COMPLEXITY REDUCTION OF NONLINEAR SYSTEMS FOR PROCESS CONTROL

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Abstract: While models of high complexity are very useful for simulation and prediction purposes in the process industry, they may not be adequate for controller design. This paper proposes a procedure for model complexity reduction for this specific purpose by combining several nonlinear identification and optimization techniques. The available complex simulation model is used as surrogate plant to generate control relevant input and output trajectories and both an optimization of the input and an iterative nonlinear identification procedure are used. The results are illustrated on a nonlinear exothermic copolymerization reactor example.

Keywords: nonlinear identification, model reduction, process control, orthogonal basis function, Volterra representation

1. INTRODUCTION

In the process sector, detailed models of very high fidelity are almost always needed during the design phase. They are usually first principles models based on physical and/or chemical considerations (Ogunnaike and Ray, 1996). Such simulation models give a good approximation of the real process behavior, but tend to be too complex to be used for advanced process controller design, mainly because they are not explicitly developed for this purpose.

Assuming a given fully characterized first principles model, this paper proposes an iterative procedure for model complexity reduction from dedicated simulations (experiments) on the complex model. The novelty in the approach lies in combining several black-box empirical modeling techniques (Ljung, 1999; Nelles, 2001; Pintelon and Schoukens, 2001) with a certain knowledge of the plant, which permits optimizing the input design phase and building the model step by step in an iterative manner. This might be unthinkable in the usual identification setting, where the number of experiments is a main limitation and furthermore all signals are corrupted by noise.

The main assumption made on the system is that it admits a Volterra representation (Schetzen, 1980). It is a sufficiently general system class to fit the input/output behavior of many system processes and furthermore, at least several model based controllers can easily be designed based on this representation (Doyle III *et al.*, 2002).

Three known results from system identification are used by the proposed procedure: frequency domain identification in the presence of nonlinearities (Schoukens *et al.*, 1998), the theory of generalized orthonormal bases for system representation (Ninness *et al.*, 1999), and the forward selection orthogonal least squares methods (Chen *et al.*, 1989; Chen *et al.*, 2003). The procedure is additionally iterative, and on each step the pole of a basis function is chosen and the input excitation for identification is optimized.

In the next section the assumptions on the input/output behavior of the system are given, which concern mainly the Volterra representation and its parametrization with respect to basis functions. Section 3 gives an overview of the linear and nonlinear identification tools used, namely frequency domain identification and orthogonal least squares techniques. The proposed procedure for model complexity reduction is fully explained in Section 4, while in Section 5 the procedure is tested on the model of a copolymerization reactor. Finally, some conclusions are presented. Throughout the text, $\mathbb{R}$ and $\mathbb{C}$ denote the real and complex sets, respectively, $\langle \cdot, \cdot \rangle$ denotes scalar product, and $\overline{\xi}$ denotes the complex conjugate of ξ.

2. ASSUMPTIONS ON THE PROCESS

Given an input time sequence $u(t)$, the output $y(t)$ can be obtained through simulation of the given complex model $\mathcal{S}$, i.e. $y(t) = \mathcal{S}[u(t)]$. It is assumed that initial conditions are always the nominal operating conditions. The aim is thus to obtain a (possibly nonlinear) model $\mathcal{M}$,

$$\hat{y}(t) = \mathcal{M}[u(t)] \approx \mathcal{S}[u(t)] = y(t), \tag{1}$$

that is useful for process control design. This paper limits the reduced model $\mathcal{M}$ to be discrete and, without loss of generality, t is taken as normalized discrete time, i.e. $\tau = tT_s$, where T_s is the sampling period and τ is real time.

Most control systems satisfy the assumption that their input/output behavior $\mathcal{S}$ can be sufficiently well approximated by a Volterra series (Lesiak and Krener, 1978). The finite discrete Volterra series is given by (Doyle III *et al.*, 2002)

$$\hat{y}(t) = y_0 + \sum_{j=1}^{n} v_m^j(t), \tag{2}$$

with $v_m^j(t) = v_m^j\left(u(t-1), \ldots, u(t-m)\right)$ the j-th order terms with memory length m, i.e.

$$v_m^j(t) = \sum_{i_1=0}^{m} \cdots \sum_{i_j=0}^{m} \alpha_j(i_1, \ldots, i_j) \prod_{k=1}^{j} u(t-i_k). \tag{3}$$

The coefficients $\alpha_j(i_1, \ldots, i_j)$ are called the j-th order kernels. It is a well known fact that *fading memory systems* (Boyd and Chua, 1985) may be approximated uniformly on bounded input sets by a finite Volterra system with n and m sufficiently large. Notice, however, that the number of terms can become quite large.

Finite Volterra models are BIBO stable, have periodic responses to periodic inputs without subharmonic generation, and can exhibit input multiplicities (same steady state output y_s for distinct steady state inputs u_s), but no output multiplicities (one u_s leads to various y_s). They also preserve asymptotic constancy, i.e. if $u(t) \to u_s$ as $t \to \infty$, then $y(t)$ also approaches a constant limit y_s.

If the memory m is allowed to be infinite, then the Volterra class may encompass more complex nonlinear behaviors, but this is impossible to implement directly. Therefore the following parametrization in terms of *generalized orthonormal basis functions* (GOBFs) is considered (Ninness and Gustafsson, 1997).

Given a set of poles $\{\xi_1, \xi_2, \ldots, \}$ satisfying

$$\begin{gathered} \sum_{k=0}^{\infty} (1 - |\xi_k|) = \infty, \qquad |\xi_k| < 1, \\ \xi_k \in \mathbb{C} \implies \xi_{k+1} = \overline{\xi_k}, \end{gathered} \tag{4}$$

the k-th GOBF is given by

$$B_k(z) = \frac{\sqrt{1 - |\xi_k|^2}}{z - \xi_k} \prod_{i=1}^{k-1} \frac{1 - \overline{\xi}_i z}{z - \xi_i}. \tag{5}$$

The basis $\{B_1(z), B_2(z), \ldots\}$ is complete and the GOBFs are orthonormal, i.e.

$$\langle B_i, B_j \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} B_i(e^{j\omega}) \overline{B_j(e^{j\omega})} d\omega = 0 \tag{6}$$

for $i \neq j$ and $\langle B_i, B_i \rangle = 1$. This set of basis functions generalizes the Laguerre basis ($\xi_k = \alpha$ for all k), the Kautz basis ($\xi_{k,k+1} = \gamma \exp(\pm j\phi)$) and the finite impulse response basis ($\xi_k = 0$).

If the Volterra kernels $\alpha_j(i_1, \ldots, i_j)$ vary regularly as functions of their arguments, then they can be expanded as

$$\alpha_j(i_1, \ldots, i_j) = \sum_{r_1=1}^{\infty} \cdots \sum_{r_j=r_{j-1}}^{\infty} \gamma_j(r_1, \ldots, r_j) \prod_{k=1}^{j} b_{r_k}(i_k). \tag{7}$$

where $\gamma_j(\cdot)$ are kernels (i.e. parameters) and $b_i(t)$ is the inverse z-transform of $B_i(z)$, i.e.

$$b_i(t) = \mathcal{Z}^{-1}\left\{B_i(z)\right\}. \tag{8}$$

Such an expansion is exact if the j-th order kernel is *stably separable* and *strictly proper* (Seretis and Zafiriou, 1997), which basically means that it can be expressed as the product of j first order kernels and $\alpha_j(i_1, \ldots, i_j) = 0$ for $i_1 \cdots i_j = 0$. A direct substitution of (7) in (3) with $m = \infty$ and regrouping leads to

$$v_\infty^j(t) = \sum_{r_1=1}^{\infty} \cdots \sum_{r_j=r_{j-1}}^{\infty} \gamma_j(r_1, \ldots, r_j) \times \prod_{k=1}^{j} \left(\sum_{i=0}^{\infty} b_{r_k}(i) u(t-i) \right). \tag{9}$$

Defining

$$\psi_r(t) = \sum_{i=0}^{\infty} b_r(i) u(t-i) \tag{10}$$

and truncating the sums up to q, the finite GOBF-Volterra series is finally given by:

$$\hat{y}(t) = y_0 + \sum_{j=1}^{n} \sum_{r_1=1}^{q} \cdots \sum_{r_j=r_{j-1}}^{q} \gamma_j(r_1, \ldots, r_j) \prod_{k=1}^{j} \psi_{r_k}(t). \quad (11)$$

Compared to the finite Volterra series (2), this parametrization usually requires only $q \ll m$ basis functions for comparable accuracy, provided the poles are chosen adequately. Notice (10) is the response of the (linear) r-th GOBF to the input $u(t)$, so it quite easy to generate all required responses $\psi_r(t)$ by implementing a realization of the linear system with transfer function matrix

$$\boldsymbol{B}(z) = \begin{bmatrix} B_1(z) & \cdots & B_q(z) \end{bmatrix}^T. \quad (12)$$

Having these responses, identification of the parameters $\gamma(\cdot)$ is simplified substantially, since (11) is linear in the parameters and usual least squares techniques may be used (Nelles, 2001). The difficulty lies in adequately selecting the poles and the number of GOBFs used. For linear systems it is known that poles close to the dominating frequencies will yield a more compact representation (Gómez, 1998). The proposed procedure iteratively selects these poles based on frequency response data, using a *best* linear identification of the error system.

3. SYSTEM IDENTIFICATION TOOLS

Linear system identification theory is a very mature subject (Ljung, 1999). Its main objective is to obtain a system model under restrictive experimental conditions, which basically pertain to measurement and generator noise. Under the framework studied here these problems do not appear, but the complexity (e.g. nonlinearity) of the system $\mathcal{S}$ makes the task no less difficult.

In the iterative procedure to be outlined in the next section, two identification techniques are used. First the identification of a *best linear* system is performed, and then an orthogonal least squares technique for structure selection and parameter estimation is implemented. The first identification has as objective to select a set of candidate GOBFs based on the significant poles of the linear approximation to the *difference* between the complex model and the current reduced model. It is based on the following result (Schoukens *et al.*, 1998).

Assume a nonlinear system with a convergent Volterra representation (2) is excited with a *random multisine*, i.e. a periodic signal with fixed amplitude spectrum, but random phase spectrum. Then the measured frequency response function (FRF) will have the following decomposition under ideal measurements

$$G(j\omega_k) = G_0(j\omega_k) + G_B(j\omega_k) + G_S(j\omega_k). \quad (13)$$

The measured FRF is the quotient $Y(k)/U(k)$ of the measured output spectrum and the measured input spectrum, whereby k is an index for the excited frequency lines. The term $G_0(j\omega_k)$ in (13) corresponds to the underlying linear system and is independent of the spectra of the input signal. The second term corresponds to the *systematic* nonlinear contributions, which depend only on the (deterministic) amplitude spectrum but not on the random phases. The third term corresponds to the *stochastic* nonlinear contributions, which are completely dependent on the (random) input spectrum, so that $Y_S(k) = G_S(j\omega_k)U(k)$ behaves like noise, uncorrelated with the input and with zero mean. For this reason, the response of the nonlinear system (2) to a random multisine can be regarded as the response through a *related linear dynamical system* $G_R(j\omega_k) = G_0(j\omega_k) + G_B(j\omega_k)$ plus some *nonlinear noise* due to the stochastic contributions.

There are a host of techniques available for system identification in the frequency domain (see e.g. the book by Pintelon and Schoukens (2001) and the references therein), given either the measured data $Y(k), U(k)$ or the measured FRF $G(jw_k)$ and certain noise assumptions. *Subspace algorithms* implemented in the frequency domain (McKelvey *et al.*, 1996) are used in the present approach. These produce a state space realization $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ for the fit $\hat{G}(z)$ to the FRF $G(j\omega_k)$. On one hand, a consistent estimate of $G_R(j\omega_k)$ with respect to the nonlinear noise is guaranteed as the number of realizations of the same random multisine tends to infinity. On the other hand, being based on principal component analysis, the dynamic order must not be fixed *a priori*. This is important in the proposed procedure, since only significant candidate poles should be tested and these are directly given by the eigenvalues of $\hat{A}$.

The other nonlinear identification scheme used in the procedure pertains structure selection and parameter estimation using orthogonal least squares techniques (Chen *et al.*, 1989). The assumed structure for the system is linear in the parameters, i.e.

$$y(t) = \boldsymbol{\varphi}^T(t)\boldsymbol{\theta} + e(t) \quad (14)$$

where $\boldsymbol{\theta} \in \mathbb{R}^r$ is the parameter vector, $\boldsymbol{\varphi} \in \mathbb{R}^r$ is the *regressor* vector, and $e(t)$ stands for the modeling error. The regressors must be functions of the measured data (e.g. filtered inputs). From (11) it is clear that grouping all possible $\gamma(\cdot)$ in $\boldsymbol{\theta}$, the regressor vector is composed of products of the type $\prod_i \psi_{r_i}(t)$. Notice that the number of regressors for (11) can be quite large, namely $r = \binom{n+q}{q}$, so estimating $\boldsymbol{\theta}$ as the least squares so-

lution may be numerically difficult. Furthermore, it would contradict the reduced complexity goal.

The identification scheme considers

$$\boldsymbol{y} = \boldsymbol{\Phi\theta} + \boldsymbol{e} = \boldsymbol{W\vartheta} + \boldsymbol{e} \tag{15}$$

where the data is given as N time domain samples:

$$\boldsymbol{y} = \big[y(1),\, y(2),\, \ldots,\, y(N)\big]^T, \tag{16}$$

and the t-th row of the $N \times r$ matrix $\boldsymbol{\Phi}$ corresponds to the regressor $\boldsymbol{\varphi}^T(t)$. Assume $\boldsymbol{\Phi}$ has been orthogonally decomposed, i.e.

$$\boldsymbol{\Phi} = \boldsymbol{WA}, \qquad \boldsymbol{\vartheta} = \boldsymbol{A\theta} \tag{17}$$

where $\boldsymbol{A} \in \mathbb{R}^{r\times r}$ is an upper triangular matrix with ones in the main diagonal and $\boldsymbol{W} = [\boldsymbol{w}_1, \ldots, \boldsymbol{w}_r]$ is a matrix with orthogonal columns.

The main aspect about orthogonal least squares techniques is that each orthogonal regressor $\boldsymbol{w}_i$ contributes independently to minimizing the quadratic cost function of the output error and $\hat{\vartheta}_i = \hat{\vartheta}_i(\boldsymbol{w}_i, \boldsymbol{y})$. Assume $\boldsymbol{w}_1$ is the most significant regressor in this sense and $\boldsymbol{w}_r$ is the least significant one. To obtain a good balance between model complexity —measured by the number of regressors— and good prediction capabilities (a small error), consider only the first, say ρ, orthogonal regressors. In essence most structure selection algorithms iteratively reorder the columns of the matrix $\boldsymbol{\Phi}$ so that the reduced model is given by

$$\boldsymbol{y} \approx \widetilde{\boldsymbol{W}}\tilde{\boldsymbol{\vartheta}} \qquad \text{and} \qquad \tilde{\boldsymbol{\vartheta}} = \widetilde{\boldsymbol{A}}\tilde{\boldsymbol{\theta}}, \tag{18}$$

where $\widetilde{\boldsymbol{W}}$ comprises the first ρ columns of $\boldsymbol{W}$ and $\widetilde{\boldsymbol{A}}$ comprises the upper left $\rho \times \rho$ sub-matrix of $\boldsymbol{A}$.

Most orthogonal least squares algorithms select in a forward manner the most significant regressors while following an orthogonalization procedure, e.g. Gram-Schmidt (Chen *et al.*, 1989). A recent paper (Chen *et al.*, 2003) proposes to combine the modified Gram-Schmidt procedure with local regularization, together with a D-optimality design in order to obtain a sparse model. The local regularization aims at minimizing over-fitting the model with spurious regressors, while the D-optimality criterion aims at maximizing the information content of the selected regressors, much as it is done with Akaike's (1974) well known information criterion (AIC).

4. ITERATIVE PROCEDURE FOR MODEL COMPLEXITY REDUCTION

The rationale behind the proposed procedure is the following: GOBF poles are selected based on measuring the FRF of the error system, the identification input is then optimally selected to *invalidate* the current model, and finally an orthogonal least squares procedure is used to select a reduced number of regressors.

The proposed methodology starts with $\kappa = 1$ and the candidate nonlinear model $\mathcal{M}^{(0)} = 0$ (iterations are denoted by parenthesized indices).

Step 1. Measure the frequency response function $G_{\mathcal{E}}(j\omega_k)$ of the error between simulation $\mathcal{S}$ and current model $\mathcal{M}^{(\kappa-1)}$ and fit a linear model $\hat{G}_{\mathcal{E}}(z)$ to it using a subspace algorithm.

Step 2. For each pole of $\hat{G}_{\mathcal{E}}(z)$ build a set of candidate filters $\boldsymbol{B}_i^{(\kappa-1)}(z)$ by adding a generalized basis function to the current filter $\boldsymbol{B}^{(\kappa-1)}(z)$ of already selected GOBFs (see (12)).

Step 3. Design an optimized control relevant input $u^{(\kappa)}(t)$ to *invalidate* the current model $\mathcal{M}^{(\kappa-1)}$ with respect to $\mathcal{S}$. Build the signals $y^{(\kappa-1)}(t)$ and $\boldsymbol{\psi}_i^{(\kappa-1)}(t)$ as the outputs of $\mathcal{S}$ and each $\boldsymbol{B}_i^{(\kappa-1)}(z)$ to this input.

Step 4. For each i construct the columns of the regressor matrix $\boldsymbol{\Phi}^{(\kappa)}$, considering a certain nonlinear order n in (11). Then perform a structure selection and parameter estimation using orthogonal least squares techniques to end up with a set of candidate models $\mathcal{M}_i^{(\kappa)}$.

Step 5. Cross-validate each candidate model and select the "best" one as $\mathcal{M}^{(\kappa)}$; also update $\boldsymbol{B}^{(\kappa)}$. If the model is adequate enough, then **stop**. Otherwise return to **Step 1** (add another GOBF) or **Step 4** (increase nonlinear degree).

Regarding **Step 1**, the error is defined as

$$\begin{aligned} e^{(\kappa)}(t) &= \mathcal{E}^{(\kappa)}[\bar{u}^{(\kappa)}(t)] \\ &= \mathcal{S}[\bar{u}^{(\kappa)}(t)] - \mathcal{M}^{(\kappa-1)}[\bar{u}^{(\kappa)}(t)] \end{aligned} \tag{19}$$

Since $\mathcal{M}^{(\kappa-1)}$ is a Volterra model and $\mathcal{S}$ is assumed to have a Volterra representation, then so does $\mathcal{E}^{(\kappa)}$ and (13) is valid for random multisines. It is recommended to design $\bar{u}^{(\kappa)}(t)$ as a random multisine with odd frequency components in order to minimize the effect of even nonlinearities (Schoukens *et al.*, 1998). Furthermore, to minimize the effect of "nonlinear noise" several realizations of $\bar{u}^{(\kappa)}(t)$ should be considered and the FRF averaged over these measurements.

With respect to **Step 2**, minimal state space realizations for a set of basis functions are easy to derive (Gómez, 1998). Furthermore, adding a candidate basis function $B_i^{(\kappa-1)}(z)$ to $\boldsymbol{B}^{(\kappa-1)}(z)$ in order to build $\boldsymbol{B}_i^{(\kappa-1)}$ is straightforward. However, care should be taken when pairs of complex conjugate poles are considered to avoid complex parameters (Ninness and Gustafsson, 1997).

Step 3 comprises one of the main features of this procedure. The previous model $\mathcal{M}^{(\kappa-1)}$ is the best possible model for a GOBF-Volterra structure of some nonlinear order and some basis functions $\boldsymbol{B}^{(\kappa-1)}(z)$. From (11) it can be seen that adding a basis function $B_i^{(\kappa)}(z)$ simply adds more terms to the structure of $\mathcal{M}^{(\kappa-1)}$, which by themselves

may compose a model for $\mathcal{E}^{(\kappa)}$. By designing an input that somehow *maximizes* the output of such an error system, the current nonlinear model $\mathcal{M}^{(\kappa-1)}$ is being invalidated. To carry on this input design, multisines are again considered, given their reported success for nonlinear Volterra system identification (Evans *et al.*, 1996).

Having chosen a frequency grid with N lines, the input signal $u^{(\kappa)}(t)$ is parametrized by its (complex) input spectrum $U(k)$. The optimization problem to solve is

$$U^*(k) = \arg\sup_{U(k)\in\mathbb{U}} \left\| \left(\mathcal{S} - \mathcal{M}^{(\kappa-1)}\right)[u(t)] \right\|^2 \quad (20)$$

whereby $\mathbb{U}$ refers to the set where the input spectrum is to be constrained, e.g. by fixing a maximal RMS value of the signal, and $\|\cdot\|$ is some norm. To solve this constrained optimization problem only a finite number of elements in the set $\mathbb{U}$ is considered and the sup-operator is replaced by a max-operator. The solution may be need to be found by exhaustive search or using global optimization strategies (Nelles, 2001).

Several candidate models $\mathcal{M}_i^{(\kappa)}$ with $\rho_i^{(\kappa)}$ parameters/regressors are obtained in **Step 4** using the same input/output data. In **Step 5** a validation is performed to select the model that has better prediction capabilities. Each model is excited with L realizations $u_\ell^{(\kappa)}(t)$ of a random multisine with the fixed amplitude spectrum of $U^*(k)$. The following performance index is then calculated:

$$J(\mathcal{M}_i^{(\kappa)}) = g(\rho_i^{(\kappa)}) + \\ + \frac{1}{L}\sum_{\ell=1}^{L}\sum_{t=1}^{N} \left\| \mathcal{S}[u_\ell^{(\kappa)}(t)] - \mathcal{M}_i^{(\kappa)}[u_\ell^{(\kappa)}(t)] \right\|^2 \quad (21)$$

where $g\colon \mathbb{N} \to \mathbb{R}_+$ is a monotonically increasing function that penalizes a large number of parameters $\rho_i^{(\kappa)}$. The "best" model is chosen as $\mathcal{M}^{(\kappa)} = \mathcal{M}_{i^*}^{(\kappa)}$ with $i^* = \arg\min_i J(\mathcal{M}_i^{(\kappa)})$.

5. EXAMPLE

The procedure is illustrated on the model of a continuous copolymerization reactor with exothermic kinetics. The model has 15 states, 6 inputs, and 7 outputs (Bindlish, 1999). The inputs are the feed rates of the inhibitor (m-dinitrobenzene), the transfer agent (acetaldehyde), and the two monomers (vinyl acetate and methyl methacrylate), plus the coolant and feed temperatures. The outputs are the effluent compositions of the two monomers and the solvent (benzene) in the separator, the copolymer composition, its intrinsic viscosity, the reactor temperature, and the polymer production rate.

Although the proposed procedure is extensible to MIMO systems, for simplicity it was applied in this example to an equivalent SISO system. The system was set to certain operating conditions and only one input was made time-varying, while only one output was measured. The main nonlinearity of the system is due to the temperature dependent kinetics of Arrhenius type, so to make the problem more interesting, the coolant temperature was chosen as input. The output chosen was the copolymer production rate.

The sampling time was set to $T_s = 2.5$ min. For **Step 1** the input used was an odd random multisine with maximum frequency $f_{\max} = 255 f_0$ and base frequency $f_0 \approx 3.9 \times 10^{-4}\text{min}^{-1}$. Input deviations of ± 15K were considered. Two periods were measured, but only the last one was considered for identification to account for transients.

The procedure was tested several times and consistently only two iterations were sufficient to obtain a satisfactory reduced complexity model. The GOBFs resulted with the following ordered poles:

$$\{0.8546 \pm j0.1934, 0.8955 \pm j0.2451\}, \quad (22)$$

which leads to a fourth order system. Before the nonlinear identification procedure was carried out, the gain of the GOBF filters was adjusted so that $\psi_i(t)$, $i = 1, \ldots, 4$, was normalized for the optimal input generated in **Step 3**. The reduced resulting structure for the system has only five terms:

$$y(t) = \theta_1\psi_1(t) + \theta_2\psi_2(t) + \theta_3\psi_3(t) + \\ + \theta_4\psi_1(t)\psi_2(t) + \theta_5\psi_1(t)\psi_4(t) \quad (23)$$

with parameter vector

$$\boldsymbol{\theta} = \left[0.148, 0.0149, 0.0513, -0.0801, -0.0627\right]^T. \quad (24)$$

After the identification, the resulting model was tested on a completely different validation input, namely a random step input (Pearson, 1998). The output of both the complex simulation model and the model (23) is shown in Fig. 1, whereas the error between both signals is shown in Fig. 2.

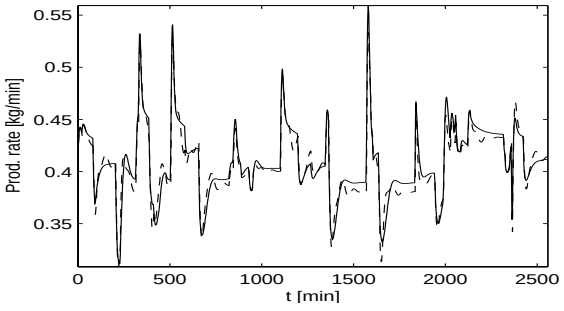

Fig. 1. Output of complex simulation model (solid) and reduced complexity model (dashed) to a random step input.

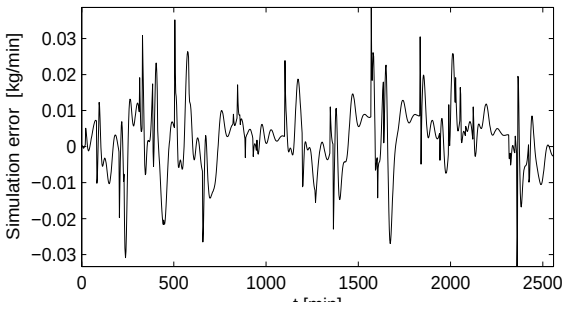

Fig. 2. Error between complex and reduced complexity models.

6. CONCLUSIONS

A procedure for complexity reduction combining frequency domain linear identification, constrained input optimization techniques, and substructure selection and parameter estimation methods has been proposed. Testing the procedure on a realistic example shows promising results and further investigations are under way to refine it.

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