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COD AND VFA´S CONTROL IN A TWO-PHASE ANAEROBIC DIGESTION PROCESS

E. Aguilar-Garnica*, D. Dochain, V. Alcaraz-González***, A.K. Dramé**, V. González-Álvarez*****

** Department of Chemistry, Autonomous University of Guadalajara*
Av. Patria 1201, Lomas del Valle, Guadalajara, México
*** CESAME, Université Catholique de Louvain*
4–6 Avenue G. Lemaître 1348, Louvain-la-Neuve, Belgium
**** Department of Chemical Engineering, University of Guadalajara*
Blvd. Marcelino García Barragán 1451, Guadalajara, México

Abstract: In this paper, a two-phase anaerobic digestion system is considered to control the outlet concentration of both the Volatile Fatty Acids (VFA´s) and the organic matter (other than the VFA's) characterized by its Chemical Oxygen Demand. The two-phase system consists of two fixed bed reactors whose dynamics are described by Partial Differential Equations. The recycle flow rates in each reactor are used as the manipulated variables to reach the control objective. The control laws designed here are decoupled and they are tested by means of simulation runs obtaining satisfactory results in spite of system input disturbances and changes on the set points. *Copyright © 2007 IFAC*

Keywords: Control laws, distributed parameter systems, partial differential equations.

1. INTRODUCTION

The Anaerobic Digestion (AD) is an organic matter degradation process used for wastewater treatment. Normally, it is carried out as a single phase system; *i.e.,* the AD takes place in a single reactor which contains a mixed culture of acidogenic and methanogenic microorganisms to degrade the organic matter (Genovesi, *et al.,* 1999; Bernard, *et al.,* 2001; Alcaraz-González, *et al.,* 2002; Steyer, *et al.,* 2002; Simeonov and Queinnec, 2006). However, in most of the AD processes, the operating conditions may impose certain limitations on the physiological and growth properties of both microorganisms (Ince, 1998; Demirer and Chen, 2005). For example, low pH values favor the growth of the acidogenic bacteria but inhibit the methanogenic bacteria growth. In fact, this was one of the reasons why Pohland and Ghosh (1971) proposed the separation of the acidogenic and the methanogenic phases of typical AD processes. They showed that a two-stage AD process offers some advantages over the conventional single-phase process: a) the use of different types of bacteria for each phase, b) the prevention of methanogens overloads; c) the improvement of the stability of the whole process by controlling the acidification phase, and d) the reduction of the reactors volume which leads to a more cost efficient process (Demirer and Chen, 2005). Although the two-phase AD process has been applied in wastewater treatment sludge (Ghosh, *et al,* 1995), brewery wastewater (Ahn, *et al.,* 2001), dairy municipal wastewater (Yu and Fang, 2002) and even in human night soil (Kunte, *et al.,* 2004), only a few of works have been focused on the control of the methanogenic phase of such a process (*e.g.,* von Sachs, *et al.,* 2003). In this paper, it is proposed to consider a two-stage AD process in order to regulate, at a certain set point, the outlet concentration of both the Volatile Fatty Acids (VFA´s) and the Chemical Oxygen Demand (COD) which is defined throughout the paper as the organic matter (other than the VFA's) characterized by its COD. The AD process under consideration consists of two fixed-bed reactors whose dynamic behavior is described by a Partial Differential Equation (PDE) model since it is assumed that a biomass concentration gradient exists in each one of these phases. The acidogenic phase is carried out in the first reactor while the methanogenic phase takes

place in the second reactor. The recycle flow rates in each one of these phases are selected as manipulated variables. By using the Salmi-Romanainen boundary conditions (Babary, *et al.,* 1999), it is shown that both control laws are conveniently decoupled, easy to implement and they reach the regulation objective previously defined. The performance of the control laws is illustrated by means of simulation runs that provide satisfactory control results.

The paper is organized as follows. In Section 2, the configuration of the two-phase AD process and its mathematical model is described. The design of the control laws is illustrated in Section 3 whereas the design of two observer–based estimators which are required in the control algorithm is presented in Section 4. Besides, some practical considerations related to the implementation of the control laws are discussed in Section 5. Finally, the performance of these laws is illustrated by means of simulation experiments in Section 5 before to establish the conclusions in Section 6.

2. THE TWO-PHASE AD PROCESS

Let us consider a two-phase AD process with two fixed-bed reactors (see Fig. 1).

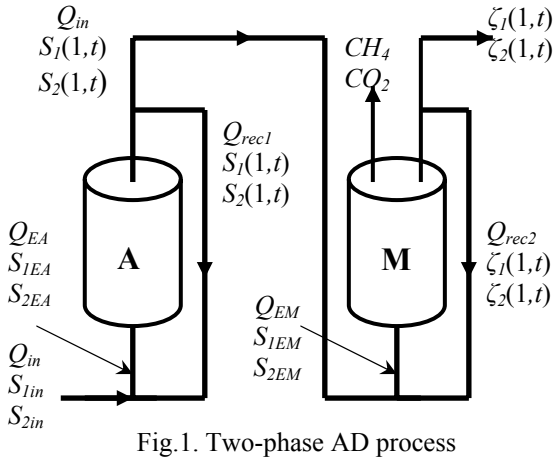

Fig.1. Two-phase AD process

Regarding this figure, the acidogenic phase is carried out in the first reactor of the system (tank A). In this phase, acidogenic microorganisms degrade a fraction of the organic matter, converting it to VFA's. The effluent of the first reactor is then fed in the second reactor where the methanogenic phase is taking place. Note that only one part of the acidogenic phase outlet flow rate is recycled whereas the rest is fed into the second reactor (tank M). In this stage, methane-producing bacteria (methanogens) use the VFA´s for methane and carbon dioxide production. It is assumed that the degradation of the COD by methanogens is negligible and as a consequence, only diffusion-convection phenomena affect the COD in this phase. Furthermore, in practice, the concentration of the bulk components may be assumed uniform within the reactors, but fixed biomass is spatially distributed and a biomass gradient can take place (Schoefs, *et al.,* 2004). These assumptions lead to the following model of the two-phase AD process:

Tank A

$$\frac{\partial X_1}{\partial t} = (\mu_1(S_1, X_1) - \varepsilon D_{EA}) X_1 \tag{1}$$

$$\frac{\partial S_1}{\partial t} = \frac{E_z}{H_A{}^2}\frac{\partial^2 S_1}{\partial z^2} - \frac{Q_{EA}}{\eta A_A H_A}\frac{\partial S_1}{\partial z} - k_1 \mu_1(S_1, X_1) X_1 \tag{2}$$

$$\frac{\partial S_2}{\partial t} = \frac{E_z}{H_A{}^2}\frac{\partial^2 S_2}{\partial z^2} - \frac{Q_{EA}}{\eta A_A H_A}\frac{\partial S_2}{\partial z} + k_2 \mu_1(S_1, X_1) X_1 \tag{3}$$

Boundary conditions

$$\frac{\partial S_k(0,t)}{\partial z} = P_e(S_k(0,t) - S_{kEA}) \qquad k = (1,2) \tag{4}$$

$$\frac{\partial S_1(1,t)}{\partial t} = -\frac{Q_{EA}}{\eta A_A H_A}\frac{\partial S_1(1,t)}{\partial z} - f_1(P_e) k_1 \mu_1(S_1, X_1) X_1 |_{z=1} \tag{5}$$

$$\frac{\partial S_2(1,t)}{\partial t} = -\frac{Q_{EA}}{\eta A_A H_A}\frac{\partial S_2(1,t)}{\partial z} + f_2(P_e) k_2 \mu_1(S_1, X_1) X_1 |_{z=1} \tag{6}$$

Tank M

$$\frac{\partial X_2}{\partial t} = (\mu_2(\zeta_2, X_2) - \tilde{\varepsilon} D_{EM}) X_2 \tag{7}$$

$$\frac{\partial \zeta_1}{\partial t} = \frac{\tilde{E}_z}{H_M{}^2}\frac{\partial^2 \zeta_1}{\partial x^2} - \frac{Q_{EM}}{\tilde{\eta} A_M H_M}\frac{\partial \zeta_1}{\partial x} \tag{8}$$

$$\frac{\partial \zeta_2}{\partial t} = \frac{\tilde{E}_z}{H_M{}^2}\frac{\partial^2 \zeta_2}{\partial x^2} - \frac{Q_{EM}}{\tilde{\eta} A_M H_M}\frac{\partial \zeta_2}{\partial x} - k_3 \mu_2(\zeta_2, X_2) X_2 \tag{9}$$

Boundary conditions

$$\frac{\partial \zeta_k(0,t)}{\partial x} = \tilde{P}_e(\zeta_k(0,t) - S_{kEM}) \qquad k = (1,2) \tag{10}$$

$$\frac{\partial \zeta_1(1,t)}{\partial x} = 0 \tag{11}$$

$$\frac{\partial \zeta_2(1,t)}{\partial t} = -\frac{Q_{EM}}{\tilde{\eta} A_M H_M}\frac{\partial \zeta_2(1,t)}{\partial x} - f_3(\tilde{P}_e) k_3 \mu_2(S_2, X_2) X_2 |_{x=1} \tag{12}$$

Initial conditions

$$\begin{aligned} &X_1(z,0) = X_1(z) \quad ; \quad X_2(x,0) = X_2(x) \\ &S_1(z,0) = S_1(z) \quad ; \quad \zeta_1(x,0) = \zeta_1(x) \\ &S_2(z,0) = S_2(z) \quad ; \quad \zeta_2(x,0) = \zeta_2(x) \end{aligned} \tag{12a}$$

In the above equations; X_1 (gVSS/L), S_1 (g/L), S_2 (mmol/L) denote respectively, the concentrations of acidogenic bacteria, COD and VFA's in the tank A, while X_2 (gVSS/L), ζ_1 (g/L), ζ_2 (mmol/L) represent, respectively, the concentrations of methanogenic bacteria, COD and VFA's, in tank M. Furthermore, t (d) is the time variable whereas z (-) and x (-) ($z, x \in [0,1]$) are dimensionless scaled space variables in tank A and tank M, respectively. It is straightforward to deduce from the mass balance that $Q_{in}+Q_{rec1}=Q_{EA}$ and $Q_{in}+Q_{rec2}=Q_{EM}$ where Q_{in}, Q_{rec1}, Q_{EA}, Q_{rec2}, Q_{EM} are the volumetric flow rates at the inlet of the system, the recycle rate in tank A, the inlet rate to tank A, the recycle rate in tank M and the inlet flow rate to tank M, respectively. Moreover , the mass balances for the species in the bulk phase at the inlet of both reactors yields $Q_{in}S_{kin}+Q_{rec1}S_k(1,t)=Q_{EA}S_{kEA}$ and $Q_{in}S_k(1,t)+Q_{rec2}\zeta_k(1,t)=Q_{EM}S_{kEM}$ which leads to $S_{kEA}=\gamma_1 S_{kin}+\chi_1 S_k(1,t)$ and $S_{kEM}=\gamma_2 S_k(1,t)+\chi_2 \zeta_k(1,t)$ with $\gamma_1=Q_{in}/Q_{EA}$, $\gamma_2=Q_{in}/Q_{EM}$, $\chi_1=Q_{rec1}/Q_{EA}$ and $\chi_2=Q_{rec2}/Q_{EM}$ (k=1,2) which describe the inlet concentration of COD and VFA´s in tank A (S_{kEA}) and the inlet concentration of COD and VFA´s in tank M (S_{kEM}). In addition, $\mu_1(S_1, X_1)$ represents the acidogenic bacteria kinetics given by a Monod-Contois model: $\mu_1(S_1, X_1) = \mu_{1\max} S_1/(K_{C1} X_1 + S_1)$ whereas $\mu_2(\zeta_2, X_2)$ denotes the methanogenic bacteria kinetics which is given by the following Haldane-Contois model: $\mu_2(\zeta_2, X_2) = \mu_{2s}\zeta_2/(K_{C2} X_2 + \zeta_2 + \zeta^2{}_2/K_I)$

where μ_{1max}, μ_{2s}, Kc_1, Kc_2 and K_I are bio-kinetic parameters. On the other hand, k_1, k_2 and k_3 are the yield coefficients. E_z, η, H_A and A_A denote the dispersion coefficient, the porosity, the height and the cross sectional area in tank A respectively whereas $\widetilde{E}_Z$, $\widetilde{\eta}$, H_M and A_M represent these same parameters but in tank M. $D_{EA} = Q_{EA} / \eta A_A H_A$ is the dilution rate in tank A and $D_{EM} = Q_{EM} / \widetilde{\eta} A_M H_M$ is the dilution rate in tank M. Besides, ε and $\widetilde{\varepsilon}$ are the bacteria fraction in the liquid phase of tank A and tank M respectively whereas P_e and $\widetilde{P}_e$ denotes the Peclet number in tank A and tank M and are defined by: $P_e=D_{EA}H_A^2/E_Z$; $\widetilde{P}_e = D_{EM}H_M{}^2 / \widetilde{E}_z$. Finally, $f_k(P_e)$ and $f_3(\widetilde{P}_e)$ are semi-empirical continuous functions represented by: $f_k(P_e)=(1+1/P_e)^{-r_k}$; $f_3(\widetilde{P}_e)=(1+1/\widetilde{P}_e)^{-r_3}$ where r_i (i=1,2,3) is a constant (see Salmi and Romananinen, 1995 for details). Notice that the so-called Salmi-Romanainen Boundary Condition (SRBC) (Babary, *et al.,* 1999) is selected to describe the output dynamical behavior of the species that are consumed or produced in tank A and tank M (see **(5), (6)** and **(12)**) while the Danckwerts Boundary Condition (DBC) is chosen to represent the behavior of the COD at the output of reactor M. The selection of these boundary conditions is motivated by the following arguments: on the one hand, for the non-reacting species it seems obvious that a concentration gradient can take place at the output of the fixed-bed. Nevertheless, this gradient is considerably smaller than the one presented for reacting species and therefore the output behavior of non-reacting species can be satisfactorily represented by a DBC. On the other hand, the reacting species can even react at the output of the fixed-bed and as a consequence this may produce a concentration gradient at the output of the reactor. The SRBC can describe not only such phenomenon but also the two extreme modes of the reactor operating: the plug flow and the complete mixing (Babary, *et al.,* 1999).

3. THE CONTROL PROBLEM

The control objective considered in this paper is to regulate the value of the concentration of COD ($\zeta_1(1,t)$) and VFA ($\zeta_2(1,t)$) at the system's output at certain desired values (S_{1sp}) and (S_{2sp}) manipulating the recycle rates Q_{rec1} and Q_{rec2} respectively. As a matter of fact, the recycle rate Q_{rec1} is the input control that will allow regulate the concentration of COD at the output of the tank A ($S_1(1,t)$). Nevertheless, it will be demonstrated by means of numerical simulations, that $\zeta_1(x,t)$ tends to $S_1(1,t)$ which is mainly due to the assumption that the COD dynamical behavior in tank M is only affected by transport phenomena. Besides, the recycle rate Q_{rec2} is considered as the manipulated variable to regulate $\zeta_2(1,t)$. In other words, Q_{rec1} will manipulate $\zeta_1(1,t)$ in an indirect form whereas Q_{rec2} will manipulate $\zeta_2(1,t)$ in a direct form. It is assumed that it is possible to measure the concentration of both the COD and the VFA's at certain positions (including at the outlet). The first of these measurements is taken in tank A whereas the second measuring procedure is carried out in tank B. From these measurements, both terms $\partial S_1(1,t)/\partial z$ and $\partial \zeta_2(1,t)/\partial x$ can be computed (*i.e.,* they are assumed as known). Then, equations **(5)** and **(12)** can be represented as:

$$\frac{dS_1(1,t)}{dt} = -\frac{Q_{EA}}{\eta A_A H_A}\frac{\partial S_1(1,t)}{\partial z} - r_{COD} \qquad (13)$$

$$\frac{d\zeta_2(1,t)}{dt} = -\frac{Q_{EM}}{\widetilde{\eta} A_M H_M}\frac{\partial \zeta_2(1,t)}{\partial x} - r_{VFA} \qquad (14)$$

where r_{COD} and r_{VFA} are the COD and the VFA consumption rates at the outlet of tank A and tank M, respectively, and are given by: $r_{COD} = f_1(P_e)k_1\mu_1 X_1\big|_{z=1}$ and $r_{VFA} = f_3(\widetilde{P}_e)k_3\mu_2 X_2\Big|_{x=1}$. Thus, the following decoupled regulation laws are proposed

$$Q_{rec1} = -\frac{(\hat{r}_{COD} + \lambda_1(S_{1sp} - S_1(1,t))\eta A_A H_A}{\dfrac{\partial S_1(1,t)}{\partial z}} - Q_{in} \qquad (15)$$

$$Q_{rec2} = -\frac{(\hat{r}_{VFA} + \lambda_2(S_{2sp} - \zeta_2(1,t))\widetilde{\eta} A_M H_M}{\dfrac{\partial \zeta_2(1,t)}{\partial x}} - Q_{in} \qquad (16)$$

which reduce **(13)** and **(14)** into

$$\frac{d\varphi_1}{dt} = -\lambda_1\varphi_1 + p_1(t) \qquad (17)$$

$$\frac{d\varphi_2}{dt} = -\lambda_2\varphi_2 + p_2(t) \qquad (18)$$

where $\hat{r}_{COD}$ and $\hat{r}_{AGV}$ denote the estimates, that can be computed by means of an observer of r_{COD} and r_{AGV} respectively. Besides, $p_1(t) = \hat{r}_{COD} - r_{COD}$, $p_2(t) = \hat{r}_{VFA} - r_{VFA}$, $\varphi_1 = S_1(1,t) - S_{1sp}$ and $\varphi_2 = \zeta_2(1,t) - S_{2sp}$. $((p_1(t), p_2(t), \varphi_1, \varphi_2) \in \Re^1)$. It is important to remark that biomass concentration sensors are either very expensive or not readily available. This explains why both terms r_{COD} and r_{VFA} are assumed as unknown. Moreover, notice that both **(17)** and **(18)** have the same mathematical structure. For this class of systems, it is well known that the dependent variable (in this case φ_1 or φ_2) remains bounded if the disturbance element (*i.e.,* $p_1(t)$ or $p_2(t)$) is bounded and if the unperturbed system is asymptotically stable. Furthermore, consider that if $\lim_{t\to\infty} p_1(t) = \xi_1$ and $\lim_{t\to\infty} p_2(t) = \xi_2$ then, it can be demonstrated that under certain assumptions $\lim_{t\to\infty}\varphi_1(t) = \xi_1/\lambda_1$ and $\lim_{t\to\infty}\varphi_2(t) = \xi_2/\lambda_2$ with (λ_1, λ_2)>0. This implies that at the limit, the values φ_1 and φ_2 are proportional to the values $p_1(t)$ and $p_2(t)$, respectively. To analyze the limit of these elements it is necessary to study the structure of the observer that provides the estimates of r_{COD} and r_{VFA} which is described in the following section.

4. THE ESTIMATION ALGORITHM

The COD and VFA consumption rates can be written as $r_{COD}=\sigma_1(1,t)S_1(1,t)$ and $r_{VFA}=\sigma_2(1,t)\zeta_2(1,t)$ with

$$\sigma_1(1,t)=\frac{f_1(P_e)k_1\mu_{\max}X_1(1,t)}{Kc_1X_1(1,t)+S_1(1,t)}$$

$$\sigma_2(1,t)=\frac{f_3(\tilde{P}_e)k_3\mu_{2s}X_2(1,t)}{Kc_2X_2(1,t)+\zeta_2(1,t)+\zeta^2{}_2(1,t)/K_I}$$

In this paper it is decided to use an Observer-Based (OB) approach to estimate the previous functions. Before to analyze the design of such observer it is necessary to establish the following assumptions:

a1) The state variables involved in the model **(1)-(12)** are positive, bounded and differentiable $\forall t \geq 0$.

a2) Due to the first assumption, $\sigma_1(1,t)$ and $\sigma_2(1,t)$, although unknown, are continuous and differentiable.

The OB estimator is designed by considering the transformations $\psi_1=-S_1(1,t)$ and $\psi_2=-\zeta_2(1,t)$ and its structure is given by: (Dochain and Vanrolleghem, 2001)

$$\begin{aligned}\frac{d\hat{\psi}_1}{dt}&=\hat{\sigma}_1(1,t)S_1(1,t)+\frac{Q_{EA}}{\eta A_A H_A}\frac{\partial S_1(1,t)}{\partial z}+\omega_1(\psi_1-\hat{\psi}_1)\\ \frac{d\hat{\sigma}_1(1,t)}{dt}&=\frac{\omega_1^2}{4S_1(1,t)}(\psi_1-\hat{\psi}_1)\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{d\hat{\psi}_2}{dt}&=\hat{\sigma}_2(1,t)\zeta_2(1,t)+\frac{Q_{EM}}{\tilde{\eta}A_M H_M}\frac{\partial\zeta_2(1,t)}{\partial x}+\omega_2(\psi_2-\hat{\psi}_2)\\ \frac{d\hat{\sigma}_2(1,t)}{dt}&=\frac{\omega_2^2}{4\zeta_2(1,t)}(\psi_2-\hat{\psi}_2)\end{aligned} \tag{20}$$

where ω_1 and ω_2 are tuning parameters. The COD and VFA consumption rates that are estimated and that will be replaced in **(15)** and **(16),** are obtained as $\hat{r}_{COD}=\hat{\sigma}_1(1,t)S_1(1,t)$ and $\hat{r}_{VFA}=\hat{\sigma}_2(1,t)\zeta_2(1,t)$. Notice that **(19)** and **(20)** are fully decoupled and as a consequence, their convergence properties can be studied separately. Let us first analyze the convergence of the observer whose structure is described in **(19)** by defining the estimation errors $e_1=\hat{\psi}_1-\psi_1$ and $e_2=\hat{\sigma}_1(1,t)-\sigma_1(1,t)$ that can be gathered as $e=\begin{bmatrix}e_1 & e_2\end{bmatrix}^T$. The dynamics of this variable can be readily obtained and has the form $\dot{e}=\Gamma e+\theta$ where

$$\Gamma=\begin{bmatrix}\Omega & \vdots & \mathrm{T}\\ \cdots & \cdots & \cdots\\ \Gamma_a & \vdots & 0\end{bmatrix}=\begin{bmatrix}-\omega_1 & \vdots & S_1(1,t)\\ \cdots & \cdots & \cdots\\ -\dfrac{\omega_1^2}{4S_1(1,t)} & \vdots & 0\end{bmatrix}\qquad \theta=\begin{bmatrix}0\\ -\dfrac{d\sigma_1(1,t)}{dt}\end{bmatrix}$$

It is a standard result of adaptive system theory (Bastin and Dochain, 1990) that the previous system is stable if the matrix Ω is constant with all its eigenvalues having strictly real parts, if the matrix T is persistently exciting and if θ is differentiable. On one hand, the first requirement is fulfilled if the tuning parameter is selected positive and constant. On the other hand, it is straightforward to demonstrate that T is persistently exciting and therefore the second requirement is also fulfilled. Finally, it can be considered that θ is differentiable due to assumption *a2*. Thus, $\dot{e}=\Gamma e+\theta$ is stable. Under assumptions *a1* and *a2* it can be demonstrated that e_1 and e_2 are asymptotically bounded and besides, both can be made asymptotically arbitrarily small by choosing a convenient value for ω_1.

Moreover, notice that the disturbance element $p_1(t)$ can be written as $p_1(t)=e_2S_1(1,t)$ and due to the previous result, the disturbance element tends to a value (*e.g.,* ξ_1) close to zero. Afterwards, since **(19)** has a mathematical structure which is identical to the one of **(20),** is possible to deduce that the disturbance element $p_2(t)$ will also tends to a number (*e.g.,* ξ_2) near to zero. Then, according to the previous section, the variables φ_1 and φ_2 will tend to a scalar number that is close to zero which implies not only that $S_1(1,t)$ will reach a value near to S_{1sp} but also that $\zeta_2(1,t)$ will tends to a number close to S_{2sp}. In other words, the controllers that are proposed here are stable but they will produce an offset whose magnitude will depend on the performance of the OB estimators.

5. SIMULATION RUNS

Before presenting the performance of the control laws, it is important to remark that the computation of $\partial S_1(1,t)/\partial z$ and $\partial\zeta_2(1,t)/\partial x$ plays a fundamental role in the implementation of the such laws since both derivative terms appear in their denominators (see **(15)** and **(16)**). In this work, it is proposed to approximate these gradients by using an approximation in terms of Backward Finite Differences (BFD) as follows:

$$\frac{\partial S_1(1,t)}{\partial z}=\frac{S_1(1,t)-S_1(z^*,t)}{\Delta z};\frac{\partial\zeta_2(1,t)}{\partial x}=\frac{\zeta_2(1,t)-\zeta_2(x^*,t)}{\Delta x}$$

where z^* and x^* are axial positions which are sufficiently near to the output of tank A and tank M, respectively whereas $\Delta z=1-z^*$ and $\Delta x=1-x$. The selection of the BFD approximation is explained as follows: physically, there exists a substrate concentration gradient in both, tank A and tank M (inclusively at the beginning of the process) which is due by a biomass concentration gradient. It can be reasonably assumed that $\partial S_1(z,t)/\partial z$ and $\partial\zeta_2(x,t)/\partial x$ fulfills with: $\partial S_1(z,t)/\partial z<0$ and $\partial\zeta_2(x,t)/\partial x<0$ $\forall t\geq 0$. Therefore, from this assumption, there will be no crossings by zero in the denominator of both **(15)** and **(16)** when a BFD approximation is used**.** In addition, BFD are cost efficient since they require only two sensors in each tank in order to make the computation of the output gradients. The performance of the control laws proposed here is

tested by means of simulations runs. These simulations are carried out using the Orthogonal Collocation Method (OCM) which allows the reduction of the PDE model into a set of Ordinary Differential Equations (ODE) that are valid at some specific axial positions along the reactors. The numerical values of these positions are obtained as a result of selecting the reduction parameters of the OCM: N, α and β. The first of these parameters denotes the number of internal collocation points (all except the boundaries) whereas α and β modify the structure of the Jacobi Polynomials depending on their values. Specifically, the roots of such polynomials are taken as the numerical values of the axial positions (Finlayson, 1980). In this paper, the reduction parameters were selected as N=3 and α=β=2 which provides the following distribution of axial positions in tank A and tank M (*i.e.,* x=z): z_1=0 (the inlet); z_2=0.21; z_3=0.5; z_4=z^*=0.79 and z_5=1 (the outlet). The simulation runs are carried out for a period of 100 *d,* with the inlet flow rate (Q_{in}) maintained in a constant value of 12.5 L/h and using the parameters (Schoefs, *et al.,* 2004): μ_{max}=1.2d^{-1}; K_{C1}=50.5(g/g); μ_{2s}=0.74d^{-1}; k_1=42.12(-); k_2=250 (mmol/g); k_3=40 (mmol/g); K_I=256 (mmol/L); K_{C2}=16.6 (mmolS$_2$/gVSS); ε= $\widetilde{\varepsilon}$ =0.5(-); $S_{1in}(t)$=10gL^{-1} and $S_{2in}(t)$=40mmolL^{-1} whereas $\eta A_A H_A = \widetilde{\eta} A_M H_M$=0.9m^3; $\widetilde{E}_z$=E_z=1m^{2}d^{-1} and H_A=H_M=3.5m, In addition, the tuning parameters of the observer-based estimator are considered as $\omega_1 = \omega_2 = 20$ both positive and constant (as it is required). The closed loop dynamics are chosen such that λ_1=λ_2=1d^{-1} in the presence of the following disturbances: the inlet COD concentration increases from 10 to 12 g/L at time t=50d whereas the inlet VFA concentration increases from 40 to 50 mmol/L at time t=75d. Besides, S_{1sp} increases from 1.5 to 2 g/L at time t=20d and S_{2sp} increases from 10 to 20 mmol/L at time t=40d. It is important to remark that the simulation results were obtained assuming that it is possible to measure $S_1(1,t)$, $S_1(z^*,t)$, $\zeta_2(1,t)$ and $\zeta_2(x^*,t)$. In this work, these measurements were taken from the model. Fig. 2 and Fig. 3 describe the evolution of the regulation laws. The effect of the first of these laws on the behavior of the COD is shown in Fig 4.

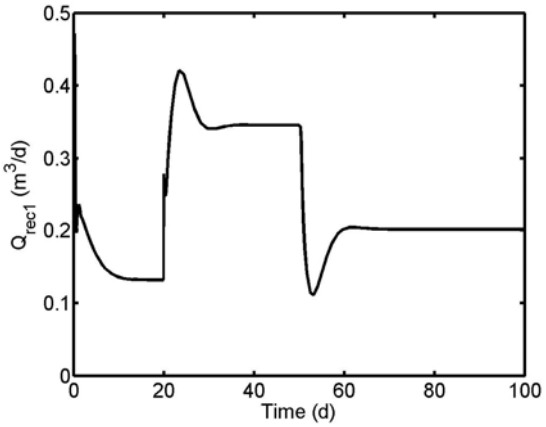

Fig. 2. Recycle rate in tank A

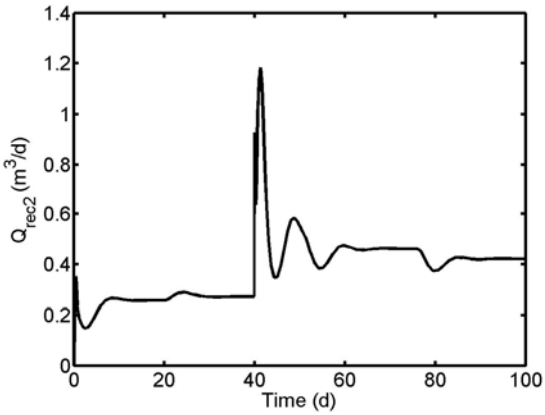

Fig. 3. Recycle rate in tank M

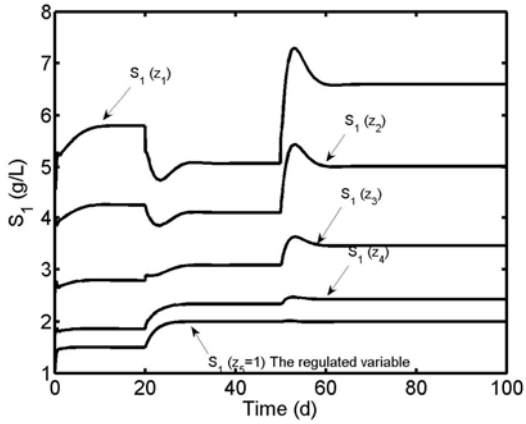

Fig. 4. Dynamical behavior of COD in tank A

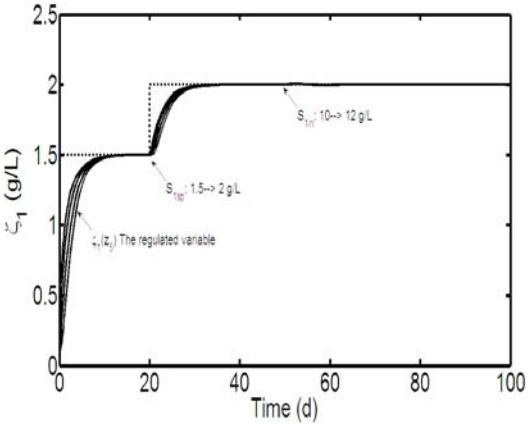

Fig. 5. Dynamical behavior of COD in tank M

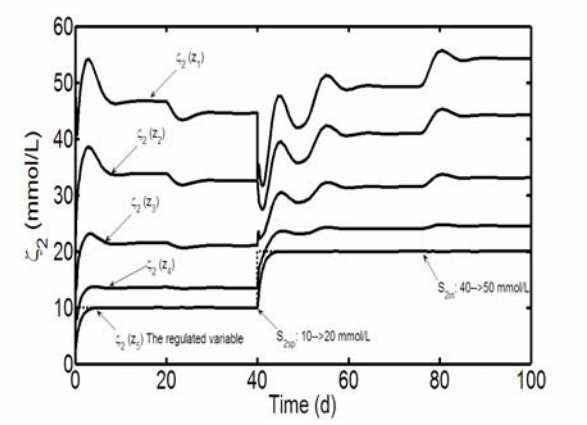

Fig. 6. Dynamical behavior of VFA's in tank M

Notice that the regulated variable follows the change in the set point and remains at such a point in spite of an increment in the inlet concentration of COD. In

Fig. 5 it is described the behavior of the COD but in the tank where the methanogenesis is taking place. As it was specified, only diffusion and convection phenomena affect the COD when it flows through tank M. This is the reason why the behavior of the regulated variable in tank A is similar to the one obtained in tank M. Afterwards, the evolution of the second regulation law is shown in Fig. 3. The effect of this law on the behavior of the concentration of VFA's is described in Fig. 6. This figure shows that the recycle rate in tank M is capable to drive the regulated variable towards certain set point and to keep it despite the disturbances in the inlet concentration of the VFA's. Finally, it is necessary to mention that a convenient selection of both tuning parameters and initial conditions allowed a satisfactory performance of the OB estimators. The behavior of these estimators is not shown here due to the lack of space but certainly it contributes to drastically reduce the offset. Obviously, if an OB estimator is not correctly initialized then it will take more time to converge and therefore, the regulation laws may not yield satisfactory results at least at the beginning of the estimation procedure. This fact may happen in practice. In such a case, it is recommended to initialize the OB estimators in an open-loop scheme for a reasonable time before to apply them in closed-loop schemes.

6. CONCLUSIONS AND PERSPECTIVES

In this paper, it was proposed a configuration for the anaerobic digestion process that facilitates the control of both the concentration of VFA's and the concentration of the organic matter. The control laws here obtained are fully decoupled and they were tested by means of numerical simulations. The simulation results show that both control laws perform adequately not only to drive the controlled variables towards a different reference value but also to keep these values despite the disturbances. Even when the simulation runs demonstrates that $\zeta_1(x,t)$ converges towards $S_1(1,t)$, it is necessary to analytically formalize such result. It is also convenient to analyze the effect of noisy measurements in the computation of $\partial S_1(1,t)/\partial z$ and $\partial \zeta_2(1,t)/\partial x$. Both topics will be the subject of our future work.

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REFERENCES

Ahn, Y.H., K.S. Min and R.E. Speece (2001). Pre-acidification in anaerobic sludge bed process treating brewery wastewater. *Water Res.,* **35** (18), 4267–4276.

Alcaraz-González, V., J. Harmand, A. Rapaport, J. P. Steyer, V. González-Alvarez and C. Pelayo-Ortiz (2002). Software sensors for highly uncertain WWTPs: a new approach based on interval observers. *Water Research,* **36** (10), 2515-2524.

Babary, J.P., S. Julien, M.T. Nihtila, J. Czeczot, M. Metzger (1999). New boundary conditions and adaptive control of fixed-bed bioreactors. *Chem. Engng. and Process.,* **38**, 35-44.

Bastin G., Dochain D. (1990) *On-line estimation and adaptive control of bioreactors.* Elsevier, The Netherlands.

Bernard, O., Z. Hadj-Sadok, D. Dochain, A. Genovesi, J.-P.Steyer (2001). Dynamical model development and parameter identification of an anaerobic wastewater treatment process. *Biotech.Bioeng.,* **75**, 424–438.

Demirer, G.N. and S. Chen (2005). Two-phase anaerobic digestion of unscreened dairy manure. *Process Biochemistry,* **40** (11), 3542-3549.

Dochain, D. and P. Vanrolleghem (2001). *Dynamical Modelling and Estimation in Wastewater Treatment Processes.* IWA publishing, London.

Finlayson, B.A. (1980). *Nonlinear analysis in chemical engineering.* Mc. Graw-Hill, New York.

Genovesi, A., J. Harmand and J.P. Steyer (1999). A fuzzy logic based diagnosis system for the on-line supervision of an anaerobic digestor pilot-plant. *Biochemical Engineering Journal,* **3** (3), 171-183.

Ghosh, S., K. Buoy, L. Dressel, T. Miller, G. Wilcox and D. Loos (1995). Pilot- and full-scale two-phase anaerobic digestion of municipal sludge. *Water Environ. Res.,* **67** (2), 206–14.

Ince, O. (1998). Performance of a two-phase anaerobic digestion system when treating dairy wastewater. *Water Res.,* **32** (9), 2707–13.

Kunte, D.P., T.Y. Yeole, D.R., Ranade (2004). Two-stage anaerobic digestion process for complete inactivation of enteric bacterial pathogens in human night soil. *Water Sci. Technol.,* **50** (6), 103-108.

Pohland, F.G., S. Ghosh (1971). Developments in anaerobic stabilization of organic wastes, the two-phase concept. *Environ Lett.,* **1**, 255–66.

Salmi, T. y Romanainen, J.J**.** (1995) A novel exit boundary condition for the axial dispersion model. *Chem. Engineering and Processing,* **34**, 359-366.

Schoefs, O., D. Dochain, H. Fibrianto, J.P. Steyer (2004). Modelling and identification of a PDE model for an anaerobic digestion wastewater treatment process. *Procc. AD 2004,* Montreal, Canadá, 1, 343-347.

Simeonov, I., and I. Queinnec (2006). Linearizing control of the anaerobic digestion with addition of acetate (control of the anaerobic digestion). *Control Engineering Practice,* **14** (**7**), 799-810.

Steyer, J.P., J.C. Bouvier, T. Conte, P. Gras, P. Sousbie (2002). Evaluation of a four year experience with a fully instrumented anaerobic digestion process. *Water Sci. Technol.,* **45** (4–5), 495–502.

von Sachs, J., U. Meyer, P. Rys, H. Feitkenhauer (2003). New approach to control the methanogenic reactor of a two-phase anaerobic digestion system. *Water Res.,* **37** (5), 973–82.

Yu, H.Q., H.H.P. Fang (2002). Acidogenesis of dairy wastewater at various pH levels. *Water Sci. Technol.,* **45** (10), 201–6.