CLOSED-LOOP TIME DELAY ESTIMATION OF SISO PROCESSES FOR CONTROL PERFORMANCE MONITORING

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Abstract: A novel time delay estimation method for SISO control loop monitoring has been introduced. The proposed method requires a process regulated by a SISO controller during routine operation to be temporarily switched to relay control. The input and output data gathered from the process under relay control can be used to produce a time delay estimate.

Keywords: performance monitoring, time delay estimation, closed-loop, SISO process

1. INTRODUCTION

Many important contributions have been made in the area of control performance monitoring in recent years. These include a method for the computation of a minimum output variance benchmark from closed-loop data of a SISO process (Harris, 1989). From this benchmark, engineers can evaluate the performance of feedback controllers of the SISO process. This algorithm assumes an *a priori* knowledge of the process time delay. Because the process time delay is a limiting factor for feedback controller performance, the computed minimum variance benchmark is sensitive to the estimated time delay. Reliable and unobtrusive techniques for estimating the process time delay in the closed-loop are necessary to utilize the existing performance assessment techniques.

To address this issue, Lynch and Dumont (1996) make use of a closed-loop time delay estimation technique presented by Elnaggar (1990) known as fixed model variable regressor estimation (FMVRE). The FMVRE method determines the delay by minimizing the expectation of the model prediction error in a least squares square sense. Candidate models for FMVRE have a known ARMA structure, but an unknown delay. The time delay and model parameters are decoupled, and any parameter estimation technique can be used to estimate these quantities. Although this method is straightforward to implement and converges quickly, it has some limitations. One limitation of this method is the ARMA model structure must be known before the time delay and model parameters can be estimated. Elnaggar (1990) demonstrates FMVRE for a first-order plus dead time model. A flexible implementation which does not require process knowledge would make time delay estimation more widely applicable. Another limitation in employing FMVRE for time delay estimation is the process input must be sufficiently rich at high frequencies. This requirement means another design factor for engineers implementing time delay estimation; it is desirable to have a method which requires minimal user input.

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Owing to the fact that relay-based auto-tuning is very popular in practice, we propose a new delay estimation method based on a relay (Yu, 1999). The new estimation method addresses the limitations of the previous methods. It requires the process, which is regulated by a feedback controller during routine operation, to be temporarily regulated by a relay controller. The input and output data collected from the process under relay control can be used to obtain a process time delay estimate. As a byproduct of this approach, an estimate and confidence limits for the minimum variance benchmark can be established.

2. CLOSED-LOOP TIME DELAY ESTIMATION

Since the closed-loop impulse response coefficients before the time delay are feedback-invariant, they should remain unchanged even if the controller is changed dramatically. Therefore, we can calculate the impulse responses from two different controllers, one from the current controller and another from the relay controller. The delay can be determined by examining the impulse response coefficients from both controllers. The proposed method for time delay estimation consists of the following steps:

(1) Collect output history from the SISO process under routine feedback control.
(2) Form several windows of output history, fit an ARMA model from the output history in each window.
(3) Calculate an impulse response from each of the ARMA models identified in Step 2.
(4) Calculate the mean and confidence interval for each coefficient of the impulse responses calculated in Step 3.
(5) Switch control of the process to a relay controller. Collect input and output history of the process under relay control.
(6) Form several windows of input and output history, fit an ARMAX model to obtain the process transfer function from each window.
(7) Extract the average time-delay estimate from the models in Step 6.
(8) Determine the number of the feedback-invariant impulse response coefficients in the mean impulse response found in Step 4 by comparing coefficients with the mean impulse response found in Step 6. This calculation will provide another time delay estimate, which will confirm the value found in Step 7.

2.1 Computation of a Mean Impulse Response for a Closed-loop Process Using Routine Operation Data

The closed-loop SISO system is described by the following equations:

$$y(t) = H(q^{-1})u(t) + N(q^{-1})a(t) \tag{1a}$$

$$u(t) = -K(q^{-1})y(t) \tag{1b}$$

where K is the linear transfer function for a PID controller:

$$K(q^{-1}) = \frac{\kappa_0 + \kappa_1 q^{-1} + \kappa_2 q^{-2}}{1 - q^{-1}} \tag{2}$$

It is assumed H is stable, N can be either stable or with integrating elements, and *a(t)* is white noise. The closed-loop transfer function, which is determined by applying feedback to the linear process, is given by the following expression:

$$y(t) = \frac{N(q^{-1})}{1 + K(q^{-1})H^*(q^{-1})q^{-1-f}} a(t) \tag{3}$$

where *b=1+f* is the time delay. As shown by Harris (1989), the structure and parameters of the closed-loop transfer function are found by fitting a time-series model, typically a model of an ARMA process, to the closed-loop output data. The impulse response coefficients of the closed-loop process can be determined by performing long division on the closed-loop transfer function. The result will have the following form:

$$y(t) = \Bigg[\underbrace{\left(1 + \psi_1 q^{-1} + ... + \psi_f q^{-f}\right)}_{\text{Feedback - Invariant}} + \underbrace{\left(\psi_{f+1} q^{-f-1} + \psi_{f+2} q^{-f-2}\right) + ...}_{\text{Feedback - Varying}} \Bigg] a(t) \tag{4}$$

The first *f+1* coefficients of the impulse response from the closed-loop transfer function are feedback-invariant. They are identical to the first *f+1* impulse response coefficients that determined by performing long division on the disturbance transfer function $N(q^{-1})$ (Harris, 1989).

We desire to obtain an estimate of the impulse response of closed-loop process under PID control from the available output history. The following assumptions are made to make this task more tractable:

(1) The plant dynamics and time delay are approximately constant over the time period over which the closed-loop data is collected.

(2) The disturbance dynamics are approximately constant over the time period in which the closed-loop data is collected.

To obtain an estimate, we first use many sample sets of the output history to obtain estimates of the closed-loop transfer function of the process. A moving window approach is applied to obtain the sample sets of output history. From these closed-loop transfer functions, samples of the impulse response are collected, and a mean impulse response and its confidence limits are determined. As more samples of the impulse response are collected, the estimation error in the mean impulse response and its confidence limits will decrease. Knowledge of the confidence limits will allow for the uncertainty in the minimum variance benchmark to be determined once the time delay is known.

2.2 Estimation of Time Delay by Switching to a Relay Controller

After a mean impulse response and its confidence-limits have been established for the closed-loop process under PID control, it is necessary to switch the process controller to an on-off relay controller. Implementation of this controller requires that a relay of magnitude h be used as the input u in the feedback control system. Initially, the input is at 0. If the output is increased above its setpoint (y=0) by a disturbance, then the input u is switched to -h. As the output decreases below its setpoint, the input is switched to h. At some time after the time delay b has elapsed, the output will begin to increase. As the output crosses zero, the input is switched back to -h, and the cycle continues in this manner (Yu, 1999). Since the output lags behind to input by at least $-\pi$ radians, a limit cycle will be present in the output. In the limit cycle, the output period is the ultimate period. The time delay of the process along with the process time constant(s) will determine the length of the ultimate period according to the standard phase angle equation (Yu, 1999). If we neglect the contribution of the time constant(s) in the standard phase angle equation, half of the period will provide an upper bound on the time delay for the SISO process:

$$Delay \leq \frac{\pi}{\omega} = \frac{1}{2} Period \qquad (5)$$

The on-off relay controller is a nonlinear controller. This characteristic of the relay controller will result in there being no linear correlation between the output and input used to provide feedback. For modeling purposes, we treat the closed-loop data as though it were collected from an open-loop process. Thus, an average time delay of the process can be extracted by fitting time series models to the closed-loop input and output history of the process under relay control.

Another means of determining the time delay involves the calculation the mean impulse response of the disturbance model of the transfer function of the closed-loop process identified under relay control. The impulse response coefficients from the disturbance model can be superimposed on the mean impulse response and its confidence limits identified from the process under PID control. Before the time delay has elapsed, the impulse response coefficients not affected by feedback should be the same for both impulse responses. An estimate of the time delay can be determined by observing where the impulse response identified under relay control has deviated two standard deviations from the mean impulse response identified under PID control. This analysis is useful for verifying the average time delay estimate extracted from fitting closed-loop data obtained relay control to time series models.

2.3 Calculation of an Estimate of the Minimum Variance Benchmark

At this point, a time delay estimate b has been obtained from the process in a non-invasive manner. According to Harris (1989), the performance of the minimum variance controller can be calculated from the first $f+1$ coefficients:

$$\mathrm{var}\left\{y(t)\right\} = \left(1 + \psi_1^2 + ... + \psi_f^2\right)\sigma_a^2 \qquad (6)$$

In addition, the uncertainty in the minimum variance benchmark can be obtained by using the mean impulse response and confidence limits found under PID control.

3. SIMULATION RESULTS

3.1 Closed-loop Time Delay Estimation for a Stable Plant with an Added Stationary Disturbance

The proposed approach for closed-loop time delay estimation was applied to a process with a stable plant model and time delay and a stationary disturbance driven by normally distributed, white noise. Assume the process model with delay is given by

$$H(q^{-1}) = H^*\left(q^{-1}\right) q^{-1-f} = \left[\frac{0.1}{1 - 0.5q^{-1}}\right] q^{-6} \qquad (7)$$

Further, assume the stationary disturbance is given by

$$N(q^{-1}) = \frac{1}{(1-0.5q^{-1})}, \ \sigma_a^2 = (0.01)^2 \quad (8)$$

In routine closed-loop operation, it is assumed that the process is regulated by a PID controller. The tuning constants for the PID controller are $(\kappa_0, \kappa_1, \kappa_2) = (2, -2, 0.2)$, and are chosen so as to give closed-loop poles that stabilize the system. Upon implementing the chosen PID controller, the process is an ARMA process. In order to estimate the impulse response and confidence limits of the closed-loop transfer function, we fit multiple ARMA models to a time series of output data collected in the closed-loop. To accomplish this task, a moving window of 1000 discrete time steps is used over 3000 time steps of the process to collect the process output history. After the first 1000 steps of output history have been collected, an ARMA model is fit to this data, and the corresponding estimate of the impulse response is calculated. The window of data is shifted forward 100 time steps each time 100 additional time steps have been collected, so that 21 time series are collected. In each window, an ARMA model is fit to the data and, and the corresponding estimate for the impulse response is calculated.

A common approach to selecting the time series model structure is to use Akaike's final prediction error (FPE) as criterion. This criterion, which consists of a loss function quantifying the candidate model fit to the training data and a penalty for a candidate model of higher complexity, is minimized to determine model structure (Ljung, 1999). A linear search over the range of possible orders of polynomials A, B, and C to find the minimum FPE for the ARMAX model (Ljung, 1995):

$$A(q^{-1})y(t) = B(q^{-1})u(t-k) + C(q^{-1})a(t) \quad (9)$$

For the particular case in which the process is known to be an ARMA process, the order of polynomial B is assigned to be 0, and the orders of A and C are varied in the minimization of the FPE. The output, white noise, and input sequences for the process under PID control are shown in Figure 1.

A moving window is applied to obtain sets of time series by which to determine the mean impulse response. To simplify the ARMAX model identification, the polynomial orders are assumed to be equal, so that it is only necessary to search for two parameters. In each window, the polynomial orders of A and C were varied from 1 to 6, and the time delay k was varied from 1 through 12. The mean impulse response and its confidence limits are displayed in Figure 2.

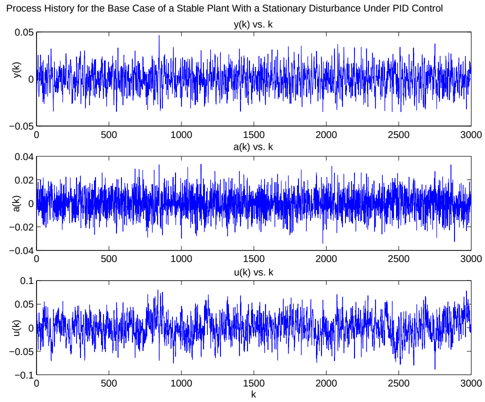

Fig. 1. Output, white noise, and input sequences for the stable plant with stationary disturbance under PID control

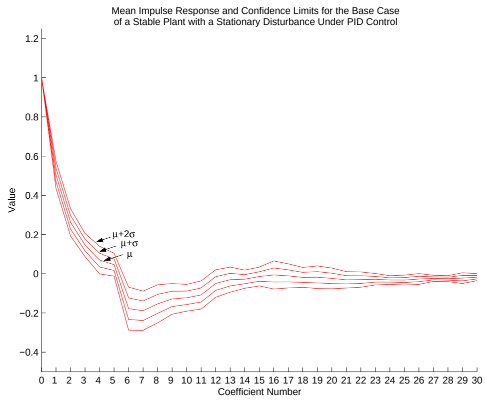

Fig. 2. Mean impulse response and its σ and 2σ confidence limits for the stable plant with stationary disturbance under PID control

In order to obtain the time delay estimate, the controller of the process is switched to an on-off relay controller. The control amplitude must chosen large enough so that the process output exhibits a limit cycle. In this simulation, the control amplitude is chosen to be 0.8 in the relay controller. The output and input history of the process under relay control, featured in Figure 3, is collected for 200 time steps.

A moving window of 100 time steps is used to obtain the mean impulse response of the disturbance model from this closed loop data. The moving window is slid every 10 samples after 100 samples have been collected, so that 11 estimates of the disturbance model are obtained. The process history within each window is fit to an ARMAX model, where the orders of polynomials A, B, C and the time delay are determined by minimizing the FPE. The polynomial orders of A, B, and C were varied from 1 through 6, and the time delay k was varied from 1 through 12. The mean impulse response of the estimated disturbance model is superimposed on the impulse responses obtained under PID control in Figure 5.

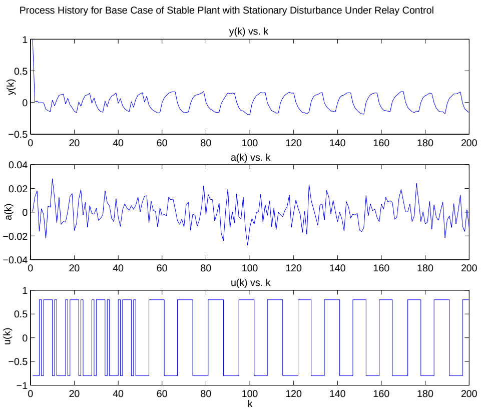

Fig. 3. Output and input history for base case of stable plant with stationary disturbance under relay control

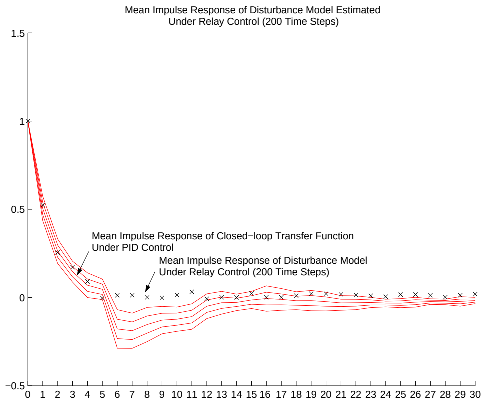

Fig. 4. Impulse response of the disturbance model estimated under relay control (200 time steps)

The 2σ confidence limit is used as the criterion for deciding which coefficients of the impulse response identified relay control are feedback-invariant. From Figure 5, it can be seen that this criterion indicates the first 5 to 6 coefficients are feedback-invariant. The average of the time delay given by the ARMAX models fit in each of the windows is found to be 5.7273.

To determine the effect of lowering the estimation error of the mean impulse response of the disturbance model on the time delay estimate, 2000 time steps of the base case for the process were simulated under relay control. A moving window of 1000 steps was implemented with the window being slid forward every 100 time steps collected after 1000. The mean impulse response of the estimated disturbance model is superimposed on the impulse responses obtained under PID control in Figure 6.

From Figure 6, it is observed that the more accurate impulse response estimate for the disturbance model yields a time delay estimate between 5 and 6. The average time delay estimated by the ARMAX models fit to process data is approximately

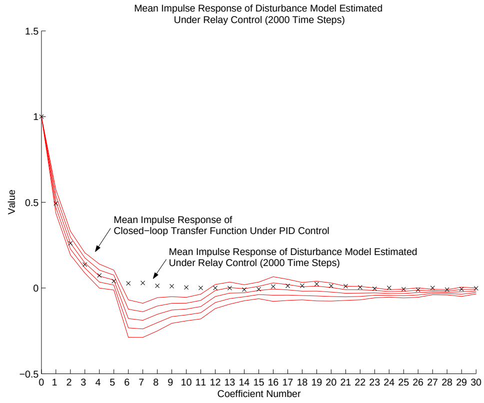

Fig. 5. Impulse response of the disturbance model estimated under relay control (2000 time steps)

6 if 200 time steps are used and 5 if 2000 time steps are used. The time delay estimates for this process are summarized in Table 1. The minimum variance benchmarks for time delays of 5 and 6 are shown in Table 2.

Table 1. Time Delay Estimate for Base Case

Amount of Data (Time Steps)	Feedback-Invariant Coeff. Approach	Average ARMAX Model Estimate
200	5-6	5.7273
2000	5-6	5.3636

Table 2. Output Variance Summary

	MV Benchmark (b=5)	MV Benchmark (b=6)
μ+2σ	0.001368	0.001657
μ	0.001184	0.001406
μ-2σ	0.001015	0.001179
	Output Variance of Current PID control:	0.002305

If a time delay estimate of 5 is considered, the current output variance is much greater than the upper 2σ confidence limit. This result indicates the current control strategy can likely be improved by retuning the PID control or by using a different controller. For the more conservative time delay estimate of 6, the current output variance is still much greater than the upper 2σ confidence limit, which provides further support to the claim that controller performance can be improved. The time delay estimates obtained from averaging the time delays extracted from the identified ARMAX models of the process underestimate the actual delay in this case. This result suggests the next highest integer delay is the better choice for the delay estimate.

3.2 Effect of Process Time Constant on Time Delay Estimate

We now consider a more general representation for the stable, minimum phase plants in the process:

$$H(q^{-1}) = \left[\frac{0.1}{1-\alpha q^{-1}}\right] q^{-6} \qquad (10)$$

If $\alpha = 0.01$ is used to simulate the process, the plant dynamics will be fast as compared to the disturbance dynamics. As a result, the disturbance dynamics will dominate the output response. If $\alpha = 0.99$ is used to simulate the process, the plant dynamics will be slow as compared to the disturbance dynamics. As a result, the plant dynamics will dominate the output response. The time delay estimates obtained by applying the proposed method to these two processes are featured in Tables 3 and 4.

Table 3. Time Delay Estimate for Case of $\alpha = 0.01$

Amount of Data (Time Steps)	Feedback-Invariant Coeff. Approach	Average ARMAX Model Estimate
200	5-6	5.7273
2000	5-6	5.2727

Table 4. Time Delay Estimate for Case of $\alpha = 0.99$

Amount of Data (Time Steps)	Feedback-Invariant Coeff. Approach	Average ARMAX Model Estimate
200	7-8	5.0000
2000	8-9	4.6364

The results of Table 3 indicate the time delay estimation method performs well when the plant dynamics are faster than the disturbance dynamics. In Table 4, we observe the accuracy of the time delay estimates are poorer when the plant dynamics are slower than the disturbance dynamics. In general, the average time delay estimate obtained from the ARMAX models is robust to processes with varying plant dynamics.

3.3 Upper Limit of Time Delay Estimate from Standard Phase Angle Equation

As mentioned previously, the standard phase angle equation gives an upper limit on the time delay as half of the period of the output response for the process under relay control. The upper limit on the time delay for the three simulated process is shown in Table 5. The period is an average over five oscillations of the output response.

Table 5. Upper Limit of Time Delay from Standard Phase Angle Equation

$\alpha = 0.01$	$\alpha = 0.5$	$\alpha = 0.99$
6.0	6.7	10.8

As observed in Table 5, the upper limit on the time delay approaches the time delay if the plant dynamics are much slower than the disturbance dynamics. In this situation, the upper limit provides a convenient technique for estimating the process time delay.

4. CONCLUSIONS

A novel time delay estimation method for SISO control loops has been introduced. The proposed method requires a process regulated by a PID controller during routine operation to be temporarily switched to relay control. The input and output data gathered from the process under relay control can be used to produce a time delay estimate.

The proposed time delay estimation method yields an average time delay estimate from the fitting of time series models to the input and output data. A time delay estimate can also be gleaned from application of the feedback-invariant property of the impulse response of the closed-loop transfer function. The latter time delay estimate is less reliable than the former estimate, but is useful for verifying the first estimate. An upper limit to time delay is obtained by calculated the period of the output response for the process under relay control.

Future studies should be focused on determining the robustness of the time delay estimates for processes with higher order plant models, higher order disturbance models, nonstationary disturbances, and higher white noise variances.

5. ACKNOWLEDGEMENTS

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