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BOUNDED POSITIVE ADAPTIVE CONTROL FOR COUNTERFLOW HEAT EXCHANGERS

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Abstract: In this work, we propose an outlet temperature control scheme for counterflow heat exchangers, that takes into account and actually exploits the analytical-stability features inherent to the open-loop dynamics. As a result, outlet temperature regulation is achieved through a simple adaptive controller which does not need to feedback the whole state vector and does not depend on the exact value of the system parameters. Furthermore, positivity and boundedness (non-saturation) of the input flow rate are additionally guaranteed through the proposed approach, without entailing complex control algorithms or stability proofs. The analytical developments are corroborated through experimental results.

Keywords: Counterflow heat exchangers, adaptive controller, global regulation, temperature control, bounded positive input

1. INTRODUCTION

Control of heat exchangers has been developed in the literature through the application of several techniques. For instance, based on a simple compartmental model, partial and total linearizing feedback algorithms have been proposed in (Alsop and Edgar 1989) and (Malleswararao and Chidambaram 1992). Unfortunately, such techniques *flatten* the system dynamics, neglecting its natural analytical-stability properties, which are consequently not exploited. Other works, like that in (Katayama, *et al.* 1990) which proposes an optimal control scheme, or that in (Lim and Ling 1989) where a generalized predictive control algorithm is developed, make use of ARX, ARMAX, or ARIMAX type models. Nevertheless, since these are (numerically) adjusted through the output response to input tests, disregarding the natural laws that determine the process behavior, such approaches also neglect the analytical and stability natural properties of the system. Moreover, none of the above mentioned works take into account the positive (unidirectional) and bounded nature of the flow rate taken as input variable.

In this work, we propose a simple adaptive-type algorithm for the outlet temperature regulation of counterflow double-pipe heat exchangers taking the opposite fluid flow rate as control input. The proposed controller takes into account the natural analytical-stability properties of the exchanger, as

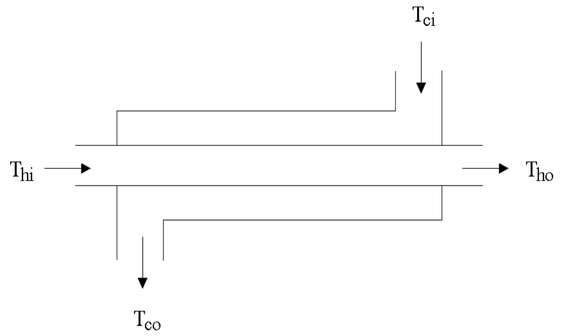

Fig. 1. Sketch of a counterflow heat exchanger.

well as the positive and bounded nature of the flow rate taken as input variable.

The text is organized as follows. In Section 2, we state the nomenclature, notation, and preliminaries that support our developments. Section 3 presents the system dynamics. In Section 4, the proposed controller is presented conventionally taking the hot fluid outlet temperature as controlled variable and the cold fluid flow rate as control input. Section 5 presents the extension to the *dual case* where the cold fluid outlet temperature is taken as controlled variable and the hot fluid flow rate as control input. Experimental results are presented in Section 6. Finally, concluding remarks are given in Section 7.

2. NOMENCLATURE AND NOTATION

We introduce the following nomenclature and notation:

F	mass flow rate
C_p	specific heat
M	total mass inside the tube
U	overall heat transfer coefficient
A	heat transfer surface area
T	temperature
t	time
ΔT	temperature difference
$\mathbb{R}$	set of real numbers
$\mathbb{R}_+$	set of positive real numbers
$\mathbb{R}^n$	set of n-tuples (x_j) with $x_j \in \mathbb{R}$
0_n	origin of $\mathbb{R}_n$
$\mathbb{R}^n_+$	set of n-tuples (x_j) with $x_j \in \mathbb{R}_+$

Subscripts:

u	upper bound	l	lower bound
c	cold	h	hot
i	inlet	o	outlet

Let ΔT_1 and ΔT_2 stand for the temperature difference at each terminal side of the heat exchanger, *i.e.* $\Delta T_1 = T_{hi} - T_{co}$ and $\Delta T_2 = T_{ho} - T_{ci}$ (see Figure 1). The *logarithmic mean temperature difference* (LMTD) among the fluids is typically expressed as

$$\Delta T_\ell \triangleq \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}}$$

Nonetheless, this expression reduces to an indeterminate form when $\Delta T_1 = \Delta T_2$. Such an indetermination is avoided if the LMTD is taken as

$$\Delta T_L \triangleq \begin{cases} \Delta T_\ell & \text{if } \Delta T_2 \neq \Delta T_1 \\ \Delta T_0 & \text{if } \Delta T_2 = \Delta T_1 = \Delta T_0 \end{cases} \tag{1}$$

This was proven in (Zavala-Río, *et al.* 2005), together with the following analytical properties:

Lemma 1. (Lemma 2 and Remark 3 in (Zavala-Río, *et al.* 2005)) ΔT_L in (1) is continuously differentiable at every $(\Delta T_1, \Delta T_2) \in \mathbb{R}^2_+$. Moreover, it is positive on $\mathbb{R}^2_+$, while $\lim_{\Delta T_1 \to 0} \Delta T_L = 0$ for any $\Delta T_2 \in \mathbb{R}_+$, and $\lim_{\Delta T_2 \to 0} \Delta T_L = 0$ for any $\Delta T_1 \in \mathbb{R}_+$. ◁

Lemma 2. (Lemma 3 in (Zavala-Río, *et al.* 2005)) ΔT_L in (1) is strictly increasing in its arguments, *i.e.* $\frac{\partial \Delta T_L}{\partial \Delta T_i} > 0$, $i = 1, 2$, $\forall (\Delta T_1, \Delta T_2) \in \mathbb{R}^2_+$. ◁

Finally, the interior and boundary of a set, say $\mathcal{B}$, will be respectively denoted as $\text{int}(\mathcal{B})$ and $\partial\mathcal{B}$.

3. THE SYSTEM DYNAMICS

Let us consider the following assumptions:

A1. The fluid temperatures and velocities are radially uniform.
A2. The heat transfer coefficient is axially uniform and constant.
A3. Constant fluid thermophysical properties.
A4. No heat transfer with the surroundings.
A5. Fluids are incompressible and single phase.
A6. Negligible axial heat conduction.
A7. There is no energy storage in the walls.
A8. Inlet temperatures, T_{ci} and T_{hi}, are constant.
A9. The flow rates are axially uniform and any variation is considered to take place instantaneously at every point along the whole length of the exchanger.
A10. The hot fluid flow rate, F_h, is kept constant, while the value of the cold fluid flow rate, F_c, can be arbitrarily varied within a compact interval $\mathcal{F}_c \triangleq [F_{cl}, F_{cu}]$, for some constants $F_{cu} > F_{cl} \geq 0$.

Under these assumptions, and taking the whole exchanger as one *bi-compartmental cell*, a simplified but suitable lumped-parameter dynamical model for a double-pipe heat exchanger is (see for instance (Zavala-Río and Santiesteban-Cos 2007)):

$$\dot{T}_{co} = \frac{2}{M_c}\left[F_c\left(T_{ci} - T_{co}\right) + \frac{UA}{C_{pc}}\Delta T_L\right] \qquad (2a)$$

$$\dot{T}_{ho} = \frac{2}{M_h}\left[F_h\left(T_{hi} - T_{ho}\right) - \frac{UA}{C_{ph}}\Delta T_L\right] \qquad (2b)$$

where ΔT_L is the LMTD (complemented) expression in (1). A physically reasonable state-space domain for system (2) is $\mathcal{D} \triangleq \{x \in \mathbb{R}^2 \mid x_j \in \mathcal{T}, j = 1, 2\}$ where $\mathcal{T} \triangleq (T_{ci}, T_{hi})$ (see for instance (Zavala-Río, *et al.* 2003)). Conventionally, the control objective consists in the regulation of the process (hot) fluid outlet temperature, T_{ho}, taking the cold fluid flow rate, F_c, as input variable with restricted range according to Assumption A10.

Remark 1. Let y denote the open-loop state vector, *i.e.* $y \triangleq (T_{co}, T_{ho})$, and let $\dot{y} = \bar{f}(y; \theta)$ represent the open-loop system dynamics (2) assuming constant flow rates, where θ is the system parameter vector, *i.e.* $\theta = (U, A, C_{pc}, C_{ph}, M_c, M_h, F_c, F_h) \in \mathbb{R}^8_+$. Considering Lemma 1, one sees (from (2)) that $\bar{f}$ is continuously differentiable in $(y; \theta)$ on $\mathcal{D} \times \mathbb{R}^8_+$. Then, the system solutions, $y(t; y_0, \theta)$ with $y_0 \triangleq y(0) \in \mathcal{D}$, do not only exist and are unique, but are also continuously differentiable with respect to initial conditions and parameters, for all $y_0 \in \mathcal{D}$ and all θ sufficiently close to any nominal parameter vector $\theta_0 \in \mathbb{R}^8_+$ (see for instance (Khalil 1996, §2.4)). ▷

In (Zavala-Río and Santiesteban-Cos 2007), it was shown that, considering constant flow rates, the system dynamics (2) possesses a unique equilibrium point $(T^*_{co}, T^*_{ho}) \in \mathcal{D}$, where

$$\begin{pmatrix} T^*_{co} \\ T^*_{ho} \end{pmatrix} = \begin{pmatrix} 1-P & P \\ RP & 1-RP \end{pmatrix} \begin{pmatrix} T_{ci} \\ T_{hi} \end{pmatrix} \triangleq \begin{pmatrix} g_c(F_c) \\ g_h(F_c) \end{pmatrix} \qquad (3)$$

with $R \triangleq \frac{F_c C_{pc}}{F_h C_{ph}}$,

$$P \triangleq \begin{cases} \dfrac{1-S}{1-SR} & \text{if } R \neq 1 \\ \dfrac{UA}{UA + F_c C_{pc}} & \text{if } R = 1 \end{cases}$$

and $S \triangleq e^{UA\left(\frac{1}{F_h C_{ph}} - \frac{1}{F_c C_{pc}}\right)}$.

Claim 1. g_h in (3) is a one-to-one strictly decreasing continuously differentiable function of F_c.

Proof. Continuous differentiability of g_h with respect to F_c follows from the arguments given in Remark 1. Hence, from (3), $g'_h(F_c) = \frac{dg_h}{dF_c}(F_c)$ is given by

$$g'_h(F_c) = \begin{cases} \dfrac{C_{pc} S\left[1 + \gamma - e^{\gamma}\right]\Delta T_i}{C_{ph} F_h \left(1 - SR\right)^2} & \text{if } R \neq 1 \\ -\dfrac{C_{pc} U^2 A^2 \Delta T_i}{2 C_{ph} F_h \left(UA + C_{ph} F_h\right)^2} & \text{if } R = 1 \end{cases}$$

where $\gamma \triangleq \frac{UA}{C_{pc}F_c} - \frac{UA}{C_{ph}F_h}$ and $\Delta T_i \triangleq T_{hi} - T_{ci}$. Thus, from Formula 4.2.30 in [1] (Abramowitz and Stegun 1972), we see that $g'_h(F_c) < 0$, $\forall F_c > 0$, showing that $g_h(F_c)$ is strictly decreasing on its domain. This, in turn, corroborates its one-to-one character. □

Observe that through Claim 1, two important facts are concluded: 1) T^*_{ho} is restricted to a *reachable steady-state space* defined by $\mathcal{R}_h \triangleq [g_h(F_{cu}), g_h(F_{cl})]$, and 2) any value of $T^*_{ho} \in \mathcal{R}_h$ is uniquely defined by a specific flow rate value $F^*_c \in \mathcal{F}_c$ (see Assumption A10), which in turn defines a unique value of T^*_{co} according to (3).

4. THE PROPOSED CONTROLLER

The analysis developed in (Zavala-Río and Santiesteban-Cos 2007), considering constant flow rates, showed that the vector field in (2) has a normal component pointing to the interior of $\mathcal{D}$ at every point on its boundary. Consequently, for all initial state vectors in $\mathcal{D}$, the system trajectories remain in $\mathcal{D}$ globally in time, and are bounded since $\mathcal{D}$ is bounded. Moreover, $\mathcal{D}$ was proven to contain a sole invariant composed by a unique equilibrium point (T^*_{co}, T^*_{ho}). Therefore, every trajectory of (2) converges to (T^*_{co}, T^*_{ho}). The idea is then to propose a dynamic controller such that the closed-loop dynamics keep the same analytical features, with F_c forced to evolve within $\text{int}(\mathcal{F}_c)$, and forcing the existence of a sole invariant composed by a unique equilibrium point $(T^*_{co}, T^*_{ho}, F^*_c)$ where $T^*_{ho} = T_{hd}$. This is achieved through the following control scheme.

Proposition 1. Consider the dynamical system (2) with $F_c \in \mathcal{F}_c$. Let the value of F_c be dynamically computed as follows

$$\dot{F}_c = k\eta_c(F_c)\left(T_{ho} - T_{hd}\right) \qquad (4)$$

for any $T_{hd} \in \text{int}(\mathcal{R}_h)$, where

$$\eta_c(F_c) \triangleq (F_c - F_{cl})(F_{cu} - F_c)$$

and k is a sufficiently small positive constant. Then, for any initial closed-loop (extended) state vector $(T_{co}, T_{ho}, F_c)(0) \in \mathcal{D} \times \text{int}(\mathcal{F}_c)$: $T_{ho}(t) \to T_{hd}$ as $t \to \infty$, with $F_c(t) \in \text{int}(\mathcal{F}_c)$, $\forall t \geq 0$, and $\left(T_{co}, T_{ho}\right)(t) \in \mathcal{D}$, $\forall t \geq 0$. ◁

[1] Formula 4.2.30 in (Abramowitz and Stegun 1972) states the following well-known inequality: $e^x > 1 + x$, $\forall x \neq 0$.

Proof. Let x denote the closed-loop (extended) state vector, *i.e.* $x \triangleq (T_{co}, T_{ho}, F_c)$, and let $\dot{x} = f(x)$ represent the closed-loop system dynamics. Based on Lemma 1, one can verify that $f_1(T_{ci}, T_{ho}, F_c) > 0$, $\forall (T_{ho}, F_c) \in \mathcal{T} \times \mathrm{int}(\mathcal{F}_c)$, $f_1(T_{hi}, T_{ho}, F_c) < 0$, $\forall (T_{ho}, F_c) \in \mathcal{T} \times \mathrm{int}(\mathcal{F}_c)$, $f_2(T_{co}, T_{ci}, F_c) > 0$, $\forall (T_{co}, F_c) \in \mathcal{T} \times \mathrm{int}(\mathcal{F}_c)$, $f_2(T_{co}, T_{hi}, F_c) < 0$, $\forall (T_{co}, F_c) \in \mathcal{T} \times \mathrm{int}(\mathcal{F}_c)$, and $f_3(T_{co}, T_{ho}, F_{cl}) = f_3(T_{co}, T_{ho}, F_{cu}) = 0$, $\forall (T_{co}, T_{ho}) \in \mathcal{D}$. This shows that there is no point on the boundary of $\mathcal{D} \times \mathcal{F}_c$ where the vector field f have a normal component pointing outwards. Consequently, for any initial extended state vector in $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$, the close-loop system solution cannot leave the system state-space domain $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$. Moreover, it is clear that the points on $\partial\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$ cannot even be approached. On the other hand, from (4) and the observations noted in the last paragraph of §3 above, one can easily see that the closed-loop system has a unique equilibrium point $x^* = (T_{co}^*, T_{ho}^*, F_c^*)$ in $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$, where $T_{ho}^* = T_{hd}$ and F_c^* takes the unique value on $\mathcal{F}_c$ through which T_{ho}^* can adopt the desired value T_{hd}. Besides, letting $x_l^* \triangleq \big(g_c(F_{cl}), g_h(F_{cl}), F_{cl}\big)$ and $x_u^* \triangleq \big(g_c(F_{cu}), g_h(F_{cu}), F_{cu}\big)$ (see (3)), with $g_h(F_{cl}) = \max\{T_{ho}^* \in \mathcal{R}_h\}$ and $g_h(F_{cu}) = \min\{T_{ho}^* \in \mathcal{R}_h\}$ (see §3), we have that $f(x_l^*) = f(x_u^*) = 0_3$. Actually, x_l^* and x_u^* are the only equilibrium points on the boundary of $\mathcal{D} \times \mathcal{F}_c$. The Jacobian matrix of f, *i.e.*

$$\frac{\partial f}{\partial x} = \begin{pmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} & \dfrac{2(T_{ci} - T_{co})}{M_c} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} & 0 \\ 0 & k\eta_c(F_c) & k\eta_c'(F_c)(T_{ho} - T_{hd}) \end{pmatrix}$$

where $\eta_c'(F_c) = \frac{d\eta_c}{dF_c}(F_c) = F_{cu} + F_{cl} - 2F_c$,

$$\frac{\partial f_1}{\partial x_1} = -\frac{2F_c}{M_c} + \frac{2UA}{M_c C_{pc}} \frac{\partial \Delta T_L}{\partial T_{co}}$$

$$\frac{\partial f_1}{\partial x_2} = \frac{2UA}{M_c C_{pc}} \frac{\partial \Delta T_L}{\partial T_{ho}}$$

$$\frac{\partial f_2}{\partial x_1} = -\frac{2UA}{M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{co}}$$

$$\frac{\partial f_2}{\partial x_2} = -\frac{2F_h}{M_h} - \frac{2UA}{M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{ho}}$$

(Lemma 1 is being considered), evaluated at x_l^* and x_u^*, *i.e.* $\frac{\partial f}{\partial x}\Big|_{x=x_l^*}$ and $\frac{\partial f}{\partial x}\Big|_{x=x_u^*}$, have eigenvalues $k(F_{cu} - F_{cl})(g_h(F_{cl}) - T_{hd}) > 0$ and $k(F_{cl} - F_{cu})(g_h(F_{cu}) - T_{hd}) > 0$, respectively. Then x_l^* and x_u^* are repulsive and consequently the points on $\mathcal{D} \times \partial\mathcal{F}_c$ cannot be asymptotically approached from the interior of the system state-space domain either. Consequently, for any $x_0 \in \mathcal{D} \times \mathrm{int}(F_c)$, $x(t; x_0) \in \mathcal{D} \times \mathrm{int}(F_c)$, $\forall t \geq 0$ (or equivalently, $F_c(t) \in \mathrm{int}(\mathcal{F}_c)$, $\forall t \geq 0$, and $(T_{co}, T_{ho})(t) \in \mathcal{D}$, $\forall t \geq 0$). Now, let us consider the Jacobian matrix of f at x^*, *i.e.* $\frac{\partial f}{\partial x}\Big|_{x^*}$. Its characteristic polynomial is given by $P(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$, where $a_2 \triangleq \Big[\frac{2F_c}{M_c} + \frac{2F_h}{M_h} - \frac{2UA}{M_c C_{pc}} \frac{\partial \Delta T_L}{\partial T_{co}} + \frac{2UA}{M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{ho}}\Big]_{x=x^*}$, $a_1 \triangleq \Big[\frac{4F_c F_h}{M_c M_h} + \frac{4F_c UA}{M_c M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{ho}} - \frac{4F_h UA}{M_h M_c C_{pc}} \frac{\partial \Delta T_L}{\partial T_{co}}\Big]_{x=x^*}$, and $a_0 \triangleq k\bar{a}_0$ with $\bar{a}_0 \triangleq \Big[\frac{4UA\eta_c(F_c)(T_{ci} - T_{co})}{M_c M_h C_{ph}} \frac{\partial \Delta T_L}{\partial T_{co}}\Big]_{x=x^*}$. From these expressions and Lemma 2, one can see that $a_2 > b_2 \triangleq \frac{2F_{cl}}{M_c} + \frac{2F_h}{M_h} > 0$, $a_1 > b_1 \triangleq -\frac{4F_h UA}{M_h M_c C_{pc}} \Big[\frac{\partial \Delta T_L}{\partial T_{co}}\Big]_{x=x^*} > 0$, and $0 < \bar{a}_0 < \bar{b}_0 \triangleq \frac{4UA\eta_c\left(\frac{F_{cl}+F_{cu}}{2}\right)(T_{ci} - T_{hi})}{M_c M_h C_{ph}} \Big[\frac{\partial \Delta T_L}{\partial T_{co}}\Big]_{x=x^*}$ (where the fact that $\eta_c(F_c) \leq \eta_c\big(\frac{F_{cl}+F_{cu}}{2}\big)$, $\forall F_c \in \mathcal{F}_c$, has been taken into account). Furthermore, let us consider that k satisfies $k \leq \frac{b_1 b_2}{\bar{b}_0} = \frac{8F_h C_{ph}(F_{cl} M_h + F_h M_c)}{M_h M_c C_{pc}(F_{cu} - F_{cl})^2 (T_{hi} - T_{ci})}$. Observe that under this consideration we have that $k\bar{a}_0 < k\bar{b}_0 \leq b_1 b_2 < a_1 a_2$, *i.e.* $a_0 < a_1 a_2$ which is a necessary and sufficient condition for the three roots of $P(\lambda)$ to have negative real part (see for instance Example 6.2 in (Dorf 2001)). Thus, x^* is asymptotically stable. Its attractivity is global on $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$ if $\{x^*\}$ is the only invariant in $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$, which is the case for a small enough value of k. Indeed, from boundedness of $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$ and its positive invariance with respect to the closed-loop system dynamics, every solution $x(t; x_0 \in \mathcal{D} \times \mathrm{int}(\mathcal{F}_c))$ has a nonempty, compact, and invariant positive limit set $\mathcal{L}^+$, and $x(t; x_0) \to \mathcal{L}^+$ as $t \to \infty$, $\forall x_0 \in \mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$ (see Lemma 3.1 in (Khalil 1996)). Then, the global attractivity of x^* on $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$ is subject to the absence of periodic orbits on $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$ (implying $\mathcal{L}^+ = \{x^*\}$). A sufficiently small value of k renders the closed loop a *slowly varying system* (see §5.7 in (Khalil 1996)). Then, the 3rd-order closed-loop dynamics can be approximated by the 2nd-order system (2) with (quasi) constant F_c. Since under such representation no closed orbits can take place (according to (Zavala-Río and Santiesteban-Cos 2007)), we deduce the absence of periodic solutions of the closed-loop (3rd-order) system on $\mathcal{D} \times \mathrm{int}(\mathcal{F}_c)$. Thus, we conclude that $T_{ho}(t) \to T_{hd}$ as $t \to \infty$. □

Remark 2. Observe that the proposed approach does not need to feedback the whole extended state vector. No measurements of T_{co} are required for its implementation. Furthermore, the exact knowledge of the accurate values of the system parameters is not needed. This characterizes the proposed algorithm as a sort of **adaptive controller** that gives rise to a control signal evolving within its physical limits, avoiding lower-bound and upper-bound input saturation. ▷

Remark 3. Notice, from the proof of Proposition 1, that $k \leq \frac{8F_h C_{ph}(F_{cl}M_h + F_h M_c)}{M_h M_c C_{pc}(F_{cu}-F_{cl})^2(T_{hi}-T_{ci})}$ may be taken as an *a priori* control gain tuning criterion. However, it is worth to note that such a condition is not necessary and that it generally turns out to be conservative. ▷

5. THE DUAL CASE

From a general viewpoint, the control objective may be stated as the regulation of the outlet temperature of one of the fluids of a counterflow heat exchanger, taking the flow rate of the other fluid as control input. The specific case considered throughout the precedent sections may be taken as the conventional case (see for instance (Alsop and Edgar 1989, Malleswararao and Chidambaram 1992)). Nevertheless, the *dual case*, where the cold fluid outlet temperature is the variable to be regulated, taking the flow rate of the hot fluid as control input, may be considered as well. For such a case, a dual Assumption A10 shall be stated as follows:

A10'. The cold fluid flow rate, F_c, is kept constant, while the value of the hot fluid flow rate, F_h, can be arbitrarily varied within a compact interval $\mathcal{F}_h \triangleq [F_{hl}, F_{hu}]$, for some constants $F_{hu} > F_{hl} \geq 0$.

Observe that in this setting, T^*_{co} and T^*_{ho} in (3) may be rather considered functions of F_h, *i.e.* $T^*_{co} = (1-P)T_{ci} + PT_{hi} \triangleq \bar{g}_c(F_h)$ and $T^*_{ho} = RPT_{ci} + (1-RP)T_{hi} \triangleq \bar{g}_h(F_h)$. Furthermore, following a procedure similar to the one developed in the proof of Claim 1, it can be shown that $\bar{g}_c$ is a strictly increasing function of F_h. Hence, we conclude two important facts: 1) T^*_{co} is restricted to a reachable steady-state space defined by $\mathcal{R}_c \triangleq [\bar{g}_c(F_{hl}), \bar{g}_c(F_{hu})]$, and 2) any value of $T^*_{co} \in \mathcal{R}_c$ is uniquely defined by a specific flow rate value $F^*_h \in \mathcal{F}_h$, which in turn defines a unique value of $T^*_{ho} = \bar{g}_h(F^*_h)$. Thus, following a procedure similar to the one developed in Section 4 for the conventional case, it is possible to prove the following proposition for its application in the dual case.

Proposition 2. Consider the dynamical system (2) with $F_h \in \mathcal{F}_h$. Let the value of F_h be dynamically computed as follows

$$\dot{F}_h = k\eta_h(F_h)\,(T_{cd} - T_{co})$$

for any $T_{cd} \in \mathrm{int}(\mathcal{R}_c)$, where

$$\eta_h(F_h) \triangleq (F_h - F_{hl})(F_{hu} - F_h)$$

and k is a sufficiently small positive constant. Then, for any initial closed-loop (extended) state vector $(T_{co}, T_{ho}, F_h)(0) \in \mathcal{D} \times \mathrm{int}(\mathcal{F}_h)$: $T_{co}(t) \to T_{cd}$ as $t \to \infty$, with $F_h(t) \in \mathrm{int}(\mathcal{F}_h)$, $\forall t \geq 0$, and $\big(T_{co}, T_{ho}\big)(t) \in \mathcal{D}$, $\forall t \geq 0$. ◁

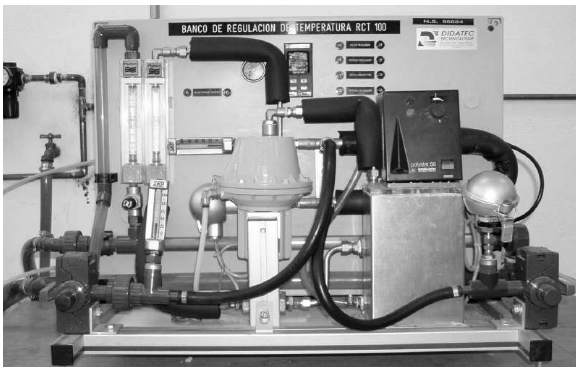

Fig. 2. Bench-scale pilot plant

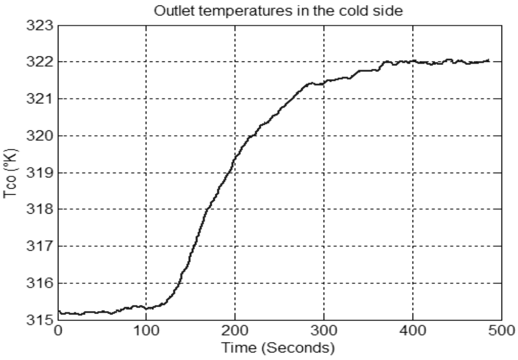

Fig. 3. Closed-loop response of T_{co}

6. EXPERIMENTAL RESULTS

In order to verify the effectiveness of the proposed controller, experiments were carried out on a bench-scale pilot plant consisting of a completely instrumented double-pipe heat exchanger; see Figure 2. The plant operates as a water-cooling process, with the hot water flowing through the internal tube and the cooling water flowing through the external pipe. However, by the way it is instrumented, in our experiments, the exchanger worked in its dual mode, *i.e.* with the hot fluid flow rate, F_h, taken as control input, and the cold fluid outlet temperature, T_c, being the controlled variable. The inlet temperatures were kept constant at $T_{ci} = 301.5\,\mathrm{K}$ (measured with a SIKA glass thermometer) and $T_{hi} = 343.1\,\mathrm{K}$ (measured *via* an Engelhard Pyro-Controle Pt-100 temperature transmitter). The flow rates were measured *via* Platon flowmeters. The cold fluid flow rate was fixed at $F_c = 5 \times 10^{-3}$ kg/sec. The hot fluid flow rate, F_h, was (arbitrarily) made vary between $F_{hl} = 5\times10^{-3}$ kg/sec and $F_{hu} = 17\times10^{-3}$ kg/sec. The cold fluid outlet temperature, T_{co}, was measured using an Engelhard Pyro-Controle Pt-100 temperature transmitter. The controller gain and initial condition were fixed at $k = 0.22\,[1/(\mathrm{kg}\cdot\mathrm{K})]$ and $F_h(0) = 6 \times 10^{-3}$ kg/sec. The desired outlet temperature was defined as $T_{cd} = 322\,\mathrm{K}$.

Figures 3 and 4 respectively show the evolution of the controlled outlet temperature, T_{co}, and the control variable, F_h. Observe that the

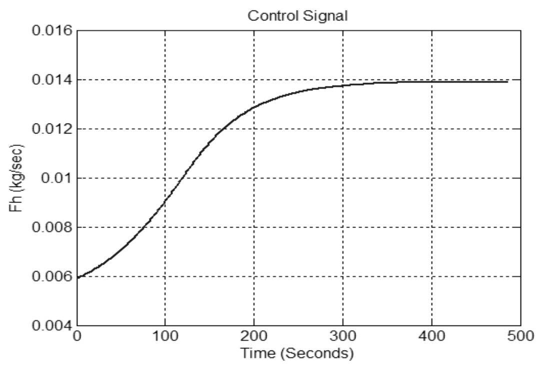

Fig. 4. Control signal F_h

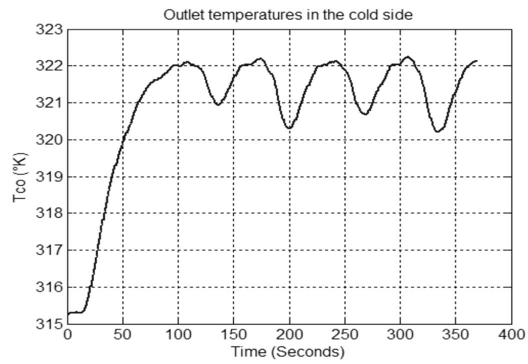

Fig. 5. Response of T_{co} under PI control

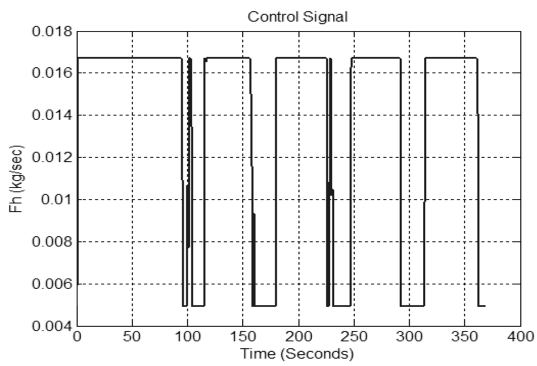

Fig. 6. PI control signal F_h

control objective is achieved avoiding input saturation. For comparison purposes, a conventional PI controller, $F_h(t) = k_P(T_{cd} - T_{co}(t)) + k_I \int_0^t (T_{cd} - T_{co}(\tau))d\tau$, was implemented with $k_P = 0.22$ [kg/(sec · K)] and $k_I = 0.22$ [kg/(sec^2 · K)]. Figures 5 and 6 respectively show the closed-loop outlet temperature response and the hot fluid flow rate under the PI control law. Observe that an oscillating response is obtained, and that lower-bound and upper-bound input saturation are not avoided. Of course, a different tuning could produce better results, but it is important to note that under a conventional PI algorithm, convergence and saturation avoidance cannot be in general guaranteed for any initial conditions, as it is the case for the proposed controller with a suitable control gain.

7. CONCLUSIONS

In this work, a bounded positive adaptive-type control scheme for the outlet temperature global regulation of counterflow heat exchangers has been proposed. The algorithm avoids lower-bound and upper-bound input saturation, guaranteeing sign invariance of the control variable, which agrees with the unidirectional nature of the corresponding flow rate. Moreover, the proposed scheme does not need to feedback the whole closed-loop state vector and does not depend on the exact knowledge of the system parameters. Experimental results corroborated the theoretical developments.

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