline{\dot{m}}_{kj}^{tI}\right\},\ k=1\ldots n_t,\quad j=1\ldots n_{O,k}^t$$

and

$$-\sum_{i=1}^{n_p}\bar{w}_i^T\bar{p}_i(z_i)<0 \tag{32}$$

PROOF. This theorem can be proved immediately from (20), (22), (24), (29), (31) and (8).

4. PASSIVITY BASED CONTROL

In Hangos and Peni (2003), it was shown that under inventory control of each unit, the entire network is stable if each node is dissipative. In this work, we show that the network is also passive if manipulated variables $u(t)$ and controlled variables $y(t)$ are properly chosen. According to the above theorem, if the production of each node is dissipative, the product flow rates and the feed flow rates to the nodes linked to terminals (such as flow rate of reactants) are manipulated and the intensive potential w_k of the terminal nodes are measured, then a passive controller can be employed to stabilize the process.

The output of the process from the point of view of the controller forms a vector containing $\bar{w}_k^T Q_k^{-1}\bar{w}_k$ and $\bar{w}_k^T c_{kj}$ for different k and j. Both terms can be calculated from w_k and the reference w_k^*. c_{kj} is a constant vector, denotes the specific quantities of different components in the feed flow (for example, concentration of reactants). If one flow carries more than one component, then what is fed into the controller is the scalar quantities, rather than vector $\bar{w}_k$. Therefore, in this case, the number of terminal flows measured and manipulated should equal the number of variables to be controlled. If multi-component flows are involved, separation units should be used so that some terminal flows only carry one component, which allows effective control of quality of one or more components.

The purpose of the proposed method is to control the potentials w_i. Under the feedback control of a passive controller, the storage function of the entire process network will approach zero, which implies $w_i \rightarrow w_i^*$.

5. ILLUSTRATIVE EXAMPLE

For this example, we only consider the state of each node as the masses of three components, A, B, C : $z=\left[\,M_A\ M_B\ M_C\,\right]^T$ with corresponding

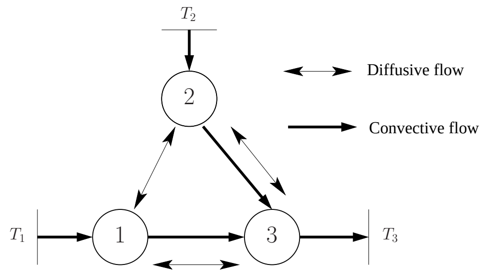

Fig. 1. Flowsheet of three node reactor network

potentials $w=\left[\,-c_A\ -c_B\ -c_C\,\right]^T$ where $c_\bullet$ is the concentration (g/L). This is proportional to the chemical potential and also to the mass of each species, assuming fixed volumes.

At each node:

$$\frac{dz_i}{dt}=p(z_i)+f_{\text{dispersion}}+f_{\text{convective}} \tag{33}$$

Reaction is present in each node, let $A\rightleftharpoons B\rightleftharpoons C$, with linear reaction kinetics (assume no temperature effects). At each node,

$$p_i(z_i)=P_i z_i=\begin{bmatrix}-k_{1f} & k_{1r} & 0\\ k_{1f} & -k_{1r}-k_{2f} & k_{2r}\\ 0 & k_{2f} & -k_{2r}\end{bmatrix} z_i \tag{34}$$

We assume $k_{1f}=5$, $k_{1r}=4$, $k_{2f}=3$, and $k_{2r}=2$. For the whole system, these matrices can be combined to form one large block diagonal P matrix such that

$$Pz=\begin{bmatrix}P_1 & 0 & 0\\ 0 & P_2 & 0\\ 0 & 0 & P_3\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} \tag{35}$$

where the subscripts refer to the production due to reaction at each node. Since P is negative definite, the reaction term in $\frac{dA}{dt}$ is also negative definite.

We assume that diffusion only occurs between the three nodes, and then convection for five of the flows (going from terminal 1 to node 1, from terminal 2 to node 2, from node 1 to node 3, node 2 to node 3, and from node 3 to terminal 3), as depicted in Figure 1. We have diffusive flows

$$f_{d,ij}=L_{ij}(w_i-w_j) \tag{36}$$

and convective flows

$$f_{c,ij}=\hat{z}_i\dot{m}_{ij} \tag{37}$$

where $\hat{z}_i=\frac{z_i}{M_{Ti}}$ represents the vector of mole fractions at node i. For the example model,

$$L_{12}=\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 4.5\end{bmatrix} \tag{38}$$

and

$$L_{13}=L_{23}=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix} \tag{39}$$

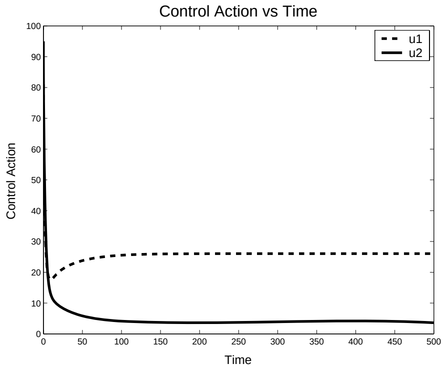

Fig. 2. Control action on the mass flow from the terminal into node 1 (u_1) and into node 2 (u_2)

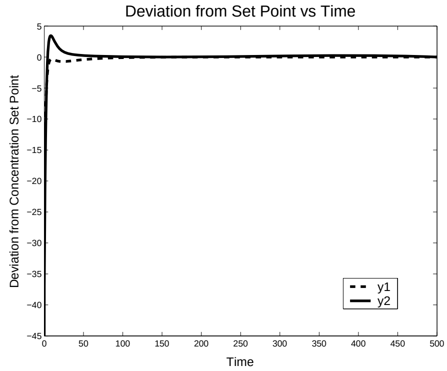

Fig. 3. Deviation to the set point at node 1 (y_1) and at node 2 (y_2)

We note that we have 5 degrees of freedom, one for each convective flow in the network. We then introduce controls on the outgoing convective flows of each node ($f^c_{13}, f^c_{23}, f^{tO}_3$) as just proportional to the total mass in their respective nodes, e.g.: $f^c_{13} = K_1 M_1$. In the example, each K_i term is equal to 0.25. Assume the feed going into node 1 is pure A and the feed going into node 2 is pure C. In this example, we would like to produce B, and we add C in order to limit the 2nd reaction. Then we can control two variables, the concentrations of A and C in nodes 1 and 2, respectively, by manipulating the bulk flows entering the nodes from their respective terminals. That is,

$$\begin{aligned} y_1 &= \hat{z}_{T1}\bar{w}_1 = \bar{w}_{1,A} \\ y_2 &= \hat{z}_{T2}\bar{w}_2 = \bar{w}_{2,C} \\ u_1 &= \dot{m}^{tI}_1 \\ u_2 &= \dot{m}^{tI}_2 \end{aligned} \tag{40}$$

According to Theorem 1, the process network is strictly state passive if the above manipulated and controlled variables are chosen. The process network can be controlled by any passive controllers. Here we illustrate the passivity condition by showing the simulation results of a PI controller, as given in Figures 2 and 3, for controller parameters of $K_{p1} = -3$, $K_{p2} = -2$, $\tau_{i1} = -4$, and $\tau_{i2} = -3$. From which it can be seen that the potentials $\bar{w}_{1,A}$ and $\bar{w}_{2,C}$ are effectively controlled. While only a very simple case study is presented here due to space limit, it should be noted that the proposed approach can be used to control very complex process networks.

6. CONCLUSION

In this paper, an approach to control of process system networks is developed based on passivity. By defining an entropy based storage function for each node and introducing total inventory control of each process unit, it has been shown that the process system network is passive for certain controlled and manipulated variables, provided the net production of each process unit is dissipative. This result is particularly useful in controlling large process system networks where control design based on a single complex model for the entire network is impossible.

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