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A NONLINEAR OBSERVER FOR BIOPROCESSES USING LMI

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Abstract: This article presents a observer based on a classic Nonlinear and an Asymptotic Observer. The values of the gain matrix are determined through the resolution of a LMI formed by the vertexes of a polytope defined by the limited values of the states. The results are compared with an asymptotic observer, a classic nonlinear observer and a hybrid observer scheme.

Keywords: Asymptotic observer, Biotechnology, Observers, LMI.

1. INTRODUCTION

The control of biotechnological processes is an interesting area due to the nonlinear time varying nature, thereby presenting a significant challenge.

Several control schemes developed over the last few years, have been applied to bioreactors, including classic control, adaptive, nonlinear, predictive, optimal, fuzzy, etc., (Schüger, 2001). At the same time, due to the nature of the variables determining the quality of the product, the sensors available on line are scarce and expensive. This has made necessary to use different schemes for state estimation and parameters such as, Luenberger, Kalman filters, neuronal networks, as well as a number of models developed to achieve this objective. (Dochain, 2003; Romero, 2003).

A knowledge of the process model is fundamental in order to develop estimators such as the Luenberger observer and the Kalman filter, in which, the determination of gain depends on the model. The concentration of the growth limiting substrate is one of the variables that determine the evolution of the system, regulating the growth of the concentration of cellular biomass and the desired product (Henson, 2006). Usually, the limiting substrate diminishes as time advances and it tends to stabilize at a very small value. So, the gain values of a Luenberger observer usually show a tendency towards a high value and thus affect the convergence of the estimation.

This article deals with the analysis of this case, using a bioprocess model. By applying the differential variant of the Mean Value Theorem (Zemouche, *et al*., 2005) to the dynamic of the estimation error, we are able to obtain a linear matrix inequality (LMI). Then by determining the working limits of the system variables we can obtain a space in which the gain of the observer is valid for the defined LMI.

Gouzé, et al, report diverse works using the analysis of interval for the design of observers of limited error and observers of interval, using for this the definition of a minimum real error that will exist between the value of the vector of rates of well-known and real reaction. On the other hand, Rapaport (2003) presents a generalization to this type of observer, denominating parallelotopic observer, which has variant limits in the time for the state variables, under the hypothesis that limits available for the uncertain terms exist.

In this work, part of the fact that the structure of the vector of rates of reaction is well-known, as well as the limits of the states of the system to define the interval of the rates of reaction, whose possible combinations form the vertices of the politope that will define the value of the gain matrix, which is solved by means of LMI.

The estimation of states is accomplished by means of classic observers, such as the asymptotic observer and the Nonlinear observer.

2. DYNAMIC MODEL OF A BIOLOGICAL REACTOR

The dynamic model of bioreactors is classified in accordance with the level of detail employed to describe an individual cell. Many descriptions are based on the models of the kinetic structure using mass balance equations (Henson, 2006). The General

Dynamic Model is a general model in state space of a reaction system (Bastin and Dochain, 1990):

$$\dot{\xi} = K\varphi(\xi) - D\xi + D\xi_i - Q \qquad (1)$$

where $\xi \in \Re^n$: vector of concentrations of the components [mass/volume], $K \in \Re^{nxm}$: matrix of coefficients of production, of range R, $\varphi(\xi) \in \Re^m$: vector of reaction rates [mass/time x volume], $D \in \Re$: specific volumetric exit flow or dilution $D= F_i/V$, $\xi_i \in \Re^n$ is the concentration of the component fed to the reactor and $Q \in \Re^n$ vector of output flow rate of the component ξ of the reactor in gaseous form per unit of volume.

3. OBSERVERS FOR BIOPROCESSES

3.1 Asymptotic Observer.

The design of the asymptotic observer is based on the general dynamic model of bioprocesses (1). If the system is exponentially observable, and when the error of the observer arrives at a point of asymptotically stable equilibrium, the process can be observed. However, its dynamics are determined partially by the experimental conditions. This is called *asymptotic observer* developed by Bastin and Dochain (1990).

The principle of the asymptotic observer is based on the transformation of the state considering the partition of $\xi = [\xi_a \quad \xi_b]^T$, $Q=[Q_a,Q_b]^T$ and $F=[F_a,F_b]^T$ induced by the partition of the matrix $K=[K_a,K_b]^T$ the submatrix being K_a of complete range (rank(K_a)=R), therefore:

$$\begin{aligned} \dot{\xi}_a &= K_a\varphi(\xi_a,\xi_b) - D\xi_{ia} - Q_a \\ \dot{\xi}_b &= K_b\varphi(\xi_a,\xi_b) - D\xi_{ib} - Q_b \end{aligned} \qquad (2)$$

Taking into account the following considerations:

- C.1 The number of states measured, q, should be greater or equal to the range of the matrix K, $q \geq R$.
- C.2 Q, D and the coefficients K are known.
- C.3 The reaction rates $\varphi(\xi)$ are unknown.

A linear change of coordinates is then defined.

$$\begin{aligned} \varsigma_a &= \xi_a \\ \varsigma_b &= A_o\xi_a + \xi_b \end{aligned} \qquad (3)$$

Where $A_o = -K_bK_a^{-1}$ and under the previously mentioned considerations, the separation of the state variables into measured, ξ_1 ,and not measured, ξ_2 is carried out, rewriting the vector ς_b as a combination of these new vectors:

$$\varsigma = A_1\xi_1 + A_2\xi_2 \qquad (4)$$

And the dynamics are independent of the reaction rates $\varphi(\xi)$:

$$\dot{\varsigma} = -D\varsigma + A_o(D\xi_{ia} - Q_a) + (D\xi_{ib} - Q_b) \qquad (5)$$

Thus, the matrix A_2, dim (N-p) x (N-q), has a left inverse , $(A_2^TA)^{-1} A_2^T$, and the asymptotic observer is defined by:

$$\dot{\hat{\varsigma}} = -D\hat{\varsigma} + A_o(\xi_{ia} - Q_a) + (\xi_{ib} - Q_b) \qquad (6)$$

$$\hat{\xi}_2 = A_2^+(\hat{\varsigma} - A_1\xi_1) \qquad (7)$$

Where A_2^+ is the left inverse of A_2, $\hat{\varsigma}$ and $\hat{\xi}_2$ are the estimates on line of ς and ξ_2.

As can be observed, there is an independence of the reaction rates. However, the speed of convergence is given by the dilution rate D. Furthermore the asymptotic observer is an open loop one.

3.2 Nonlinear observers

Consider the nonlinear state space model (Dochain, 2003) defined by:

$$\frac{dx}{dt} = f(x,u) \qquad (8)$$

The measured variables are givenl by $y=h(x)$. Thus, the general scheme of an observer of state is:

$$\frac{d\hat{x}}{dt} = f(\hat{x},u) + \Omega(\hat{x})(y - \hat{y}) \qquad (9)$$

where $\hat{x}, \hat{y}$ are the estimation of x, y by means of the state observer $\hat{y} = h(\hat{x})$ with $\Omega(\hat{x})$ as the gain of the observer.

If we define the observation error as $e = x - \hat{x}$, the dynamics of the error will be:

$$\frac{de}{dt} = f(\hat{x}+e,u) - f(\hat{x},u) - \Omega(\hat{x})\big(h(\hat{x}+e) - h(\hat{x})\big) \qquad (10)$$

Designing the state observer involves choosing an appropriate gain so that the dynamics of the error has the desired properties. In order to accomplish this, deterministic considerations (Luenberger observer) or stochastic (Kalman filter) ones can be used, among others.

One general type of state observer for the nonlinear system that defines the General Dynamic Model ecc. (1), is:

$$\frac{d\hat{\xi}}{dt} = K\varphi(\hat{\xi}) - D\hat{\xi} + D\xi_i - Q(\xi) + \Omega(\hat{\xi})L[\xi - \hat{\xi}\ ] \qquad (11)$$

where $\hat{\xi}$ is the state estimation , $\Omega(\hat{\xi}) \in \Re^{qxn}$ is the gain matrix that depends on $\hat{\xi}$ and L is the matrix which selects the measured components of ξ defined from the analysis of observability. Thus, $\xi_l=L\xi$. This equation is a copy of the model with a correction term proportional to the observation error of the measured states. The error of observation is defined as $\varepsilon = \xi - \hat{\xi}$ and the dynamics of the error follows the differential equation:

$$\frac{d\varepsilon}{dt} = K[\varphi(\xi) - \varphi(\hat{\xi})] - D\varepsilon - \Omega(\hat{\xi})L\varepsilon \qquad (12)$$

4. LINEARIZATION OF THE GENERAL DYNAMIC MODEL

4.1 Problem of the linearization.

As the model is nonlinear, one option is to linearize (8), for example, around the observation error e=0:

$$\frac{de}{dt} = [A(\hat{x}) - \Omega(\hat{x})C(\hat{x})]e \qquad (13)$$

where $A(\hat{x}) = \frac{\partial f}{\partial x}$, $C(\hat{x}) = \frac{\partial h}{\partial x}$ are the Jacobians obtained when $x = \hat{x}$.

There are some conditions that can define the behavior of the observers, such as the linearization around $\varepsilon = 0$ which is based on the Jacobian of the reaction rates and the observability of the model.

$$\frac{d\varepsilon}{dt} = [A(\hat{\xi}) - \Omega(\hat{\xi})L]\varepsilon \qquad (14)$$

where:

$$A(\hat{\xi}) = K\left[\frac{\partial\varphi(\xi)}{\partial\xi}\right]_{\xi=\hat{\xi}} - DI_n \qquad (15)$$

The gain matrix Ω of the observer is defined in such a way that the roots of the equation will be negative. In every biotechnological process, the equilibrium of all the variables in fedbatch fermentations is only reached mathematically when the substrate reaches 0, but, according to the type of nutrition, certain variables may reach stabilization, while others are not stabilized (for example, volume). Thus, if the value of the substrate has decreased to a stable value and this value is low (close to 0), then some of the Jacobians will be null when the system reaches equilibrium, therefore, the inverse of the Jacobian will be poorly conditioned.

Example: As an example of this proposal, we can consider the case proposed by Bastin and Dochain, 1990:

$$\begin{bmatrix}\dot{S}\\ \dot{X}\end{bmatrix} = \begin{bmatrix}-k_1\\ 1\end{bmatrix}\varphi(\xi) - D\begin{bmatrix}S\\ X\end{bmatrix} + \begin{bmatrix}DS_{in}\\ 0\end{bmatrix}$$

where the reaction rate is defined by the law of Contois:

$$\varphi(\xi) = \frac{\mu_{\max}XS}{K_cX + S}$$

A situation is presented where $\xi_1 = S$. Real negative values are desired for the Luenberger observer gains, thus the proper values of $A(\hat{\xi}) - \Omega(\hat{\xi})L$, $\{\lambda_1, \lambda_2\} < 0$:

$$\omega_1 = -\lambda_1 - \lambda_2 - k_1\phi_S + \phi_X - 2D$$

$$\omega_2 = \frac{1}{k_1\phi_X}(-\lambda_1\lambda_2 - (\lambda_1 + \lambda_2)(D - \phi_X) - (D - \phi_X)^2 + k_1\phi_X\phi_S)$$

Where

$$\begin{bmatrix}\phi_X\\ \phi_S\end{bmatrix} = \begin{bmatrix}\frac{\partial\varphi}{\partial X}\\ \frac{\partial\varphi}{\partial S}\end{bmatrix}_{\xi=\hat{\xi}} = \begin{bmatrix}\frac{\mu_{\max}\hat{S}^2}{(K_c\hat{X} + \hat{S})^2}\\ \frac{\mu_{\max}K_c\hat{X}^2}{(K_c\hat{X} + \hat{S})^2}\end{bmatrix}$$

And the dynamic is defined as:

$$\dot{\varepsilon} = \left\{\begin{bmatrix}-k_1\\ 1\end{bmatrix}\begin{bmatrix}\phi_S & \phi_X\end{bmatrix} - DI_n - \begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix}\right\}\varepsilon$$

If the estimated value of the substrate tends to low value,

$$\hat{S} \to 0$$

$$S = S_{in} - \frac{K\varphi(S_\infty)}{D}$$

then there will be a term of the gain matrix (ω_2) that tends to infinitum.

$$\dot{\varepsilon} = \begin{bmatrix}-k_1\phi_S - D - \omega_1 & 0\\ \infty & -D\end{bmatrix}\varepsilon$$

with a consequent effect on the convergence of the observer.

4.2 Theorem of the Differential Mean Value

The main idea to avoid this problem is to find a polytope that covers the family of all possible situations of the model, and then design the gains of the observer that guarantees global convergence by using Linear Matrix Inequalities (LMI).

It will be used the defferential mean value theorem to obtain the polytope from the observer error equations.

The Differential Mean Value Theorem in $\Re^n$ is defined as:

Theorem 1 Being $f : \Re^n \to \Re$. Being $a, b \in \Re^n$ Assuming that f is differentiable in *Co(a,b)*. Then, there is a constant $c \in Co(a,b)$, $c \neq a$, $c \neq b$ thus:

$$f(a) - f(b) = f'(c)(a - b) \qquad (16)$$

Where

$$f' = \left[\frac{\partial f}{\partial x_1} \quad \cdots \quad \frac{\partial f}{\partial x_n}\right] \tag{17}$$

Since DMVT is not applicable to a vector function, in Zemouche *et al* (2005) the following is proposed. Being $f:\Re^n \rightarrow \Re^q$ a vector function, if $f(x)=[f_1(x),\dots,f_q(x)]^T$, where $f_i:\Re^n \rightarrow \Re$ is the *i-th* component of *f*. then:

$$f(x) = \sum_{i=1}^{q} e_q(i) f_i(x) \tag{18}$$

Where the canonic base E_q is defined as:

$$E_q = \left\{ e_q(i) \mid e_q(i) = 0,\cdots,0,1,0,\cdots,0)^T, i = 1,\cdots,s \right\}$$

Based on the above, Zemouche, *et al* (2005) propose:

Theorem 2. Being $f:\Re^n \rightarrow \Re^q$. Being $a,b \in \Re^n$. Assuming that *f* is differentiable in *Co(a,b)*. Then, there are constant vectors $c_1,\dots,c_q \in Co(a,b)$, $c_i \neq a$, $c_i \neq b$, *i=1,..,q,* thus:

$$f(a) - f(b) = \left(\sum_{i,j=1}^{q,n} e_q(i) e_n^T(j) \frac{\partial f_i}{\partial x_j}(c_i) \right)(a-b) \tag{19}$$

Applying this concept to the dynamics of the observation error for the model of (13), by DMVT there exists $\xi^* \in Co(\xi,\hat{\xi})$:

$$\varphi(\xi) - \varphi(\hat{\xi}) = \frac{\partial \varphi(\xi^*)}{\partial \xi}(\xi - \hat{\xi}) \tag{20}$$

As $\frac{\partial \varphi(\xi^*)}{\partial \xi} \in \Re^n$ then:

$$\frac{\partial \varphi(\xi^*)}{\partial \xi} = \sum_{i=1}^{n} e_n^T(i) \frac{\partial \varphi(\xi^*)}{\partial \xi_i}\left(\xi - \hat{\xi}\right) \tag{21}$$

Where ξ_i, {i=1,…,n} are the individual states of the model. If we define:

$$h_i(t) = \frac{\partial \varphi(\xi^*)}{\partial \xi_i}$$

the dynamic of the error is defined as:

$$\frac{d\varepsilon}{dt} = K \sum_{i=1}^{n} e_n^T(i) h_i(t)\varepsilon - D\varepsilon - \Omega(\hat{\xi}) L\varepsilon \tag{22}$$

If we redefine :

$$\gamma = \Omega(\hat{\xi})$$
$$h(t) = \left(h_1(t),\cdots,h_n(t)\right)$$
$$\mathrm{A}(h(t)) = K \sum_{i=1}^{n} e_n^T(i) h_i(t) - D$$

Then the dynamic of the error will be:

$$\dot{\varepsilon} = \left(\mathrm{A}(h(t)) - \gamma L\right)\varepsilon \tag{23}$$

As the states are positive, limited variables and are in relation to the maximum value of the entering concentration (Bastin and Dochain, 1990):

$$\xi_i \leq a_{i(n)} S_{\max} \mid a_{i(n)} = \frac{\bar{k}_{ij}}{\bar{k}_{nj}}, \tag{24}$$

Where $\bar{k}_{ij}$, *i=1,...n, j=1,...m,* is an element of the production coefficient matrix, and $\bar{k}_{nj}$ is the element of the matrix relating to S.

Therefore, if the assumption is that the reaction kinetics follow a continuous and differentiable model, we can deduce that $h_i(t)$ is limited:

$$h_{i(\max)} = \max_t(h_i(t)) \text{ and } h_{i(\min)} = \min_t(h_i(t) \tag{25}$$

Therefore, if we have a polytope of 2^n vertexes composed of all possible combinations of the values of $h_{i(min)}$ and $h_{i(max)}$:

Applying the Lyapunov function

$$V(t) = V(\varepsilon(t)) = \varepsilon^T P \varepsilon$$

Where *P>0* is a symmetric matrix. The error of observation will converge exponentially to zero if *V(t)>0* and $\dot{V}(t)<0$, $\forall \varepsilon \neq 0$:

$$\dot{V}(t) = \dot{\varepsilon}^T P \varepsilon + \varepsilon^T P \dot{\varepsilon}$$
$$\dot{V}(t) = \varepsilon^T \left(\mathrm{A}(h(t))^T P - L^T \gamma^T P + P\mathrm{A}(h(t)) - P\gamma L \right)\varepsilon$$

If $R = \gamma^T P$ y $P=P^T>0$ then:

$$\dot{V}(t) = \varepsilon^T (\mathrm{A}(h(t))^T P - L^T R + P\mathrm{A}(h(t)) - R^T L)\varepsilon$$

If the LMI

$$\begin{cases} \mathrm{A}(h(t))^T P - L^T R + P\mathrm{A}(h(t)) - R^T L < 0 \\ P > 0 \end{cases} \tag{26}$$

is feasible in $a_i \in \{h_{i(min)}, h_{i(max)}\}$, then $\dot{V}(t)<0$ and the gain is determined by:

$$\gamma = P^{-1} R \tag{27}$$

Since LMI is feasible in the domain defined $\alpha=\alpha_1,\ldots,\ \alpha_n|\ a_i \in \{h_{i(min)},\ h_{i(max)}\}$, the values of gain will allow the convergence of the observer.

5. EXAMPLE

In order to show the results of the application of main results of Zemouche, *et al.,* (2005) in general dynamic model for bioprocesses, let consider the following system (Farza, 1997).

$$\begin{bmatrix}\dot{X}\\ \dot{S}\\ \dot{P}\end{bmatrix}=\begin{bmatrix}1 & 0\\ -k_1 & -k_2\\ 0 & 1\end{bmatrix}\begin{bmatrix}\mu X\\ \upsilon X\end{bmatrix}-D\begin{bmatrix}X\\ S\\ P\end{bmatrix}+\begin{bmatrix}0\\ DS_{in}\\ 0\end{bmatrix} \qquad (3$$

Where $n=3$, k_1 y k_2 are production coefficients and the specific rates of growth and biosynthesis are defined by

$$\mu=\frac{\mu_{\max}S}{K_{S1}+S+\dfrac{S^2}{K_I}} \quad \text{and} \quad \upsilon=\frac{\upsilon_{\max}S}{K_{S2}+S}\,\frac{K_p}{K_p+P}$$

K_{S1} y K_{S2} being constants of saturation and K_I y K_P being constants of inhibition. The values of these parameters used for the simulation of the model are indicated in Table 1:

Table 1. Simulation values

k_1=5	K_{S1}=1 g/l	S_{in}=45 g/l
k_2=10	K_{S2}=5 g/l	$X(0)$=0.5 g/l
$\mu_{\max}$ =0.25 h^{-1}	K_I= 150 $(g/l)^2$	$S(0)$= 40 g/l
$\upsilon_{\max}$ =0.1 h^{-1}	K_P=10 g/l	$P(0)$=0.25 g/l

Taking into consideration that $L=[1\ 0\ 0]$, in other words, that the concentration of the biomass (X) is measured on line. The evolution of the states of the model is shown in the following figure, as well as the behavior of the dilution rate D.

Asymptotic Observer

In order to satisfy the condition C1, the measurement of the product is considered (P), thus ξ_1=[X,P] and ξ_2=S:

$$\begin{cases}\dot{\varsigma}=-D\hat{\varsigma}+A_0D\xi_{in}\\ \hat{\xi}_2=\hat{S}=A_2^{-1}\hat{\varsigma}-A_0\xi_1\end{cases} \qquad (33)$$

Nonlinear Observer

The observer for the model of the ecc (32) is defined as:

$$\dot{\hat{\xi}}=K\varphi(\hat{\xi})-D\hat{\xi}+D\xi_i-Q+\Omega(\xi_1-\hat{\xi}_1) \qquad (34)$$

Where $\xi_1=X$ and the gain matrix Ω can be defined by different methods such as Ackermann (Luenberger Observer) or the solution of the Riccati equation (Kalman Filters) may be used.

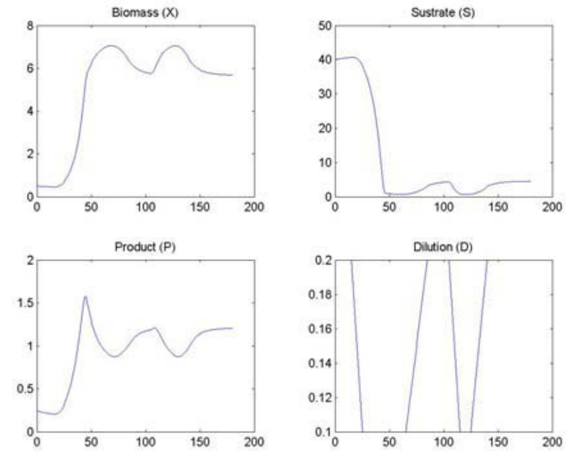

Fig. 1 Evolution of the states of the model and the dilution rate.

As we can see in figure 1, the substrate tends to low values due to the behavior of the dilution rate, therefore, in this case we calculate the gain matrix value $\gamma=\Omega$ applying the previous concepts of DMVT.

If:

$$\mathrm{A}(h(t))=K\sum_{i=1}^{n}e_n^T(i)h_i(t)-DI_n \qquad (35)$$

Where $n=3$ and $h_i=\begin{bmatrix}\dfrac{\partial\mu(\xi)X}{\partial\xi_i}\\ \dfrac{\partial\upsilon(\xi)X}{\partial\xi_i}\end{bmatrix}$, $i=1,\ldots,n$

Since $S_{max}=S_{in}$, then the limits of the concentrations are: $X=\{0,\ 9\}g/l,\ S=\{0,\ 45\}g/l,\ P=\{0,\ 4.5\}g/l$. The maximum and minimum values of h_i are:

$$h_{i(\min)}=\begin{bmatrix}0.0227 & 0.0207 & 0\\ 0.0019 & 0.0019 & -9.5\text{x}10^{-8}\end{bmatrix}$$

$$h_{i(\max)}=\begin{bmatrix}0.1891 & -0.0079 & 0\\ 0.0621 & 0.0012 & -0.0385\end{bmatrix}$$

The LMI

$$\begin{cases}\mathrm{A}(h(t))^TP-C^TR+P\mathrm{A}(h(t))-R^TC<0\\ \qquad\qquad\qquad\qquad\qquad\qquad P>0\end{cases}$$

where $h(t)=(h_{i(min)},h_{i(max)})$, $R=\gamma^TP$ and $P=P^T$, it is feasible in accordance with the development using Matlab and the solution provides the gain matrix values:

$$\gamma=[\ 0.9736\ \ -1.5926\ \ -0.1314]^T.$$

Figure 2 shows the error of observation using the development of the asymptotic observer and the nonlinear observer which uses the results obtained by means of the LMI as the gain matrix, thus presentir a better behaviour over a period of time.

Figure 3 shows the evolution of the error when tl existence of a variation in the parameter is simulate $\mu_{\max}$ =0.255 h^{-1} and noise is present in tl measurement, where we can observe that tl asymptotic observer does not present a goc response in the presence of noise.

6. CONCLUSIONS

Bioprocesses are nonlinear systems in which tl estimation of states is important in order to obtain better production. To this end, the asymptot observer, Luenberger and Kalman filter are the mo widely used. The Differential Mean Value Theore: was applied to calculate the gain matrix of the observer and to determine its performance by comparing it with the asymptotic observer. Furthermore, the asymptotic observer requires that the states measured be at least equal to the number of reaction rates, for the example, two are required, while the nonlinear observer only requires the measurement of one state.

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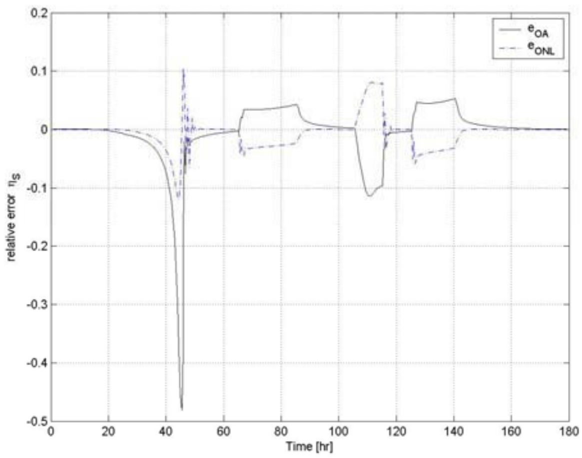

Fig. 2. Behavior of the error in the estimation of the substrate e_{OA}: error using an Asymptotic Observer, e_{ONL} error using a Nonlinear Observer.

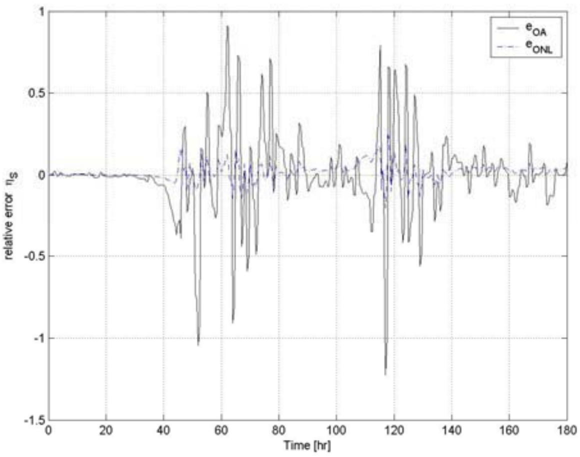

Fig. 3. Evolution of the observation error in the presence of noise in the measurement and variation in the parameter $\mu_{\max}$.

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