Evaporation rate independent state estimation for a spray drying process

Arthur Lepsien [*,**], **Héctor Hernández-Escoto** [***], **Alexander Schaum** [*,**]

[*] *Department of Process Analytics, University of Hohenheim, Stuttgart, Germany*
e-mail:{arthur.lepsien;alexander.schaum}@uni-hohenheim.de
[**] *Computational Science Hub, University of Hohenheim, Stuttgart, Germany*
[***] *Chemical Engineering Department, Guanajuato University, Guanajuato, México*
e-mail: hhee@ugto.mx

Abstract: The problem of model-based state estimation in absence of a reliable model for the liquid evaporation rate in a spray drying process is addressed. The process is described by a three-state model consisting of the product moisture, air humidity and temperature in the spray drying chamber with measurements of the temperature and air humidity. The state estimation problem is approached within the framework of unknown-input observer design, considering the evaporation rate as unknown time-varying bounded source term. It is shown that using an adequate integral state transformation based on energy and mass conservation mechanisms it is possible to obtain reliable estimates for the product moisture by model-based sensor fusion. The performance of the proposed scheme is evaluated using numerical simulations for a previously validated process model.

Keywords: Spray drying, Nonlinear observer design, Unknown input observer, Sensor fusion

1. INTRODUCTION

Spray drying is a dominant approach to obtain powdered products from liquids (Wrzosek et al., 2013) involving the combination of different mechanisms, starting from heat transfer in air and liquids, over evaporation to structure formation and morphological changes in the final particles. Besides the Particle Size Distribution (PSD) and morphology, moisture content is a central characteristic of the obtained powder. One obstacle is that it is difficult and cost intensive to measure inline (e.g., using Near-Infrared spectrometry), calling for adequate state estimation schemes for online monitoring of this property.

One central problem for reliable state estimation lies in the fact that process models come along with different sources of uncertainty. On one side, spray towers are typically large units leading to spatio-temporal profiles for temperature and humidity that are not directly measurable and difficult to predict in real time. On the other side the process itself is determined by a complex interplay of macroscopic and microscopic mechanisms, which in particular influence the evaporation of liquid in the particles. This is particularly due to the fact that a crust is forming during the last phase of particle drying, and that this process is subject to considerable variations between the resulting particles (see, e.g., Fritsching (2016); Lepsien et al. (2024)).

In consequence, the possibility of obtaining reliable estimates for moisture content will mainly depend on reliable models for evaporation. Such models can be obtained within experimental certainty ranges employing dynamic vapor sorption and single particle drying (Fritsching, 2016), i.e., cost and time intensive analytic tools.

These considerations lead to the question whether it is possible to design an unknown input observer for a spray drying process that considers the evaporation rate as unknown time-varying bounded input.

The ideas and concepts for the design of unknown input observers have been subject to intensive research starting from the early work of Hautus (see, e.g., (Hautus, 1983; Darouach et al., 1994)). For nonlinear systems different approaches have been presented, including dissipative observers (Rocha and Moreno, 2010), more general matrix-inequality based approaches (Ha and Trinh, 2004), sliding mode observers (Edwards and Spurgeon, 1998; H. Niederwieser et al., 2022), high-gain observers (Khalil, 2007; Astolfi et al., 2016) and continuous-discrete Kalman Filter adaptations (Rocha-Cozatl et al., 2015, 2012). In case that the dynamics of the input are known, using a so-called exosystem model more structured approaches can be followed (Goodwin and Seron, 2020; Francis and Wonham, 1976; Rocha-Cozatl and Vande Wouwer, 2011; Schaum and Meurer, 2015; Schaum et al., 2024).

In this regard it should be recalled that for linear systems reduced order unknown input observers can be designed in case that an unknown input affects directly the dynamics of the measured variable (Hautus, 1983). In this case a state transformation exists that allows to define

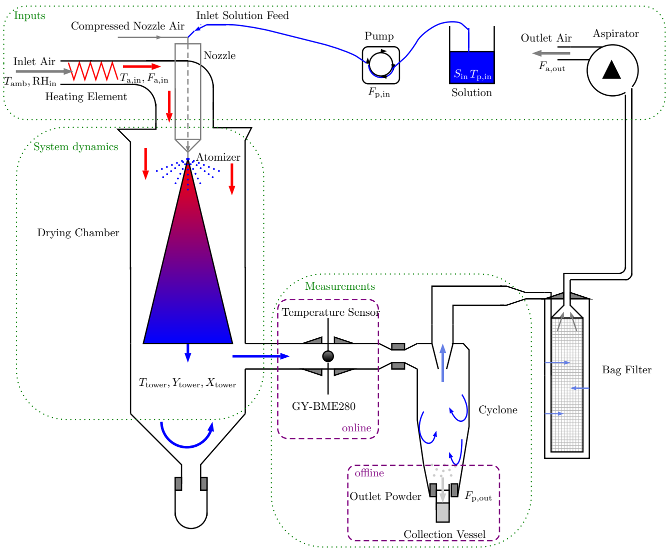

Fig. 1. Principle arrangement of a spray dryer, motivated by the case of the Mini Büchi B290 (similar to (Lepsien and Schaum, 2024a)).

basis vectors of the tangent space which correspond to state combinations whose dynamics is independent of the unknown input. This idea has also been extended to some classes of nonlinear systems (Ha and Trinh, 2004; Rocha and Moreno, 2010).

Having the above studies as point of departure, in the present paper we focus on the design of an observer for a spray drying process considering the evaporation rate as unknown input. One central difficulty in applying the approaches mentioned above consists in the fact that the effect of the unknown evaporation rate on the states depends on the states themselves, meaning that we have state dependent input gains, rendering the underlying transformation non-trivial. It is shown that by using integral state transformations depending on the specific heat of evaporation and changes in the density of vapor due to the increase of water content an input-independent state transformation can be found, such that the resulting differential equations are independent of the unknown input. Based on this transformation a geometric unknown input observer is designed extending ideas from (Alvarez and Lopez, 1999; Jerono et al., 2021) and tested in numerical simulations for an experimentally validated model of a spray dryer (Lepsien and Schaum, 2024a).

2. PROBLEM DESCRIPTION

A typical spray dryer setup is shown in Figure 1, inspired by the Mini Büchi used for the validation of the model employed in the present study. The basic working principle is as follows. A bi–component solution of water and solvent is pumped by a peristaltic pump and it is atomized by the nozzle. The created droplets fall through the drying chamber within a concurrent air flow field, set by the aspirator volumetric flow rate. The air is heated by the inlet heating element. The dried particles are collected via the outlet cyclone. Too small particles find their way into the bag filter outlet.

The dynamics of the product moisture content (X), air water content (Y) and temperature (T) can be modeled using standard arguments from thermodynamic modeling (see, e.g., (Petersen et al., 2015; Lepsien and Schaum, 2024b)) with state vector $\boldsymbol{x}(t) = [X,\ Y,\ T]^{\mathrm{T}} \in \mathcal{M} \subseteq \mathbb{R}^3_{\geq 0}$ for $t \geq 0$, solid mass m_{s}, dry air mass m_{da}, heat capacity C as known parameters, volumetric liquid feed flow rate $u = \dot{v}_{\mathrm{p}} \in \mathbb{R}_{\geq 0}$ as known system input and unknown state–dependent evaporation rate $R_{\mathrm{w}}(\boldsymbol{x})$. In compact notation this leads to a system in the form

$$\dot{X} = u\sigma_1(X) + g_1 R_{\mathrm{w}}(\boldsymbol{x}), \tag{1a}$$

$$\dot{Y} = \sigma_2(Y,T) + g_2(T) R_{\mathrm{w}}(\boldsymbol{x}), \tag{1b}$$

$$\dot{T} = u\sigma_3(X,T) + \sigma_4(Y,T) + g_3(T) R_{\mathrm{w}}(\boldsymbol{x}), \tag{1c}$$

with

$$\sigma_1(X) = \frac{1}{m_{\mathrm{s}}}\left(F_{\mathrm{p,in}} X_{\mathrm{in}} - F_{\mathrm{p,out}}(X) X\right),$$

$$\begin{aligned}\sigma_2(Y,T) = &\frac{\dot{v}_{\mathrm{a}}}{m_{\mathrm{da}}}\left(\rho_{\mathrm{a,in}} Y_{\mathrm{in}} - F_{\mathrm{a,out}}(Y,T) Y\right)\\ &+ \frac{1}{m_{\mathrm{da}}}\left(F_{\mathrm{add}} Y_{\mathrm{add}} - F_{\mathrm{add}} Y\right)\end{aligned}$$

$$\sigma_3(X,T) = \frac{1}{C}\left(F_{\mathrm{p,in}} h_{\mathrm{p,in}} - F_{\mathrm{p,out}}(X) h_{\mathrm{p,out}}(X,T)\right),$$

$$\begin{aligned}\sigma_4(Y,T) = &\frac{1}{C}(\dot{v}_{\mathrm{a}}\left[\rho_{\mathrm{a,in}} h_{\mathrm{a,in}} - \rho_{\mathrm{a,out}}(Y,T) h_{\mathrm{a,out}}(Y,T)\right]\\ &- Q_{\mathrm{l}}(T) + F_{\mathrm{add}}\left(h_{\mathrm{add}} - h_{\mathrm{a,out}}(Y,T)\right)),\end{aligned}$$

and

$$g_1 = -\frac{1}{m_{\mathrm{s}}}, \tag{2a}$$

$$\begin{aligned}g_2(T) &= \frac{1}{m_{\mathrm{da}}}\left(1 - \frac{\rho_{\mathrm{a,in}} Y_{\mathrm{in}}}{\rho_{\mathrm{v}}(T)}\right)\\ &= \frac{1}{m_{\mathrm{da}}}\left(1 - \frac{R Y_{\mathrm{in}} \rho_{\mathrm{a,in}}}{M_{\mathrm{w}} P_0} T\right) = \sigma_5\left(1 - \sigma_6 T\right),\end{aligned} \tag{2b}$$

$$\begin{aligned}g_3(T) &= -\frac{1}{C}\left(\frac{h_{\mathrm{a,in}} \rho_{\mathrm{a,in}}}{\rho_{\mathrm{v}}(T)} + \lambda(T)\right)\\ &= -\frac{1}{C}\left(\frac{M_{\mathrm{a,in}} h_{\mathrm{a,in}}}{M_{\mathrm{w}} T_{\mathrm{amb}}} T + \lambda(T)\right),\end{aligned} \tag{2c}$$

where

$$F_{\mathrm{p,in}} X_{\mathrm{in}} = u\rho_{\mathrm{s}}\left(1 - S_{\mathrm{in}}\right) + u\rho_{\mathrm{w}} \frac{\left(1 - S_{\mathrm{in}}\right)^2}{S_{\mathrm{in}}} = uc_1,$$

with c_1 being a known constant, and

$$F_{\mathrm{p,out}}(X) = uS_{\mathrm{in}}\left(\rho_{\mathrm{s}} + \rho_{\mathrm{w}} X\right) = uf_1(X).$$

The temperature dependent latent heat of vaporization for water $\lambda(T)$ is depicted in Fig. 2 and the density of water vapor $\rho_{\mathrm{v}}(T)$ in Fig. 3. The system evolution in (1) leads

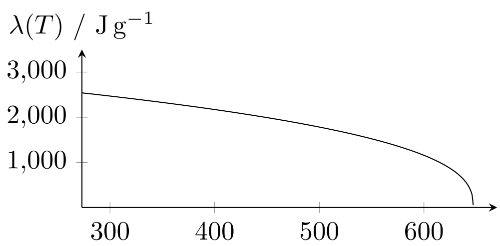

Fig. 2. Latent heat of vaporization $\lambda(T)$.

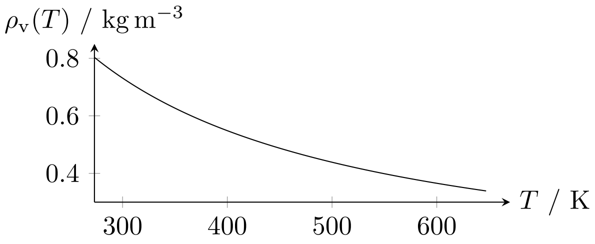

Fig. 3. Water vapor density $\rho_\mathrm{w}(T)$ at atmospheric pressure.

to the compact state-space description

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u) + \boldsymbol{g}(x_3)w = \begin{bmatrix} f_1(\boldsymbol{x}, u) \\ f_2(\boldsymbol{x}, u) \\ f_3(\boldsymbol{x}, u) \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2(x_3) \\ g_3(x_3) \end{bmatrix} w, \quad \text{(3a)}$$

$$\boldsymbol{y} = [x_2 \ x_3]^\mathrm{T} \ , \quad \text{(3b)}$$

where $\boldsymbol{f}, \boldsymbol{g}$ are defined accordingly and the evaporation rate $w(t) \coloneqq R_w(\boldsymbol{x}(t)) \in \mathbb{R}_{\geq 0}$ is considered as unknown but bounded source term in the following. The measured output is denoted by $\boldsymbol{y}$ and consists of the air humidity $x_2 = Y = y_1$ and the temperature $x_3 = T = y_2$.

The problem consists in providing an estimate $\hat{\boldsymbol{x}}(t)$ of the state vector $\boldsymbol{x}(t)$ in the sense that $\lim_{t\to\infty} \|\hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)\| = 0$ in spite of the unknown evaporation rate w, by combining the model (3a) and the measurements (3b).

3. REDUCED ORDER OBSERVER DESIGN

In the following, an reduced order unknown input observer is designed, employing a concept similar to one of reaction invariants (Aris, 1965). To reduce the system order, the temperature measurement $y_2 = x_3$ is used directly instead of solving the ODE (1c), which is especially useful, as g only depends on x_3.

A state transformation $\boldsymbol{z} = \boldsymbol{\varphi}(\boldsymbol{x})$, being sufficiently often differentiable, satisfies

$$\dot{\boldsymbol{z}} = \frac{\mathrm{d}\boldsymbol{\varphi}(\boldsymbol{x})}{\mathrm{d}t} = \boldsymbol{\nabla}\boldsymbol{\varphi}(\boldsymbol{x}) \cdot [\boldsymbol{f}(\boldsymbol{x}, u) + \boldsymbol{g}(\boldsymbol{x}_3)w] \,. \quad (4)$$

For unknown input independence it is required that the gradient of $\boldsymbol{\varphi}(\boldsymbol{x})$ is perpendicular to the evaporation rate everywhere on $\mathcal{M}$, i.e.,

$$\boldsymbol{\nabla}\varphi_i(\boldsymbol{x}) \cdot \boldsymbol{g}(\boldsymbol{x}) = 0, \ \forall \boldsymbol{x} \in \mathcal{M}. \quad (5)$$

With this, mutually linear independent basis vectors of the tangent space $\mathcal{T}_{\boldsymbol{x}}\mathcal{M}$ can be determined at every point $\boldsymbol{x} \in \mathcal{M}$. As long as $\varphi_i, i = 1, 2$ depend on x_1, x_2 this means that the set

$$\mathcal{B}_{\boldsymbol{x}} = \{\boldsymbol{\nabla}\varphi_1(\boldsymbol{x}), \boldsymbol{\nabla}\varphi_2(\boldsymbol{x}), \boldsymbol{e}_3\}, \quad (6)$$

with $\boldsymbol{e}_3 = [0 \ 0 \ 1]^\mathrm{T}$ forms a basis of $\mathcal{T}_{\boldsymbol{x}}\mathcal{M}$. It follows that

$$\boldsymbol{\Phi}(\boldsymbol{x}) = [\varphi_1(\boldsymbol{x}) \ \varphi_2(\boldsymbol{x}) \ x_3]^\mathrm{T} \quad (7)$$

is a diffeomorphism, i.e., in particular that it holds that

$$\frac{\partial \boldsymbol{\Phi}(\boldsymbol{x})}{\partial \boldsymbol{x}} = \begin{bmatrix} \boldsymbol{\nabla}^\mathrm{T} \varphi_1(\boldsymbol{x}) \\ \boldsymbol{\nabla}^\mathrm{T} \varphi_2(\boldsymbol{x}) \\ \boldsymbol{e}_3^\mathrm{T} \end{bmatrix}, \quad (8)$$

has rank= 3 everywhere on $\mathcal{M}$.

Considering the associated state transformation $\boldsymbol{z} \coloneqq \boldsymbol{\varphi}(\boldsymbol{x})$, it turns out that

$$\dot{\boldsymbol{z}} = \boldsymbol{\Psi}(\boldsymbol{z}, y_2, u), \quad (9)$$

is independent of the unknown input w. Note that this approach extends the ones typically found in the literature (see, e.g., Ha and Trinh (2004); Rocha and Moreno (2010)) as there the gains for the unknown input are typically independent of the states.

The proposed condition is similar to the construction of the Byrnes-Isidori normal form for an input-output linearizing controller (Isidori, 1995). Note, that for an affine input SISO system it is always possible to find a diffeomorphism, which is independent of the affine input, here the unknown input w. Condition (5) is equivalent to

$$g_1 \frac{\partial \varphi_i}{\partial x_1} + g_2(x_3)\frac{\partial \varphi_i}{\partial x_2} + g_3(x_3)\frac{\partial \varphi_i}{\partial x_3} = 0, \quad (10)$$

what can be solved with the methods of characteristics. There are two different solutions for (10), that fulfill (5), which can be constructed in the following way. If one chooses φ_1 to be independent of x_2 and sets the left hand side derivative to $\frac{\partial \varphi_1}{\partial x_1} = 1$, then (10) simplifies to

$$\frac{\partial \varphi_1}{\partial x_3} = -\frac{g_1}{g_3(x_3)}, \quad (11)$$

leading to

$$\varphi_1(\boldsymbol{x}) = x_1 - g_1 \int_{x_{3,\mathrm{ref}}}^{x_3} \frac{1}{g_3(s)} \mathrm{d}s \quad (12)$$

with suitably defined reference temperature $x_{3,\mathrm{ref}}$. For the second solution, one can choose φ_2 to be independent on x_1 and set $\frac{\partial \varphi_2}{\partial x_2} = 1$, what leads to

$$\frac{\partial \varphi_2}{\partial x_3} = -\frac{g_2(x_3)}{g_3(x_3)}, \quad (13)$$

and thus

$$\varphi_2(\boldsymbol{x}) = x_2 - \int_{x_{3,\mathrm{ref}}}^{x_3} \frac{g_2(s)}{g_3(s)} \mathrm{d}s. \quad (14)$$

This leads to the choice of $\boldsymbol{\varphi}$ given by

$$\boldsymbol{z} = \boldsymbol{\varphi}(\boldsymbol{x}) = \begin{bmatrix} x_1 - g_1 \int_{x_{3,\mathrm{ref}}}^{x_3} \frac{1}{g_3(s)} \mathrm{d}s \\ x_2 - \int_{x_{3,\mathrm{ref}}}^{x_3} \frac{g_2(s)}{g_3(s)} \mathrm{d}s \end{bmatrix}. \quad (15)$$

Now, the system order is reduced, as the third state is directly measured, i.e. $y_2 = x_3 = T$. Taking the derivative of $\boldsymbol{z}$ with respect to time, one obtains the dynamics

$$\begin{aligned} \dot{\boldsymbol{z}} &= \left. \begin{bmatrix} f_1(\boldsymbol{x}, u) - g_1 \frac{1}{g_3(x_3)} f_3(\boldsymbol{x}, u) \\ f_2(\boldsymbol{x}, u) - \frac{g_2(x_3)}{g_3(x_3)} f_3(\boldsymbol{x}, u) \end{bmatrix} \right|_{\boldsymbol{x} = \boldsymbol{\varphi}^{-1}(\boldsymbol{z}, y_2)} \\ &\eqqcolon \boldsymbol{\Psi}(\boldsymbol{z}, u, y_2), \end{aligned} \quad (16)$$

that are independent on the unknown input w, as proposed above. The inverse of (7) resulting in

$$\boldsymbol{x} = \begin{bmatrix} z_1 - g_1 \int_{y_{2,\mathrm{ref}}}^{y_2} \frac{1}{g_3(s)} \mathrm{d}s \\ z_2 - \int_{y_{2,\mathrm{ref}}}^{y_2} \frac{g_2(s)}{g_3(s)} \mathrm{d}s \\ y_2 \end{bmatrix} = \boldsymbol{\Phi}^{-1}(\boldsymbol{z}, y_2), \quad (17)$$

can directly be evaluated using the temperature measurement y_2 and the known functions $g_i, i = 1, 2, 3$ from (2).

Remark 1. The temperature-dependent transformations are based on the factors g_i that quantify the sensitivities of the rate of change in time in X, Y, T with respect to the water evaporation during the drying process. The main part of the integrals basically has the form

$$\int \frac{1}{h + \lambda} \mathrm{d}T$$

with specific enthalpy h of the humid air at the tower inlet and latent heat of evaporation λ. It can be interpreted as cumulative resistance to temperature changes, reflecting how much temperature change corresponds to the energy required to evaporate water and heat up the humid air, because the combination of $h + \lambda$ accounts for the total specific energy associated to a phase change of water into gas at a given temperature T.

3.1 Observability of the reduced dynamcis

After the coordinate change and given (17) the first measurement can be rewritten as

$$y_1 = x_2 = z_2 + \int_{y_{2,\mathrm{ref}}}^{y_2} \frac{g_2(s)}{g_3(s)} \mathrm{d}s := h(\boldsymbol{z}, y_2), \tag{18}$$

giving rise to a new measurement

$$\xi_1 := y_1 - \int_{y_{2,\mathrm{ref}}}^{y_2} \frac{g_2(s)}{g_3(s)} \mathrm{d}s = z_2 = \boldsymbol{c}^\mathrm{T}\, \boldsymbol{z}, \tag{19}$$

with $\boldsymbol{c}^\mathrm{T} = [0 \;\; 1]$. The associated observability map yields

$$\boldsymbol{\mathcal{O}}(\boldsymbol{z}, u, y_2) = \begin{bmatrix} c^\mathrm{T}\, \boldsymbol{z} \\ c^\mathrm{T}\, \boldsymbol{\Psi}(\boldsymbol{z}, u, y_2) \end{bmatrix} = \begin{bmatrix} z_2 \\ \Psi_2(\boldsymbol{z}, u, y_2) \end{bmatrix} \in \mathbb{R}^{n_z}, \tag{20}$$

with $n_z = 2$. The local observability can be analyzed via the inverse function theorem, resulting in the condition

$$\det\left(\frac{\partial \boldsymbol{\mathcal{O}}}{\partial \boldsymbol{z}}(\boldsymbol{z}, u, y_2)\right) = -\frac{\partial \Psi_2(\boldsymbol{z}, u, y_2)}{\partial z_1} \neq 0. \tag{21}$$

As $\Psi_2(\boldsymbol{z}, u, y_2)$ is a function of z_1, this implies the structural observability. Moreover, one can conclude the local observability, if $S_\mathrm{in} \neq 0, u \neq 0$, $y_2 = T \neq 0$ and four lengthy other conditions are fulfilled, which to check can be analytically challenging for any feasible $\boldsymbol{z}$.

3.2 Observer construction

Motivated by the above result, the following reduced order geometric unknown input observer is proposed in the transformed coordinates

$$\dot{\hat{\boldsymbol{z}}} = \boldsymbol{\Psi}(\hat{\boldsymbol{z}}, u, y_2) - \left(\left.\frac{\partial \boldsymbol{\mathcal{O}}}{\partial \boldsymbol{z}}\right|_{\boldsymbol{z}=\hat{\boldsymbol{z}}}\right)^{-1} \boldsymbol{l}\left(\boldsymbol{c}^\mathrm{T}\, \hat{\boldsymbol{z}} - \xi_1\right). \tag{22}$$

Note that once the dynamics are written in the evaporation rate independent basis, the geometric observer has the typical structure as proposed, e.g., in (Alvarez and Lopez, 1999) and further exploited, e.g., in Hernández and Alvarez (2003); Hernández-Escoto et al. (2010); Jerono et al. (2021).

In the next step, the observer gain $\boldsymbol{l}$ should be designed. Introducing a new state transformation based on (20)

$$\boldsymbol{\zeta} := \mathcal{O}(\boldsymbol{z}, u, y_2),$$

the dynamics

$$\dot{\zeta}_1 = \zeta_2,$$
$$\dot{\zeta}_2 = \omega\left(\boldsymbol{\zeta}, u, y_2\right),$$

follows. This gives rise to the transformed observer state dynamics

$$\dot{\hat{\zeta}}_1 = \hat{\zeta}_2 - l_1\left(\hat{\zeta}_1 - \zeta_1\right),$$
$$\dot{\hat{\zeta}}_2 = -l_2\left(\hat{\zeta}_1 - \zeta_1\right) + \omega\left(\hat{\boldsymbol{\zeta}}, u, y_2\right),$$

with the nonlinear function ω. Defining the observer errors as $\tilde{\zeta}_i := \hat{\zeta}_i - \zeta_i,\; i \in \{1, 2\}$ the observer error dynamics read

$$\dot{\tilde{\zeta}}_1 = -l_1\tilde{\zeta}_1 + \tilde{\zeta}_2, \tag{23}$$
$$\dot{\tilde{\zeta}}_2 = -l_2\tilde{\zeta}_1 + \tilde{\omega}\left(\hat{\boldsymbol{\zeta}}; \boldsymbol{\zeta}, u, y_2\right) \tag{24}$$

$$\tilde{\omega}\left(\hat{\boldsymbol{\zeta}}; \boldsymbol{\zeta}, u, y_2\right) := \omega\left(\boldsymbol{\zeta} + \tilde{\boldsymbol{\zeta}}, u, y_2\right) - \omega\left(\boldsymbol{\zeta}, u, y_2\right) \tag{25}$$

where it holds that $\tilde{\omega}\left(\mathbf{0}; \boldsymbol{\zeta}, u, y_2\right) = 0$ for all $\boldsymbol{\zeta}, u, y_2$.

For the subsequent design the following assumption is introduced.

Assumption 1. The function $\tilde{\omega}$ is Lipschitz continuous with respect to $\tilde{\boldsymbol{\zeta}}$, i.e., there exists a constant $L > 0$ such that $\left\|\tilde{\omega}\left(\hat{\boldsymbol{\zeta}}; \boldsymbol{\zeta}, u, y_2\right)\right\| \leq L\left\|\hat{\boldsymbol{\zeta}}\right\|, \forall\, \boldsymbol{\zeta}, u, y_2$.

Now, the observer gains can be designed from the linear part of the error dynamics, so that the destabilization potential of the nonlinearity, measured by the constant L is compensated. The linear subsystem of the error dynamics are described by the matrix

$$A - \boldsymbol{l}\boldsymbol{c}^\mathrm{T} = \begin{bmatrix} -l_1 & 1 \\ -l_2 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \tag{26}$$

where now either the eigenvalues can be directly assigned and the observer gains computed or the system can be brought into second order form

$$\ddot{\tilde{\zeta}}_1 + l_1\dot{\tilde{\zeta}}_1 + l_2\tilde{\zeta}_1 = 0, \tag{27}$$

where the parameters of this equation change the observer error dynamics. Solving (27) results in

$$\tilde{\zeta}_1(t) = e^{-\frac{l_1}{2}t}\left(C_1 e^{t\sqrt{\frac{l_1^2}{4} - l_2}} + C_2 e^{-t\sqrt{\frac{l_1^2}{4} - l_2}}\right), \tag{28}$$

where the critically damped case occurs if $\frac{l_1^2}{4} - l_2 = 0$, leading to $l_2 = \frac{l_1^2}{4}$. Now, if Assumption 1 is fulfilled, choosing l_1 sufficiently large to dominate the Lipschitz constant L the observer is exponentially stable. Using the designed observer gains, the observer (22) is used and the state transformation (17) is employed to bring the system into original coordinates.

4. CASE STUDY

In the following the above proposed design is tested for the previously experimentally validated process model with parameters from (Lepsien and Schaum, 2024a).

To implement the integral transformations (15) either a numerical integration, e.g. with the trapezoidal rule can be used, or the function $g_3(s)$ can be approximated with a second order polynomial, as that integral can be analytically computed for the case that $b^2 - 4ac > 0$ to be

$$\int \frac{1}{as^2 + bs + c}\mathrm{d}s = \frac{1}{\Delta}\ln\left|\frac{2as + b - \Delta}{2as + b + \Delta}\right| + C,$$
$$\Delta = \sqrt{b^2 - 4ac}.$$

Because of the above condition, the roots s_1 and s_2 of the quadratic polynomial are distinct and real given by

$$s_1 = \frac{-b - \Delta}{2a^2}, \quad s_2 = \frac{-b + \Delta}{2a^2},$$

leading to the solution of the integral by decomposition using partial fractions

$$\int \frac{s}{as^2+bs+c}\mathrm{d}s = \frac{s_1}{a^2(s_1-s_2)}\ln|s-s_1| + \frac{s_2}{a^2(s_2-s_1)}\ln|s-s_2| + C,$$

which is used in the following. The function $g_3(y_2)$ from (2c) is depicted in Fig. 4 with its second order approximation

$$g_3^{\mathrm{app}}(y_2) = g_3^2(y_2)^2 + g_3^1 y_2 + g_3^0, \tag{29}$$

giving the parameters

$$g_3^2 = 1.9173\cdot 10^{-4},\ g_3^1 = -0.0831,\ g_3^0 = -167.3416,$$

and roots

$$\mathcal{R}_{1,2} = \frac{-g_3^1 \pm \sqrt{\left(g_3^1\right)^2 - 4g_3^2 g_3^0}}{2\left(g_3^2\right)^2}. \tag{30}$$

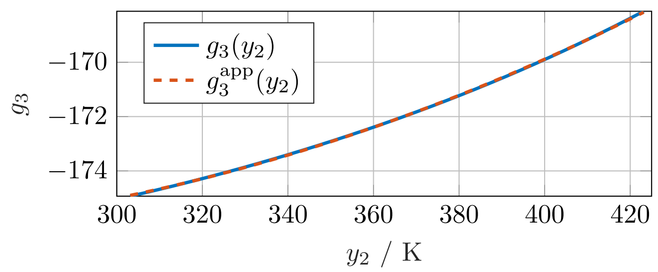

Fig. 4. State dependent gain $g_3(y_2)$ and its approximation $g_3^{\mathrm{app}}(y_2)$ in the region of interest.

This leads to the integral approximations

$$\mathcal{G}_1(y_2) := g_1 \int_{y_{2,\mathrm{ref}}}^{y_2} \frac{1}{g_3^2 s^2 + g_3^1 y_2 + g_3^0}\mathrm{d}s \tag{31}$$

$$= \frac{g_1}{\Delta_g}\left(\ln\left|\frac{2g_3^2 y_2 + g_3^1 - \Delta_g}{2g_3^2 y_2 + g_3^1 + \Delta_g}\right| - \ln\left|\frac{2g_3^2 y_{2,\mathrm{ref}} + g_3^1 - \Delta_g}{2g_3^2 y_{2,\mathrm{ref}} + g_3^1 + \Delta_g}\right|\right),$$

$$\Delta_g = \left(g_3^1\right)^2 - 4g_3^2 g_3^0,$$

$$\mathcal{G}_2(y_2) := \sigma_5 \int_{y_{2,\mathrm{ref}}}^{y_2} \frac{(1-\sigma_6 s)}{g_3^2 s^2 + g_3^1 y_2 + g_3^0}\mathrm{d}s \tag{32}$$

$$= \sigma_5\left(\mathcal{G}_1(y_2) - \sigma_6 \int_{y_{2,\mathrm{ref}}}^{y_2} \frac{s}{g_3^2 s^2 + g_3^1 y_2 + g_3^0}\mathrm{d}s\right)$$

$$= \sigma_5\mathcal{G}_1(y_2) - \frac{\sigma_5\sigma_6\mathcal{R}_1}{(g_3^2)(\mathcal{R}_1-\mathcal{R}_2)}\ln|y_2-\mathcal{R}_1|$$

$$- \frac{\sigma_5\sigma_6\mathcal{R}_2}{(g_3^2)^2(\mathcal{R}_2-\mathcal{R}_1)}\ln|y_2-\mathcal{R}_1|$$

$$+ \frac{\sigma_5\sigma_6\mathcal{R}_1}{(g_3^2)(\mathcal{R}_1-\mathcal{R}_2)}\ln|y_{2,\mathrm{ref}}-\mathcal{R}_1|$$

$$+ \frac{\sigma_5\sigma_6\mathcal{R}_2}{(g_3^2)^2(\mathcal{R}_2-\mathcal{R}_1)}\ln|y_{2,\mathrm{ref}}-\mathcal{R}_1|.$$

The numerical approximation of the integrals and their polynomial approximations in (31) and (32) are depicted in Fig. 5, showing that both lead to similar descriptions.

4.1 Observer design and numerical results

Choosing the observer gains as $l_1 = 10$ and $l_2 = \frac{l_1^2}{4} = 25$, the results shown in Fig. 6 are obtained. As can be seen

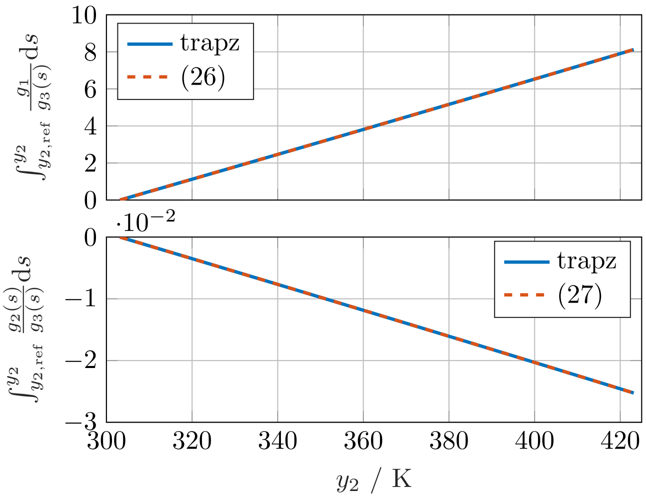

Fig. 5. Numerical integration and polynomial approximation of the integrals in (15).

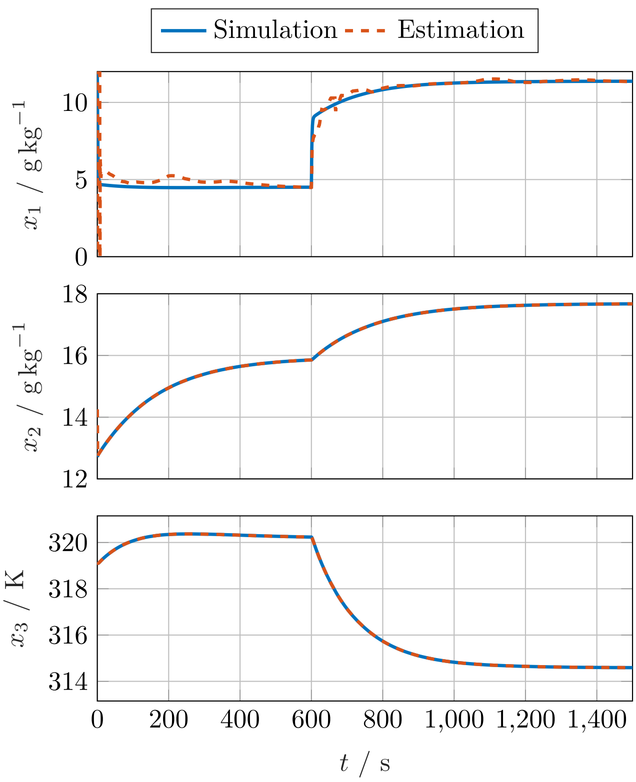

Fig. 6. Numerical results of the unknown input observer in original coordinates.

from Fig. 6, after an initial overshoot in the estimate of the product moisture a good convergence behavior is obtained that can enable a reliable monitoring of the product moisture content during process runtime. At the same time, temperature is directly fed through the estimator by construction, and the value for the air humidity is quickly reconstructed as expected. Further tuning beyond the chosen values could potentially improve product moisture (x_1) estimation, what should be studied in more detail in future studies.

5. CONCLUSION

By the preceding analysis it is shown that it is possible to obtain reliable model-based state estimates in spray drying processes without the need of a detailed model for the evaporation rate if the tower (or tower outlet) air temperature and humidity are measured, which are easily accessible process variables. It is demonstrated that by an adequate model-based sensor fusion, involving an integral transformation that requires knowledge about the specific heat of evaporation, an unknown input observer can be designed. For the purpose of the present study a geometric observer design was employed for this task, which extends previous ones in the sense that it explores the particular state and sensor signal transformations. The performance of the proposed scheme is illustrated using numerical simulations for a model that was previously validated using experimental data.

Future studies should further involve the additional estimation of the evaporation rate as well as experimental testing of the proposed scheme under different process conditions. Furthermore, a more detailed exploration of the structure of the nonlinear terms appearing in the geometric observer design could be considered that in the present study were handled using a Lipschitz continuity argument leading to sufficient condition for the convergence of the estimates to the actual states. Additionally, the approach will be extended to consider sampled and delayed measurements, following ideas from (Hernández and Alvarez, 2003; Hernández-Escoto et al., 2010).

REFERENCES

Alvarez, J. and Lopez, T. (1999). Robust dynamic state estimation of nonlinear plants. AIChE Journal, 45 (1), 107–123.

Aris, R. (1965). Introduction to the analysis of chemical reactors. Prentice Hall.

Astolfi, D., Marconi, L., Praly, L., and Teel, A. (2016). Sensitivity to high-frequency measurement noise of nonlinear high-gain observers. IFAC-PapersOnLine, 49(18), 862–866.

Darouach, M., Zasadzinski, M., and Xu, S. (1994). Full-order observers for linear systems with unknown inputs. IEEE Transactions on Automatic Control, 39(3), 606–609.

Edwards, C. and Spurgeon, S. (1998). Sliding mode control: theory and applications. Crc Press.

Francis, B. and Wonham, W. (1976). The internal model principle of control theory. Automatica, 12(5), 457–465.

Fritsching, U. (2016). Process-spray: functional particles produced in spray processes. Springer.

Goodwin, G. and Seron, M. (2020). Fundamental performance properties of a general class of observers for linear systems having predictable disturbances. Automatica, 113, 108717.

H. Niederwieser, M. Tranninger, R. Seeber, and M. Reichhartinger (2022). Unknown input observer design for linear time-invariant multivariable systems based on a new observer normal form. International Journal of Systems Science, 53(10), 2180–2206.

Ha, Q.P. and Trinh, H. (2004). State and input simultaneous estimation for a class of nonlinear systems. Automatica, 40(10), 1779–1785.

Hautus, M.L.J. (1983). Strong detectability and observers. Linear Algebra Appl., 50, 353–368.

Hernández, H. and Alvarez, J. (2003). Robust estimation of continuous nonlinear plants with discrete measurements. Journal of Process Control, 13(1), 69–89.

Hernández-Escoto, H., López, T., and Alvarez, J. (2010). Estimation of alkyd reactors with discrete-delayed measurements. Chemical engineering journal, 160(2), 698–707.

Isidori, A. (1995). Nonlinear Control Systems. Springer-Verlag, London.

Jerono, P., Schaum, A., and Meurer, T. (2021). Observability analysis and robust observer design for a continuous yeast culture. Journal of Process Control, 104, 62–73.

Khalil, H. (2007). High-gain observers in nonlinear feedback control. In New directions in nonlinear observer design, 249–268. Springer.

Lepsien, A., Jüptner, A., Scherließ, R., and Schaum, A. (2024). Control oriented modeling for particle size distributions in a spray drying process. IFAC-PapersOnLine, 58(15), 438–443.

Lepsien, A. and Schaum, A. (2024a). Thermodynamic model identification for a one-stage spray dryer. IFAC-PapersOnLine, 58(14), 228–234.

Lepsien, A. and Schaum, A. (2024b). Trajectory planning and tracking control for a one-stage spray dryer. IFAC-PapersOnLine, 58(14), 354–359.

Petersen, L.N., Jørgensen, J.B., and Rawlings, J.B. (2015). Economic optimization of spray dryer operation using nonlinear model predictive control with state estimation. IFAC-PapersOnLine, 48, 507–513.

Rocha, E. and Moreno, J. (2010). Dissipative design of unknown input observers for systems with sector nonlinearities. Int. J. Robust Nonlinear Control, 21 (14), doi:10.1002/rnc.1656., 1623–1644. doi: 10.1002/rnc.1656.,.

Rocha-Cozatl, E., Moreno, J.A., and Wouver, A.V. (2012). Application of a continuous-discrete unknown input observer to estimation in phytoplanktonic cultures. IFAC Proceedings Volumes, 45(15), 579–584.

Rocha-Cozatl, E., Sbarciog, M., Dewasme, L., Moreno, J., and Wouver, A.V. (2015). State and input estimation of an anaerobic digestion reactor using a continuous-discrete unknown input observer. In Proceedings of the 9th International Symposium on Advanced Control of Chemical Processes, Canada.

Rocha-Cozatl, E. and Vande Wouwer, A. (2011). State and input estimation in phytoplanktonic cultures using quasi-unknown input observers. Chemical Engineering Journal, 175, 39–48.

Schaum, A. and Meurer, T. (2015). A nonlinear quasi-unknown input estimator for the chemostat droop model. Computer Aided Chemical Engineering, 37, 1565–1570.

Schaum, A., Koch, S., and Moreno, J.A. (2024). Nonlinear quasi-unknown input observer design using dissipativity. Proceedings of the IEEE CDC, Milan, Italy, accepted.

Wrzosek, K., Moravčík, J., Antošová, M., Illeová, V., and Polakovič, M. (2013). Spray drying of the mixtures of mono-, di-, and oligosaccharides. Acta Chimica Slovaca, 6(2), 177–181.