A Smith Predictor Enhanced PID Controller

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Abstract: In this work, a Smith predictor enhanced PID controller, SP-PID, is proposed. A tuning parameter K_{sp} is devised which gradually transforms the controller from a PID controller to a Smith predictor as K_{sp} changes 0 to 1. Properties of the SP-PID are explored and design procedure is given to ensure a certain degree of robustness. Simulation results clearly indicate that the SP-PID takes advantage of the SP when small modeling error is encountered and it is gradually detuned to a PID controller, a user friendly controller, when the model quality degrades. Moreover, the proposed concept can be implemented to current process control computer with virtually no extra hardware cost.

Keywords: dead time compensation, Smith predictor, PID control, robust performance

1. Introduction

The Smith predictor (SP) introduced by Otto Smith (1957) provides a nice controller structure for deadtime compensation and it has received considerable attention over past 40 years (Meyer et al., 1976; Ogunnaike and Ray, 1979; Morari and Zafiriou, 1989; Huang et al., 1990; Palmer and Blau, 1994; Lee et al., 1996; Kwak et al., 1999; Majhi and Atherton, 2000; Kaya, 2001; Ingimundarson and Hägglund, 2002). However, the modeling requirement, non-trivial tuning, and unfamiliarity prevent wide-spread applications. A typical scenario is that, in order to maintain robust stability, the associated PI controller is detuned to such a degree that control performance is no better than a simple PI controller and this is especially true for systems with small deadtime to time constant ratio (D/τ) (Ingimundarson and Hägglund, 2002). Despite the clear advantage for deadtime dominant processes, the Smith predictor again finds limited applications throughout process industries. On the other hand, we have found PID controllers remain as a standard feature in process industries (Åström and Hägglund, 1995; Yu, 1999; Tan et al., 1999).

The modeling problem of the SP can be overcome using relay feedback tests which also becomes a standard feature in many process control computers (Åström and Hägglund, 1984; Luyben, 1987; Chang et al., 1992; Palmer and Blau, 1994; Yu, 1999; Majhi and Atherton, 2000; Wang et al., 2003). Based on the shape information, the model structure as well as model parameters can be identified in a single relay feedback test (Luyben, 2001; Thyagarajan and Yu, 2003). Once the process model is available, one can proceed with controller tuning. In this work, a new type of controller is proposed which includes the Smith predictor as an enhanced feature of a standard PID controller. Next, a new tuning parameter, K_{sp}, is devised which provides a gradual transition between the SP (K_{sp}=1) and PID (K_{sp}=0) controller. That is the Smith predictor is functioning at its full capacity when the model quality is good and the deadtime compensation feature is gradually turned off when plant-model mismatch develops, and, ultimately, the controller reduced to the familiar PID controller.

The remainder of this paper is organized as follows. The concept of SP-PID is depicted and design procedure is also given in section 2. Improved PID design is proposed and robust performance of SP-PID is explored in section 3 followed by the conclusion.

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2. Smith predictor enhanced PID control

. 2.1 Concept

Ingimundarson and Hägglund (2002) present an interesting paper showing that the performance of the SP (Fig. 1b) is inferior than a PI (Fig. 1A) controller for first order plus deadtime systems (FOPDT) with the deadtime to time constant ratio (D/τ) less than 0.2. Moreover, for PID controller, the advantage of the SP can only be seen for FOPDT systems with D/τ greater than 10. It is troublesome because how can a modeled-based approach, with correct controller structure, fail to achieve improved performance. The reason is obvious that the SP is significantly detuned to achieve certain degree of robustness (or the PI settings were tightened to obtain better performance). In other words, if a model-based controller is loosely

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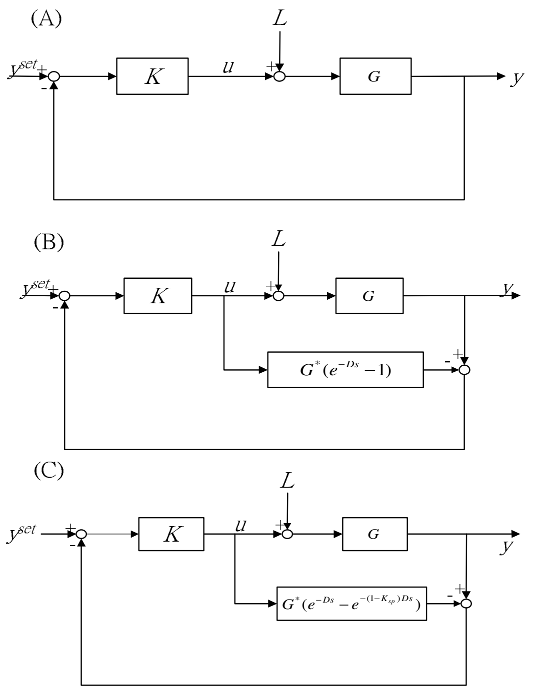

Fig. 1 Structures of: (A) PI control, (B) Smith predictor, and (C) Smith predictor enhanced PI control (SP-PID).

tuned, the performance can be no better than a PI controller. This scenario is often encountered in practice, especially when the users are not familiar with the controller and its tradeoff between the robustness and performance.

On the other hand, the progress in integrated circuits provided much increased computing ability for most controllers. From the hardware perspective, the implementation cost of a Smith predictor is virtually the same as a standard PID controller. How can we integrated the model-based SP into the standard PID controller while possesses some degree of user friendliness. Figure 1C shows the Smith predictor enhanced PID controller via a new tuning constant K_{sp}. Figure 1C indicates that when K_{sp} is set to one, it becomes the SP (Fig. 1B) and when K_{sp} is turned to 0, the feedback reduced to the conventional one, a PID type of controller. That is the amount of deadtime in the feedback (D_R, remaining deadtime) can be adjusted via K_{sp} from high performance/less robust/less familiarity (K_{sp}=1) to nominal performance/increased robustness/familiarity (K_{sp}=0). Certainly, this provides a gradual transition from a SP to a PID controller.

2.2 Analysis

Consider the block diagram in Fig.1 where the process G is expressed in terms of the delay-free part G^* and the deadtime portion e^{-Ds}.

$$G = G^* e^{-Ds} \tag{1}$$

The model also has a similar structure.

$$\tilde{G} = \tilde{G}^* e^{-\tilde{D}s} \tag{2}$$

when $\tilde{G}^*$ is the delay free part of the model and $\tilde{D}$ is the deadtime in the model. The closed-loop relationship of the SP can be expressed as

$$y = \frac{KG}{1 + K\tilde{G}^* + KG - K\tilde{G}} y^{set} + \frac{(1 + K\tilde{G}^* - K\tilde{G})G}{1 + K\tilde{G}^* + KG - K\tilde{G}} L \tag{3}$$

where y^{set} is the set point, K is the feedback controller and L is the load variable. With the perfect model assumption (i.e., $G = \tilde{G}$), we have :

$$y = \frac{KG}{1+KG^*} y^{set} + \frac{(1+KG^* - KG)G}{1+KG^*} L \tag{4}$$

The advantage of the SP can clearly be seen from the characteristic equation (denominator of Eq. 4) that the controller K can be designed aggressively to achieve a large bandwidth. For the proposed SP-PID controller in Fig. 1c, the closed-loop relationship becomes:

$$y = \frac{KG}{1 + K\tilde{G}^* e^{-(1-K_{sp})\tilde{D}s} + KG - K\tilde{G}} y^{set} + \frac{(1 + K\tilde{G}^* e^{-(1-K_{sp})\tilde{D}s} - K\tilde{G})G}{1 + K\tilde{G}^* e^{-(1-K_{sp})\tilde{D}s} + KG - K\tilde{G}} L \tag{5}$$

Similarly, when $G = \tilde{G}$, one obtains:

$$y = \frac{KG}{1 + KG^* e^{-(1-K_{sp})Ds}} y^{set} + \frac{(1 + KG^* e^{-(1-K_{sp})Ds} - KG)G}{1 + KG^* e^{-(1-K_{sp})Ds}} L \tag{6}$$

Eq. 6 clearly contrasts the difference between the SP and the SP-PID where, for the later, the remaining deadtime (D_R=(1- K_{sp})D) in the feedback loop is adjustable via K_{sp}. It becomes a SP on one end (K_{sp} =1) and resumes the role of a PID controller on the other end (K_{sp} =0).

Let us use the first order plus deadtime (FOPDT) process to illustrate the performance and robustness characteristics of SP-PID control. Consider

$$G(s) = \frac{K_p e^{-Ds}}{\tau s + 1} \tag{7}$$

Here, K_p is the steady-state gain, τ is the time constant, and D is the deadtime. Without loss of generality, we use IMC type of tuning rule for a PI controller (Morari and Zafiriou, 1989; Chien and Fruehauf, 1990). First, a closed-loop time constant λ is selected and the controller gain becomes:

$$K_c = \frac{\tau_I}{K_p \lambda} \tag{8}$$

and the rest time τ_I is simply set to:

$$\tau_I = \tau \tag{9}$$

Note that this assumption will be relaxed in a later section. In doing this, the closed-loop relationship for the SP-PID becomes:

$$y = \frac{e^{-Ds}}{\lambda s + e^{-(1-K_{sp})Ds}} y^{set} + \frac{\lambda s + e^{-Ds}\left(e^{K_{sp}Ds} - 1\right)}{\lambda s + e^{-(1-K_{sp})Ds}} \frac{K_p e^{-Ds}}{\tau s + 1} L \tag{10}$$

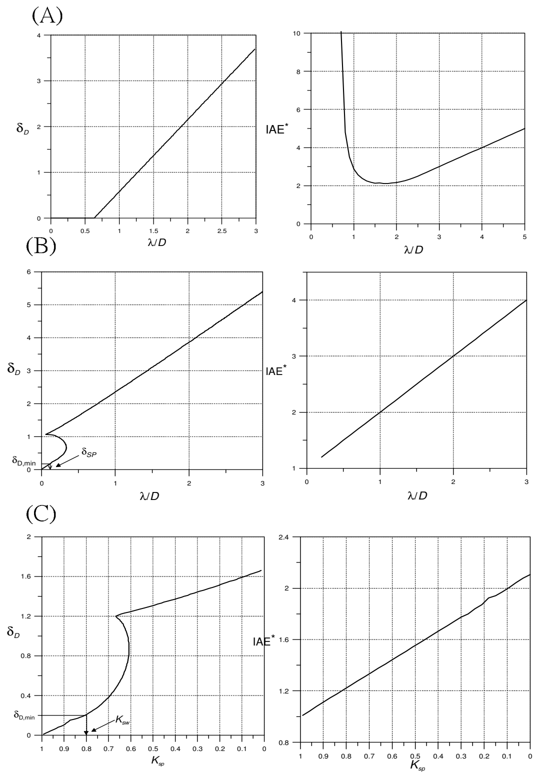

Fig. 2 Tolerable deadtime error and IAE for: (A) PI control, (B) Smith predictor, and (C) Smith predictor enhanced PI control.

where λ is the closed-loop time constant. The integrated error (IE) for a unit step setpoint change can be derived analytically using the final-value theorem.

$$(\text{IE})_{\text{setpoint}} = \lim_{s \to 0} s \frac{\lambda s + (e^{-(1-K_{sp})Ds} - e^{-Ds})}{\lambda s + e^{-(1-K_{sp})Ds}} \frac{1}{s} \cdot \frac{1}{s} \quad (11)$$

$$= \lambda + K_{sp} D$$

Eq. 10 shows that the IE is a function of the closed-loop time constant (λ) as well as K_{sp}. If λ is chosen to be a function of the remaining deadtime (D_R) in the feedback loop and following the IMC-PI tuning rule, we have:

$$\lambda = 1.7 D_{\text{R}} = 1.7(1 - K_{sp})D \quad (12)$$

and IE thus becomes:

$$(\text{IE})_{\text{setpoint}} = (1.7 - 0.7 K_{sp})D$$

Despite the fact that the IE is a good performance measure only for monotonic step response, it provides a qualitative indication of the closed-loop performance for the SP-PID. Figure 2 show a more realistic performance measure, the normalized integrated absolute error (IAE*), for the proposed controller. For a FOPDT system, the normalized IAE is defined as:

$$\text{IAE}^* = \frac{\text{IAE}}{\text{IAE}_{\text{min}}} \quad (13)$$

where IAE_{min} is the minimum IAE and for a FOPDT system it is simply IAE_{min} =D for a unit step change. Figure 2c shows that the performance is improved as we turn on the deadtime compensation portion of the

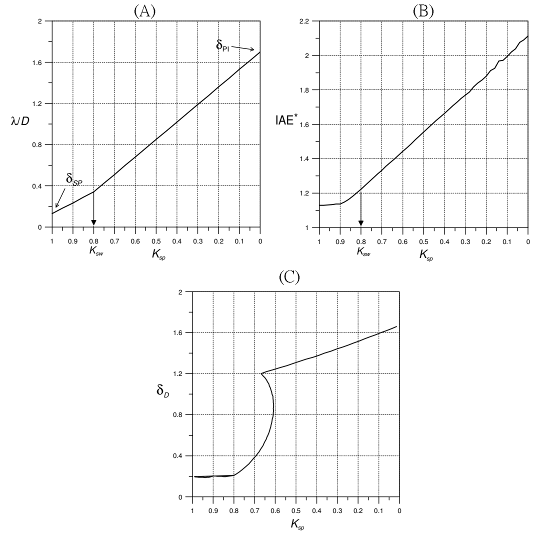

Fig. 3 Closed-loop time constant (A), IAE (B), and tolerable deadtime error (C) for SP-PID.

controller via an increased K_{sp}. Qualitative correct trend is also seen as compared to the simple performance measure in Eq. 12. Similarly, the performance of the SP can be conjectured from Eq. 10 by setting K_{sp} =1 and the normalized IAE (IAE*) is also shown in Fig. 2b. However, the performance degrades significantly for a PI controller at region of small λ (Fig. 2a) for an obvious reason: the stability constraint is almost violated and oscillatory responses result at small λ region. The performance part (IAE*) of Fig. 2 indicates that the SP-PID takes the advantage of the Smith predictor at the high performance region and it is reduced to a PID controller when performance requirement is not high.

The characteristic equation in Eq. 5 can be used to evaluate robust stability with respect to deadtime error. Assume uncertainty occurs in the deadtime part (i.e., $D \neq \tilde{D}$) the closed-loop characteristic equation (CLCE) becomes:

$$\lambda s + e^{-(1-K_{sp})\tilde{D}s}(e^{-(K_{sp}+\delta_D)\tilde{D}s} - e^{-K_{sp}\tilde{D}s} + 1) = 0 \quad (14)$$

where δ is the multiplicative deadtime error, i.e. $\delta = (D - \tilde{D}) / \tilde{D}$. Taking the extremes ($K_{sp}$ =0 and K_{sp} =1), the CLCE's for the PI and SP are:

$$\lambda s + e^{-(1+\delta)\tilde{D}s} = 0 \quad (15)$$

$$\lambda s + e^{-(1+\delta)\tilde{D}s} - e^{-\tilde{D}s} + 1 = 0 \quad (16)$$

Eqs. 14~16 can be used to compute tolerable deadtime errors (δ_D) for different closed-loop time constants (Cheng and Hwang, 1999). By tolerable deadtime error we mean the amount of deadtime error leads to the limit of stability. Interesting enough, conditional stability is observed for the SP as well as SP-PID over certain ranges of controller settings when is not observed for the PI controller. Figure 2 also reveals that extreme sensitivity for deadtime error when λ approaches zero or K_{sp} approaches one. Certainly, such settings are not acceptable in practice, despite almost perfect performance.

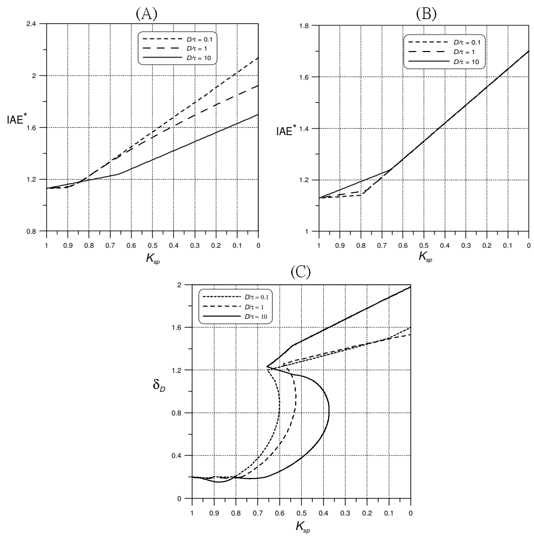

Fig. 4 IAE's for unit step setpoint (A) and load (B) changes and tolerable deadtime error (C) using SP-PID.

2.3 Design

The design principle is to turn the SP gradually toward the PID controller when the model quality degrades while maintain a certain of robustness at the SP end with respect to deadtime error. Let us consider controller tunings at two extremes, K_{sp} =0 and K_{sp} =1. At the PI control end (K_{sp} =0), the closed-loop time constant λ is set proportional to the deadtime. That is:

$$\lambda = \delta_{PI} \cdot D \qquad (17)$$

A typical value of $\delta_{PI} = 1.7$ is often employed according to the IMC-PI rule. At the SP side, λ is often selected to handle a pre-determined degree of deadtime error. Note that the setting does not have to be conservative because the tradeoff between robustness/performance is handled by the controller via K_{sp}. A typical value of 20% tolerable deadtime error ($\delta_{D,\min} = 0.2$) is recommended at the performance end. Therefore, we can read-off δ_{SP} directly from Fig. 2b given $\delta_{D,\min}$ (or computed δ_{SP} from Eq. 16). Thus, we have

$$\lambda = \delta_{SP} \cdot D \qquad (18)$$

In theory, Eqs 17 and 18 will be sufficient to formulate the tuning constant for the SP-PID. However, in order to ensure the tolerable deadtime error is larger than $\delta_{D,\min}$ over the entire K_{sp} range (0~1), a switching point of K_{sp} has to be located that gives the tolerable deadtime error to be $\delta_{D,\min}$. Then, a linear interpolation is employed to set the closed-loop time constant for K_{sp} greater than K_{sw} (Fig. 3). In other words, we have:

$$\begin{cases} \dfrac{\lambda}{D} = \left(1-K_{sp}\right)\delta_{PI}, & K_{sp} \le K_{sw} \\ \dfrac{\lambda}{D} = \left(1-K_{sp}\right)\delta_{PI} + \left(\dfrac{K_{sp}-K_{sw}}{1-K_{sw}}\right)\delta_{SP}, & K_{sp} > K_{sw} \end{cases} \qquad (19)$$

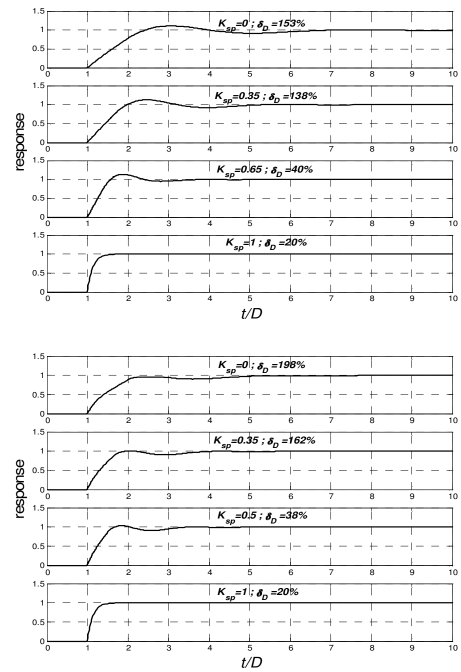

Fig. 5 Unit step setpoint responses usung different Ksp settings for systems with D/τ=1 (top) and 5 (bottom).

Therefore, the controller design procedure becomes:

S1. Select δ_{PI} (e.g., from IMC-PI tuning rule).

S2. Choose $\delta_{D,\min}$ (a typical value is $\delta_{D,\min}$ =0.2, 20% deadtime error).

S3. Find δ_{SP} from Eq.16 (or Fig. 2B) and K_{sw} from Eq. 15 (or Fig. 2C).

S4. Set the closed-loop time constant to $\frac{\lambda}{D} = \max\left[\left(1-K_{sp}\right)\delta_{PI}, \left(1-K_{sp}\right)\delta_{PI} + \left(\frac{K_{sp}-K_{sw}}{1-K_{sw}}\right)\delta_{SP}\right]$ If IMC-PI tuning rule is employed (i.e., $\tau_I = \tau$ and δ_{PI} =1.7), for $\delta_{D,\min}$ =0.2, we have: K_{sw} =0.8 and $\delta_{SP} = \lambda / D$ =0.129. Figure 3 shows the relationships among closed-loop time constant λ / D , performance (IAE*), and robustness to deadtime error (δ_D). Note that Fig.3 remains the same for FOPDT systems with all possible D/τ ratios provided with the plain IMC-PI tuning, i.e., $\tau_I = \tau$ and $K_c K_p = \tau_I / \lambda$. Fig. 3B also shows that the IAE at the PI and (K_{sp} =0) is at 210% of the minimum value as opposed to 170% predicted by Eq. 9 using the integrated error (IE). The reason is obvious that the area above and below the set point cancelled out each other, a non-monotonic set point response. Control performance can be improved using improved IMC-PI tuning rules (Morari and Zafiriou, 1989; Chien and Fruehauf, 1990).

3. Performance

3.1 Improved PID tuning

The improved IMC-PI tuning rule takes the effect of

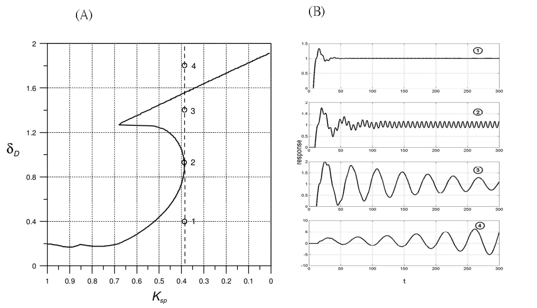

Fig. 6 Deadtime error resulting in conditional stability (A) and corresponding setpoint responses (B).errors

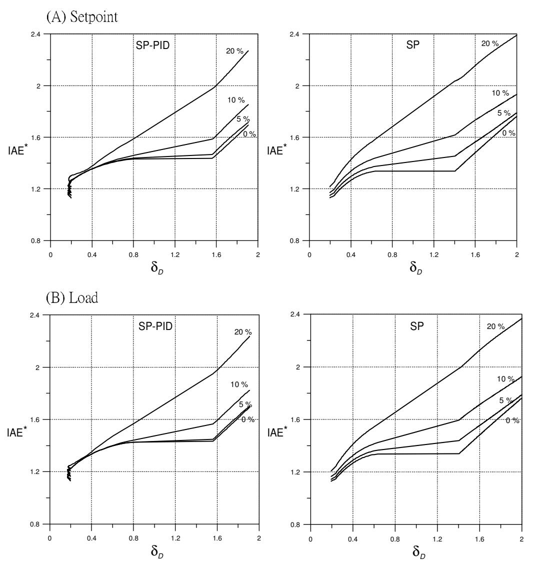

Fig. 7 IAE's for SP-PID and SP designed for the same amount of tolerable deadtime error under the same percentage (0, 5, 10, 20%) of deadtime errors.

deadtime into consideration and, thus, the reset time is adjusted according to $\tau_I = \tau + D_R/2$. Following the same line, the controller gain becomes:

$K_c K_p = \tau_I /(\delta_{PI} \cdot D_R) = \tau_I /(1.7 \cdot D_R)$

Using this setting, the closed-loop characteristic equation for the SP-PID can be expressed as (cf. Eq. 14):

$$\lambda s + \frac{\left\{1+\left[\tau+(1-K_{sp})\tilde{D}/2\right]s\right\}}{\tau S+1} e^{-(1-K_{sp})\tilde{D}s}\left[e^{-(K_{sp}+\delta)\tilde{D}s} - e^{-K_{sp}\tilde{D}s}+1\right] = 0 \tag{20}$$

Eq. 20 can be used to computed the switching value of K_{sp} (K_{sw}) and δ_{SP} given $\delta_{D,\min}$. Again, procedure in sec. 2.3 can be employed to design the SP-PID for systems with different D/τ values. Results reveal that the setpoint performance and robust stability are functions of D/τ while the regulatory performance is almost independent of the deadtime to time constant ratio as shown in Fig. 4. Furthermore, improved performance (e.g., IAE* =170% for D=10 with K_{sp} =0) actually can be achieved using a better tuning rule for the PI controller, especially at high D/τ region.

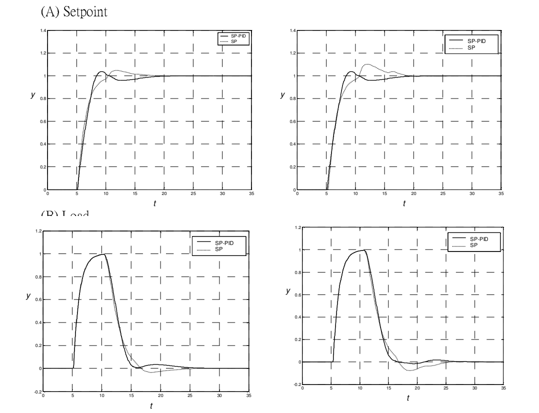

Fig. 8 Setpoint (A) and load (B) responses for +10% (left) and +20% (right) deadtime errors for D/τ=5 using SP and SP-PID (designed with the same amount of tolerable deadtime error)

Let us use a system with D=1 and D=5 to illustrate the performance of the SP-PID control. Figure 5 shows that better performance becomes evident as we gradually turn on the Smith predictor. But this is achieved with the expense of decreased robustness as also shown in Fig. 5,comparing the tolerable deadtime error (δ_D). Figure 5 clearly illustrates the advantage of the SP-PID control where aggressive tuning (provided with correct controller structure) can be applied when the model quality is good and the controller structure is detuned (via K_{sp}) as the model quality degrades.

3.2 Robust Performance

Up to this point, we have only studied the nominal performance of the SP-PID. One may wonder how the proposed one differs from a detuned Smith predictor? The SP-PID control has two clear advantages over a typical SP. First, it provides a transparent tuning constant K_{sp} in a finite range and the user can use the on-off concept to adjust the performance/robustness tradeoff while, for the SP, we really do not know to what degree the corresponding PI controller should be detuned when the model quality degrades. The second, but not an obvious one, is that the SP-PID gives a better robust performance. Let use design both the SP-PID (via K_{sp}) and the SP (via closed-loop time constant λ) with the same amount of tolerable deadtime error over a wide range of δ_D , 20%~200%. Again, let us use D=5 case as an example. Nominally, the SP gives better performance over the range of small δ_D as shown in Fig. 7. However, as the deadtime error increases to 5%, 10%, 20% of the nominal value, the SP-PID shows a slower rate of increase in the integrated absolute error (IAE*) as shown in Fig. 7. This is the case for both set point and load changes. Figure 8 compares the performance of the SP-PID and SP for 10 & 20% deadtime for the case of δ_D =40% (K_{sp}=0.52 for SP-PID and λ / D =0.264 for the SP). Similar behavior is observed for systems with different D/τ values.

Conclusion

In this work, a new type of controller, Smith predictor enhanced PID controller SP-PID, is proposed. A tuning parameter (K_{sp}) gradually transforms the controller from a PID controller to a Smith predictor as K_{sp} varies from 0 to 1, a controller with a well-defined range of settings. The PID parameters are adjusted according to the remaining deadtime in the feedback loop. The characteristics and stability of the SP-PID is explored and a design procedure is proposed to ensure a pre-specified degree of robustness. Following the design procedure with a minimum deadtime error of 20%, we have the following recommendations for 4 distinct settings:

(1) Set K_{sp}=1 right after autotuning. This correspond to tolerable deadtime error (δ_D) of 20% and normalized integrated absolute error (IAE*) of 1.13 for all D/τ.

(2) Set K_{sp}=0.65 for $D/\tau\leq1$ and K_{sp}=0.5 for $D/\tau>1$ when oscillatory behavior occurs. This corresponds to δ_D = 40% & IAE* =1.38 for D/τ=1 and δ_D = 38% & IAE*=1.35 for D/τ=10.

(3) Set K_{sp}=0.35 when oscillatory behavior is again observed. This corresponds to δ_D = 138% & IAE* =1.65 for D/τ=1 and δ_D = 162% & IAE*=1.45 for D/τ=10.

(4) Turn the SP-PID to a PID controller, K_{sp} =0, when significant oscillation still exists. This gives δ_D = 153% & IAE* =1.92 for D/τ=1 and δ_D = 198% & IAE*=1.70 for D/τ=10.

Certainly, when the PID control (K_{sp}=0) still cannot provide satisfactory control performance, a further detuning and/or autotune identification should be initiated. Simulation results show that the SP-PID indeed takes advantage of the theoretically correct SP structure while maintaining the familiar PID structure, user friendliness. More importantly, it can be implemented on current process control computers with virtually no extra hardware cost.

Acknowledgement

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