USING A DITHERING SIGNAL IN THE REFERENCE TO IMPROVE THE ESTIMATES FROM SUBSPACE IDENTIFICATION METHODS ON CLOSED LOOP DATA

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Abstract: It is shown that when an appropriate dithering signal is used in the reference of a specific process DSR gives eigenvalue estimates with smaller bias and variance than PEM on finite closed loop data. As dithering signal a sinusoid signal at different frequencies have been tried. Comparing the optimums of DSR and PEM we find that DSR provide eigenvalue estimates with smaller bias and variance at its optimum than PEM does at its. When using an external dithering signal in the reference the choice of the frequency of the dithering signal is important when using the input and output data for direct system identification in closed loop.

Keywords: Subspace methods; identification; closed loop; finite data sets; dithering signal.

1. INTRODUCTION

The reason for the problems that can occur when applying subspace identification (SID) algorithms for direct identification of closed loop data is the projection of the future outputs onto the future inputs. Future inputs and future outputs are assumed to be uncorrelated. In closed loop operation this assumption is not necessarily fulfilled. This is the cause of the bias. Di Ruscio (2003) shows that using a filter in the feedback loop is a method to reduce the bias problem. It is also stressed that biased estimates may be more reliable than estimates from an unbiased algorithm because the variance may be small. Van Overschee and De Moor (1996), (1997) are suggesting to use the Markov parameters of the controller in the algorithm to avoid the problem. The controller is assumed to be linear. Chou and Verhaegen (1997) have developed a SID algorithm for Errors In Variables (EIV) problems. They have shown that this algorithm will give consistent estimates on closed loop data, if a persistence of excitation requirement is satisfied. This leads to the use of signals with relatively high order of persistent excitation in the reference. Gustafsson (2001) has proposed an Instrument Variable (IV) approach SID algorithm as an improvement of the already existing EIV algorithm of Chou and Verhaegen (1997). The modified algorithm is named SIV (Subspace-based Identification using instrumental Variables). Our work is based on the already

existing SID algorithm named DSR, Aoki and Havenner (1997), Di Ruscio (1996). We want to examine if an external dithering signal used in the reference of a system operating in closed loop can lead to improved parameter estimates. This is an idea presented in Di Ruscio (2003) together with the experience that an optimal experiment for the subspace methods is in general not a white noise input, but rather a colored input where the frequency spectrum is optimized to excite the parameters in the system as well as possible. The estimates from DSR will be compared to estimates from PEM (Prediction Error Method) in the system identification toolbox in Matlab 6.5. A description of prediction error methods in general and their properties can be found in amongst others Ljung (1999).

The rest of this paper is organized as follows. A model of a chemical reactor operating in closed loop is presented in Section 2.1. This model will be used throughout the paper. The effect of using a sinusoid signal as dithering signal in the reference of the chemical reactor operating in closed loop is investigated in Section 2.2. Closed loop eigenvalue estimates are presented i Section 2.2.1. In Section 2.2.2 the parameter estimates from DSR are considered as a function of the frequency of the sinusoid signal used as dithering signal and the prediction horizon used in DSR. In Section 2.2.3 the parameter estimates from DSR, with a constant prediction horizon, and the parameter estimates from PEM are presented as functions of the frequency of the sinusoid signal used as dithering signal. Alternative quality measures are presented in section 2.2.4. The results are compared. Some concluding remarks follows in Section 3.

2. A CASE STUDY IN CLOSED LOOP SYSTEM IDENTIFICATION

2.1 A model of a chemical reactor operating in closed loop

In order to generate closed loop data we choose to use a model of a chemical reactor operating in closed loop. The reaction mechanism of the reactor is given by:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C, \tag{1}$$

$$2A \xrightarrow{k_3} D. \tag{2}$$

The reaction from body A to body D is of order two, while the other reactions are of order 1. The manipulated variable is the feed flow (flow rate) u $[\frac{1}{hours}]$. The concentration of body A in the feed flow, u, is θ. The concentration of body A and body B in the tank is respectively x_1 and x_2. The connection from u to $y = x_2$ is then given by the model:

$$\dot{x}_1 = -k_1 x_1 - k_3 x_1^2 + (\theta - x_1)u, \tag{3}$$

$$\dot{x}_2 = k_1 x_1 - k_2 x_2^2 - x_2 u, \tag{4}$$

$$y = x_2, \tag{5}$$

where the reaction rate constants are given by $k_1 = 50$, $k_2 = 100$ and $k_3 = 10$. The stationary values of the states, disturbances, parameters and manipulated variable are given: $x_1^s = 2.5$, $x_2^s = 1$, $\theta^s = 10$ and $u^s = 25$. We assume that θ is constant and known.

Defining:

$$\delta u_k = u_k - u^s, \tag{6}$$

$$\delta x_k = x_k - x^s, \tag{7}$$

$$\delta y_k = y_k - y^s, \tag{8}$$

$$\delta \theta_k = \theta_k - \theta^s, \tag{9}$$

a linearized discrete model can be expressed on the form:

$$\delta x_{k+1} = A\delta x_k + B\delta u_k + C\delta\theta_k + v_k, \tag{10}$$

$$\delta y_k = D\delta x_k + E\delta u_k + w_k, \tag{11}$$

where we in addition have added additive process and measure noise, v_k and w_k, respectively.

After linearization and discretization the system is expressed by:

$$A = \begin{bmatrix} 0.8750 & 0 \\ 0.0500 & 0.8750 \end{bmatrix}, B = \begin{bmatrix} 0.0075 \\ -0.0010 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0250 \\ 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 1 \end{bmatrix} \tag{12}$$

and the matrix E is the zero matrix. The linearization is done at the stationary point. The discretization is done by explicit Euler with uniform sampling interval $\delta t = 0.001$. We consider a time series from 0 to 1, with N=1001 discrete data points, $k = 0, 1, \ldots, N$.

The body B is controlled by a PI-controller with $k_p = 50$ and $T_i = \frac{1}{75}$. The PI-controller on discrete form is given by:

$$u_k = k_p e_k + z_k, \tag{13}$$

$$z_{k+1} = z_k + \delta t \frac{k_p}{T_i} e_k, \tag{14}$$

where e_k is given by:

$$e_k = r_k - y_k. \tag{15}$$

The linearized reactor model, (10) and (11), and the PI-controller, (13) and (14), are used in order to generate closed loop data which will be used throughout this paper for closed loop system identification. The reference will be constant $r_k = 1$ at each time instant k, superposed a dithering signal. The noise used is significant. The process noise variance used is $E(v_k v_k^T) = 0.1 \cdot I$ and the measuring noise variance used is $E(w_k^2) = 0.0001$. The reason for using a linearized discrete model of the chemical reactor is that we want to compare the estimated eigenvalues to the eigenvalues of the linearized discrete model. The closed loop system is illustrated in Figure 1.

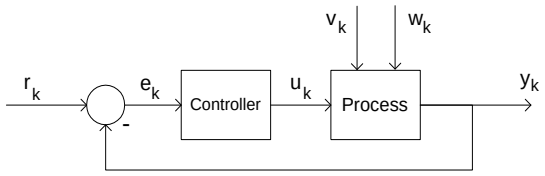

Fig. 1. System operating in closed loop

2.2 The effect of using a sinusoid signal as dithering signal in the reference of a chemical reactor operating in closed loop

2.2.1. Closed loop eigenvalue estimates A sinusoid signal with frequency $\omega = 600$ $[rad/s]$ and with magnitude ± 0.1 is chosen as dithering signal. The reference at time instant k is given by:

$$r_k = 1 + 0.1 \cdot \sin(\omega \delta t k). \tag{16}$$

We find it natural to compare DSR with PEM implemented in the system identification toolbox in Matlab 6.5. Throughout the paper the stationary values in both inputs and outputs will be removed prior to identification. Initial parameter estimates to PEM is provided by **canstart** in the Matlab system identification toolbox with the parameters chosen as: $orders = 2$, $D = 1$, $K = 1$ and $X = 1$.

The system was simulated 100 times. The same input was used each time but the noise realization was changed each time. Figure 2 shows the estimated eigenvalues from the Monte Carlo simulation. The linearized discrete system have multiple poles, actually two real eigenvalues at 0.875. The parameters in DSR are chosen as: $n = 2$, $g = 0$, $L = 12$ and $J = 13$. The default parameters are used in PEM. Figure 2 shows that by using an appropriate dithering signal DSR can provide an eigenvalue estimate with smaller bias and variance than PEM on data from a system operating in closed loop.

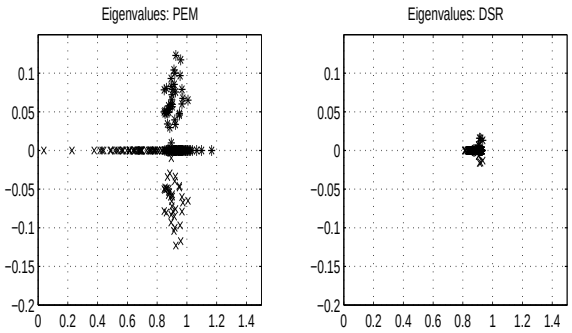

Fig. 2. Monte Carlo simulation using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$ with $\omega = 600$, using DSR with $n = 2$, $g = 0$, $L = 12$ and $J = 13$ and PEM with default parameters for identification.

We have to make a remark that PEM in this case actually gives some estimates with unstable eigenvalues. To improve the estimates from PEM we have tried to reduce the tolerance (Tol) from the default value 0.01 and increase the maximum number of iterations (MaxIter) from the default value 20. Several values were tried with Tol as low as 0.0001 and MaxIter as high as 20000. This did not improve the estimates from PEM. By increasing the tolerance to 0.02 and reducing the maximum number of iterations to 15 we observed a marginal improvement of the estimates from PEM. Because of the insignificant improvement of the estimates from PEM when changing the algorithm parameters from the default values we have chosen to use the default values in PEM. An additional comment on the estimates from PEM with unstable eigenvalues is that when using Matlab 6.1 PEM does not only give estimates with eigenvalues larger than 1, but also eigenvalues with more negative values than -1. When using Matlab 6.0 we get approximately the same values as when using Matlab 6.5.

Table 1 contents the mean and the standard deviation (Std) of the eigenvalues of the estimated system matrices from the Monte Carlo simulation when using DSR. Table 2 contains corresponding data from PEM. The data in Table 1 and Table 2 support the conclusion from the visual inspection of Figure 2, that DSR gives estimates with smaller bias and variance than PEM in this case.

2.2.2. The parameter estimates from DSR as a function of the prediction horizon and the frequency of the sinusoid signal used as dithering signal in the reference In DSR there are four parameters g, n, L and J that can be chosen by the user. If the structure parameter g is 1, which is the default, the data matrix E in the state space model is identified. If g is put to zero the matrix

Table 1. Mean and standard deviation from the Monte Carlo simulation using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$ with $\omega = 600$, and using DSR with $n = 2$, $g = 0$, $L = 12$ and $J = 13$ for identification.

DSR	Pole 1	Pole 2
	$[Re \quad Im]$	$[Re \quad Im]$
Mean	[0.9203 0.0006]	[0.8612 -0.0006]
Std	[0.0043 0.0027]	[0.0202 0.0027]

Table 2. Mean and standard deviation from the Monte Carlo simulation using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$ with $\omega = 600$, and using PEM with default parameters for identification

PEM	Pole 1	Pole 2
	$[Re \quad Im]$	$[Re \quad Im]$
Mean	[0.9325 0.0241]	[0.7692 -0.0241]
Std	[0.0538 0.0349]	[0.1758 0.0349]

E is forced to be the zero matrix. The parameter n specify the model order. The parameter L is the number of block rows in the extended observability matrix. L can be interpreted as the identification horizon used to predict the number of states. This again limits the maximal system order which can be identified. The order must be chosen in the interval $1 \leq n \leq L \cdot m$, where m is the number of outputs. The parameter J is the number of time instants in the past horizon which is used defining the instrument variable matrix which are used to remove noise.

The experience so far in open loop cases is that the parameter L should be chosen as small as possible in order to reduce the variance of the estimates. This is especially important in case of poorly excitating input signals. The parameter J is usually chosen as $J = L$ or $J = L + 1$. It is normally not crucial which of these two alternatives which are chosen.

Related to this it is interesting to consider the quality of the parameter estimates from DSR as a function of the frequency, ω, of the dithering signal and the identification horizon, L, in the algorithm when data is collected from a system operating in closed loop. The parameter J is chosen as $J = L + 1$. The system used is still the chemical reactor operating in closed loop (10), (11), (13) and (14). Now we do not only consider a Monte Carlo simulation at one frequency but Monte Carlo simulations in 25 $[rad/s]$ steps from 25 $[rad/s]$ up to the Nyquist frequency which is the half of the sampling frequency. To evaluate the quality of the estimated model parameters we choose to use a quadratic criterion on the eigenvalues of the estimated model. The criterion used sums up the square of the deviation between the eigenvalues of the estimated model and the actual eigenvalues. The quality criterion (17) is presented using Matlab notation where $\lambda(A)\ R \in \mathbb{R}^n$ is the vector of the eigenvalues of the true state transition matrix and $\lambda(\overline{A}_i)\ R \in \mathbb{R}^N$ is the vector of the estimated eigenvalues of the state transition matrix at Monte Carlo run number i. M is the total number of Monte Carlo runs.

$$V = \sum_{i=1}^{M} \|\mathrm{Re}(\lambda(\overline{A}_i)) - \mathrm{Re}(\lambda(A))\|_F^2 + \sum_{i=1}^{M} \|\mathrm{Im}(\lambda(\overline{A}_i)) - \mathrm{Im}(\lambda(A))\|_F^2 \qquad (17)$$

The squared Frobenius norm of a matrix $A \in \mathbb{R}^{m \times n}$ is equal to the trace of the product $A^T A$, and defined as follows:

$$\|A\|_F^2 = \mathrm{tr}(A^T A) = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij}^2. \qquad (18)$$

For a complex number $c = a + \mathrm{j} \cdot b$ we define:

$$\mathrm{Re}(c) = a \text{ and } \mathrm{Im}(c) = b. \qquad (19)$$

Figure 3 shows the criterion (17) as a function of ω and L. Each point in the figure is the sum given by (17) from a Monte Carlo simulation at a specific ω with a specific L.

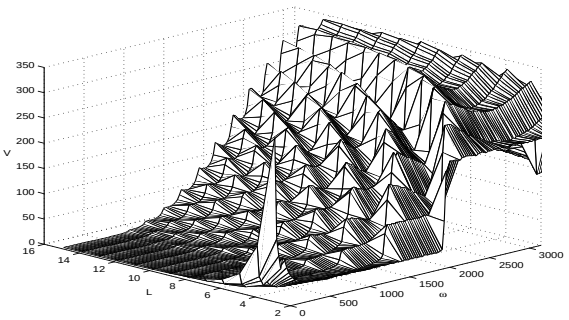

Fig. 3. The quality criterion (17) as a function of ω and L when using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$, and DSR with $n = 2$, $g = 0$ and $J = L + 1$ for identification.

From Figure 3 it is obvious that the choice of ω is the most crucial parameter. When zooming in Figure 3 for ω in the interval up to 1000 $[rad/s]$ we get Figure 4. Figure 4 shows that in this closed loop data set it is not favorable to choose L as small as possible as in open loop cases.

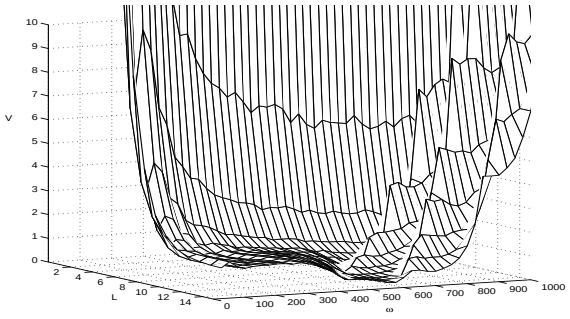

Fig. 4. The quality criterion (17) as a function of ω and L when using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$, and DSR with $n = 2$, $g = 0$ and $J = L + 1$ for identification.

2.2.3. Comparing the parameter estimates from DSR and PEM as functions of the frequency of the sinusoid signal used as dithering signal in the reference In Figure 5 PEM and DSR are compared as functions of ω of the sinusoid signal used as dithering signal in the reference. The criterion used is given by (17). DSR is used with $L = 12$ and $J = L + 1$. From Figure 5 two important notes have to be done. The estimates from PEM and DSR have optimums for dithering signals in different frequency ranges. DSR can actually give estimates with smaller bias and variance than PEM by choosing an appropriate frequency of the dithering signal.

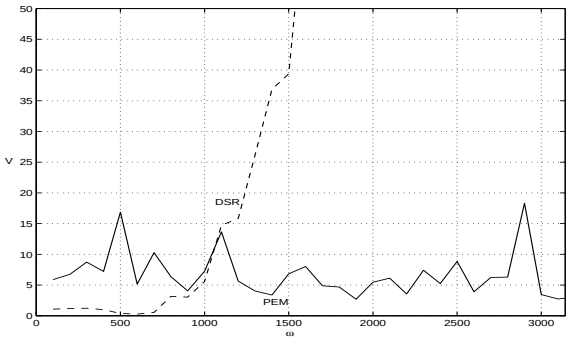

Fig. 5. The quality criterion (17) as a function of ω, when using a constant reference superposed a sinusoid signal as dithering signal $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$, and the estimates from PEM with default parameters and DSR with $n = 2$, $g = 0$, $L = 12$ and $J = 13$. PEM is illustrated by the solid line and DSR is illustrated by the broken line

2.2.4. Alternative closed loop quality measures Transforming (12) to observable canonical form gives system matrices with the following structure:

$$A = \begin{bmatrix} 0 & 1 \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad (20)$$

and E is till the zero matrix. The parameters to be estimated in the observable canonical form are collected in a parameter vector:

$$\theta = [a_{21} \ a_{22} \ b_{11} \ b_{21}]. \quad (21)$$

The true parameter vector is:

$$\theta = [-0.7656 \ 1.7500 \ -0.0010 \ -0.0005]. \quad (22)$$

In figure 6 the estimated parameters in the parameter vector from DSR and PEM are compared as a function of the runs in the Monte Carlo simulation. The reference is given by (16) with $\omega = 600$. DSR is used with the parameters $n = 2$, $g = 0$, $L = 12$ and $J = 13$. The parameter estimates from DSR all have small variance. The parameters b_{11} and b_{21} have a bias, but considering the scale of the axes the bias is marginal. The parameter estimates from PEM are characterized by large variance compared to the estimates from DSR. The mean and standard deviation of the parameters in the parameter vector are listed in Table 3 and Table 4 for respectively DSR and PEM.

Table 3. Mean and standard deviation from the Monte Carlo simulation using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$ with $\omega = 600$, and using DSR with $n = 2$, $g = 0$, $L = 12$ and $J = 13$ for identification

DSR	a_{21}	a_{22}	b_{11}	b_{21}
Mean	-0.7925	1.7815	-0.0057	-0.0037
Std	0.0185	0.0201	0.0003	0.0001

Table 4. Mean and standard deviation from the Monte Carlo simulation using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$ with $\omega = 600$, and using PEM with default parameters for identification.

PEM	a_{21}	a_{22}	b_{11}	b_{21}
Mean	-0.7194	1.7024	-0.0014	-0.0004
Std	0.1732	0.1814	0.0027	0.0018

A common quality measure is the simulated error (SE) which is the deviation between the real outputs in Y and the simulated outputs in Y^d. Using the squared Frobenius norm (18) we introduce the Squared Simulated Error (SSE):

$$SSE = \|Y - Y^d\|_F^2. \quad (23)$$

The SSE is plotted for DSR and PEM in figure (7) for the data generated to evaluate the estimation of the parameters in the parameter vector.

The SSE from the parameter estimates from DSR have smaller bias and variance than the SSE from

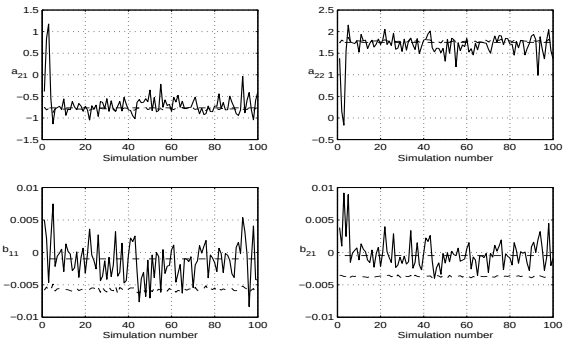

Fig. 6. The parameter vector (21) as a function of the runs in the Monte Carlo simulation using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$ with $\omega = 600$, using DSR with $n = 2$, $g = 0$, $L = 12$ and $J = 13$ and PEM with default parameters for identification. PEM is illustrated by the solid line, DSR is illustrated by the broken line and the true value is illustrated by the dash dotted line.

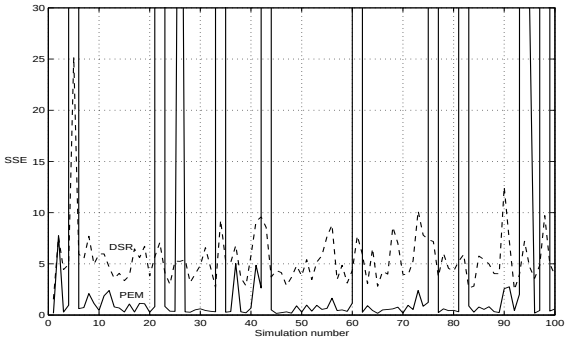

Fig. 7. Squared Simulated Error (SSE) for each of the runs in the Monte Carlo simulation using a constant reference superposed a sinusoid signal as dithering signal, $r_k = 1 + 0.1 \cdot \sin(\omega \delta t k)$ with $\omega = 600$, using DSR with $n = 2$, $g = 0$, $L = 12$ and $J = 13$ and PEM with default parameters for identification. PEM is illustrated by the solid line and DSR is illustrated by the broken line.

the parameter estimates from PEM. Actually the mean and standard deviation of the SSE is respectively approximately 5.5 and 2.8 for DSR. The corresponding values from PEM are both exceeding working precision. This because of the unstable models from PEM shown in Figure 2.

3. CONCLUDING REMARKS

Our simulations on finite data sets from a system operating in closed loop have shown that by using an appropriate dithering signal in the reference the DSR algorithm can provide eigenvalue estimates with smaller bias and variance than PEM, from the system identification toolbox in Matlab 6.5. As dithering signal we have tried sinusoid signals at different frequencies. We have observed that DSR and PEM have optimums in different frequency ranges. Comparing the optimums of DSR and PEM we find that DSR gives eigenvalue estimates with smaller bias and variance at its optimum than PEM does at its.

An other observation is that DSR provide eigenvalue estimates with smallest bias and variance when the dithering signal is low frequent.

When using an external dithering signal in the reference the choice of the frequency of the dithering signal is the most crucial parameter when using DSR on closed loop data, except the choice of model order which is not treated in this work. When choosing the prediction horizon L in DSR it is important to use an L that is large enough. This is opposite of what is used in identification of open loop data. Then keeping L as small as possible helps keeping the variance small, especially when input signals are poorly excited.

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