Adaptive Optimal Control of Lettuce Growth in Greenhouses Using Sensitivity-Driven Measurement Collection

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Abstract: This paper presents a novel workflow for the design of an adaptive model-based controller to optimize the time and energy consumption for plant cultivation combined with online analysis and estimation of model parameters based on scarce data. A non-linear model of lettuce growth is subject to sensitivity analysis of selected parameters to determine the effective sequence and time horizon of infrequent data sampling of plant physiological properties. In the designed measurement campaign, the parameter estimation is performed to update the model parameter space, improving the accuracy of plant growth predictions and control efficiency. The implementation of run-time-updated model in a predictive control framework leads to minimization of the energy-related cost and the full-growth time of the plant. Simulations show promising results in minimizing the time required to the desired plant yield.

Keywords: Greenhouse model, Sensitivity analysis, Parameter estimation, Optimal control

1. INTRODUCTION

The Earth's population is growing ever faster and it is estimated that up to two thirds of the population will live in cities by 2050 (United Nations, 2014). This puts large pressure on the efficient production of high-quality food in order to maintain sustainable health conditions among the community (Brears, 2023). Concurrently, the cultivation of plants is closely intertwined with the influence of external conditions, global climate changes, which is also reflected in the non-linearities and negative trend of their growth. To minimize these limitations, data analysis and optimal tuning of crop growth control in greenhouses creates opportunities for increasing crops production efficiency while reducing resource consumption (Shimizu et al., 2008; Graamans et al., 2018; Eaton et al., 2023).

Among the different plants, lettuce cultivation has emerged as a suitable plant type for growth in greenhouses because of its ability to fast growing and the minimum area requirements for space occupation. Several models of lettuce have been proposed, while a frequently used model is by Van Henten (1994). The dynamic model of lettuce growth is based on the evolution of the dry weight of the plant, which is affected by (I) the growth of the physical parts of the plant such as leaves, roots, stem and its structure and (II) the reserves of substances and nutrients that the plant needs to carry out physiological processes. The analysis of the effect of external resources on the growth of lettuce plants is discussed by Mahmood et al. (2024), where the electricity, water, light, and CO_2 requirements are considered in estimating the yield of the cultivated plant. Stochastic modelling approaches were addressed by Chang et al. (2021) using fuzzy-logic and neural networks to efficiently predict the growth properties of lettuce, including transpiration and photosynthetic processes of the plant.

Models predicting the lettuce growth are typically characterised by a large number of parameters whose values are difficult to obtain, i.e., have been measured under specific conditions (Van Holsteijn, 1981; Sweeney et al., 1981) and the global climate change has had a significant impact on altering their values. This causes significant problems in the manipulation (control) of lettuce growth when suboptimal yield occurs. For this reason, the estimation of selected parameters of the lettuce model is investigated by Ioslovich et al. (2006) who achieved good results when applying the Dominant Parameter Selection method. Ojo et al. (2024) reported results of an R^2 index above 90 % when predicting lettuce phenotypic parameters using a deep learning regression model, while Gang et al. (2022) obtained similar results for the estimation of several growth indices using a convolutional neural network.

The estimation of lettuce model parameters is often preceded by an analysis of parameter estimability to increase the relevance of the information provided by the available data. The sensitivity of model parameters is widely calculated using relative parameter sensitivity indices (Van Henten and Van Straten, 1994; Lopez-Cruz et al., 2004), where the effect of a parameter change on the measured state is normalized parameter- and state-wise. Another method involves the normalized deviation ratio (Tan et al., 2022), based on root-mean squared deviation of the model dynamics before and after parameter perturbation. However, neither of the approaches clearly explains how the parameter perturbation affects the model dynamics across the whole time horizon. Simultaneously, the literature provides a limited representation of the application of such information in adaptive optimal control of lettuce growth, where the authors exclusively focus on implementing the predictive control (van Straten et al., 2000; Padmanabha et al., 2020) or distributed control (Rohde and Forni, 2023) without considering the time-evaluation of the model parameters and the overall production time for the desired product yield of the plant. Moreover, a non-negligible challenge in the effectiveness of model-based design of experiments is the efficient acquisition of the required information from collected data, burdened with measures of uncertainties and inaccuracies (Kusumo et al., 2022). Our work is concerned with the sensitivity analysis of the lettuce growth model that guides the in-cultivation data collection schedule with a subsequent on-line parameter update to adapt the model for the energy-aware optimization. A workflow dealing with these challenges is proposed.

This paper is further divided into the following sections: In Section 2, the model of the lettuce growth model is defined. Optimal plant growth control, together with the model sensitivity analysis and parameter estimation, is described in Section 3. The workflow for lettuce growth is specified in Section 4, with simulated control results in Section 5.

2. PLANT GROWTH MODEL

We present the plant growth model of lettuce formulated by Van Henten (1994). The proposed first-principles model is based on the plant's physiology, with the main state variable being the dry weight of the plant. The dry weight $\boldsymbol{x}(t) = (x_1(t), x_2(t))^\intercal$ (in $\mathrm{g \cdot m^{-2}}$) consists of two main components – a structural dry weight $x_1(t)$ (SDW, i.e., a physical structure and support of the plant) and a nonstructural dry weight $x_2(t)$ (NSDW, i.e., a reserve of energy, nutrients, and other resources). The model consists of a set of nonlinear ordinary differential equations:

$$\frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = r_\mathrm{g}(t)x_1(t), \tag{1}$$

$$\begin{aligned}\frac{\mathrm{d}x_2(t)}{\mathrm{d}t} &= c_\mathrm{C}\psi_\mathrm{p}(t) - r_\mathrm{g}(t)x_1(t) - \psi_\mathrm{r}(t) \\ &- \frac{1-c_\mathrm{y}}{c_\mathrm{y}} r_\mathrm{g}(t)x_1(t),\end{aligned} \tag{2}$$

where $r_\mathrm{g}(t)$, $\psi_\mathrm{p}(t)$, $\psi_\mathrm{r}(t)$, and c_C denote the rates of growth (specific), gross canopy photosynthesis, maintenance respiration, carbon dioxide CO_2 to carbohydrate CH_2O conversion, respectively. c_y is the yield factor and t represents a time instant.

The output vector coincides with the state vector, while the destruction of a significant part of the plant is necessary to measure the dry weight. The control input vector $\boldsymbol{u}(t) = (u_1(t), u_2(t), u_3(t))^\intercal$ consists of canopy temperature $u_1(t)$ (in °C), greenhouse CO_2 concentration $u_2(t)$ (in $\mathrm{g \cdot m^{-3}}$), and incident photosynthesis active radiation $u_3(t)$ (in $\mathrm{W \cdot m^{-2}}$). The value of c_C is the ratio of the molecular weight of CH_2O and CO_2.

The SDW $x_1(t)$ is related to the specific growth rate $r_\mathrm{g}(t)$, representing the transformation of NSDW to SDW with

$$r_\mathrm{g}(t) = c_\mathrm{g,max}\frac{x_2(t)}{c_\mathrm{g}x_1(t) + x_2(t)} c_\mathrm{Q10,g}^{\frac{u_1(t)}{10}-2}, \tag{3}$$

where $c_\mathrm{g,max}$, c_g, $c_\mathrm{Q10,g}$ are, respectively, the saturation growth rate at 20 °C, growth rate coefficient, and growth rate sensitivity $\mathrm{Q_{10}}$ factor w.r.t. the canopy temperature.

The photosynthesis rate $\psi_\mathrm{p}(t)$ specifies the growth rate of the nonstructural dry weight. It is formulated as

$$\psi_\mathrm{p}(t) = \left(1 - e^{-c_\mathrm{e}c_\mathrm{a}(1-c_\mathrm{r})x_1(t)}\right)\psi_\mathrm{p,max}(t), \tag{4}$$

where $\psi_\mathrm{p,max}(t)$, c_e, c_a, and c_r represent the gross CO_2 assimilation rate for a canopy with $1\,\mathrm{m^{-2}}$ of effective surface area at the full soil covering, the light extinction coefficient, the structural leaf area ratio, and the ratio of the root dry mass to the dry mass of the whole plant.

The value of the gross CO_2 assimilation rate for a canopy $\psi_\mathrm{p,max}(t)$ in (4) is further calculated as

$$\psi_\mathrm{p,max}(t) = \frac{\epsilon(t)u_2(t)G(t)\left(u_3(t) - \Gamma(t)\right)}{\epsilon(t)u_2(t) + G(t)\left(u_3(t) - \Gamma(t)\right)}, \tag{5}$$

where $\epsilon(t)$, $\Gamma(t)$, and $G(t)$ denote the light use efficiency, the CO_2 compensation point, and the canopy conductance related to the diffusion of the CO_2, respectively.

The light use efficiency $\epsilon(t)$ is related to the CO_2 compensation point $\Gamma(t)$ as follows

$$\epsilon(t) = c_\epsilon\frac{u_3(t) - \Gamma(t)}{u_3(t) + 2\Gamma(t)}, \tag{6}$$

where c_ϵ is the light use efficiency at high CO_2 concentration. The compensation point $\Gamma(t)$ is obtained as

$$\Gamma(t) = c_\Gamma c_\mathrm{Q10,\Gamma}^{\frac{u_1(t)}{10}-2}, \tag{7}$$

where c_Γ is the CO_2 compensation point at 20 °C and $c_\mathrm{Q10,\Gamma}$ is the temperature-dependent $\mathrm{Q_{10}}$ factor of $\Gamma(t)$.

The overall ability to diffuse the CO_2 from the air to the chloroplast in the plant is described by the canopy conductance $G(t)$, which is calculated as

$$\frac{1}{G} = \frac{1}{G_\mathrm{b}} + \frac{1}{G_\mathrm{s}} + \frac{1}{G_\mathrm{c}(t)}, \tag{8}$$

where G_b, G_s, and $G_\mathrm{c}(t)$ are the boundary layer, stomatal, and carboxylation conductances, respectively.

The $G_\mathrm{c}(t)$ is a polynomial function in the temperature range 5 °C to 40 °C with

$$G_\mathrm{c}(t) = a_\mathrm{c}u_1^2(t) + b_\mathrm{c}u_1(t) + c_\mathrm{c}, \tag{9}$$

where a_c, b_c, and c_c represent the tabulated parameters.

The NSDW relates to the maintenance respiration rate as

$$\psi_\mathrm{r}(t) = \left(c_\mathrm{s}\left(1 - c_\mathrm{r}\right) + c_\mathrm{m}c_\mathrm{r}\right)x_1(t)c_\mathrm{Q10,r}^{\frac{u_1(t)}{10}-\frac{5}{2}}, \tag{10}$$

where c_s and c_m are the shoot and the root maintenance respiration coefficient at 25 °C, $c_\mathrm{Q10,r}$ represent the $\mathrm{Q_{10}}$ factor of the maintenance respiration. The parameter values are displayed in Table 1.

3. ADAPTIVE MODEL PREDICTIVE CONTROL

We introduce the essential theoretical concepts that support the proposed advanced control solution.

3.1 Sensitivity Analysis

The relative sensitivities of the model states in (1) and (2) w.r.t. the selected parameters are given by:

Table 1. Growth model parameters.

Parameter	Value	Unit	Source
c_C	6.82×10^{-1}	1	—
c_y	0.80	1	1
c_g	1.20	1	1
c_ϵ	1.70×10^{-5}	$\mathrm{g} \cdot \mathrm{J}^{-1}$	2
c_Γ	4.00×10^{1}	$\mathrm{g} \cdot \mathrm{m}^{-3}$	2
c_e	6.75×10^{-2}	1	2
c_r	0.15	1	3
c_a	1.00	$\mathrm{g}^{-1} \cdot \mathrm{m}^{-2}$	3
$c_{\mathrm{g,\,max}}$	5.00×10^{-6}	s^{-1}	4
$c_{\mathrm{Q10},\Gamma}$	2.00	1	2
$c_{\mathrm{Q10,g}}$	1.60	1	1
$c_{\mathrm{Q10,r}}$	2.00	1	5
c_s	3.47×10^{-7}	s^{-1}	6
c_m	1.16×10^{-7}	s^{-1}	6
G_b	7.20×10^{-4}	$\mathrm{m} \cdot \mathrm{s}^{-1}$	7
G_s	5.00×10^{-3}	$\mathrm{m} \cdot \mathrm{s}^{-1}$	7
a_c	-1.32×10^{-5}	$\mathrm{m} \cdot \mathrm{s}^{-1} \cdot {}^\circ\mathrm{C}^{-2}$	5
b_c	5.94×10^{-4}	$\mathrm{m} \cdot \mathrm{s}^{-1} \cdot {}^\circ\mathrm{C}^{-1}$	5
c_c	-2.64×10^{-3}	$\mathrm{m} \cdot \mathrm{s}^{-1}$	5

[1] – Sweeney et al. (1981), [2] – Goudriaan et al. (1985), [3] – Lorenz and Wiebe (1980), [4] – Van Holsteijn (1981), [5] – Van Henten (1994), [6] – Van Keulen et al. (1982), [7] – Stanghellini (1987)

$$\boldsymbol{S}_{\boldsymbol{x}_i,\boldsymbol{p}_j}(t) = \frac{\partial \boldsymbol{x}_i(t)}{\partial \boldsymbol{p}_j}\frac{\boldsymbol{p}_j}{\boldsymbol{x}_i(t)}, \quad \begin{cases} i \in \{1,2\}, \\ j \in \{1,\dots,N_p\}, \end{cases} \tag{11}$$

where $\boldsymbol{p}$ denotes a vector of analysed parameters of size N_p, indices i and j represent the state and parameter number in $\boldsymbol{x}(t)$ and $\boldsymbol{p}$ vectors. The sensitivity calculation can be simplified by replacing the term $\partial \boldsymbol{x}_i(t)/\partial \boldsymbol{p}_j$ with $\Delta \boldsymbol{x}_i(t)/\Delta \boldsymbol{p}_j$ in (11), where $\Delta \boldsymbol{x}_i(t) = \boldsymbol{x}_{i,\Delta \boldsymbol{p}_j}(t) - \boldsymbol{x}_i(t)$ represents the difference of i-th state value before and after the model perturbation of j-th parameter $\Delta \boldsymbol{p}_j$. The simplified relative sensitivity is selected, as it classifies the influence of the parameter change to model evaluation (France and Thornley, 1984).

3.2 Parameter Estimation

The parameter estimation is done using the measurements of SDW and NSDW of the plant. The least-squares approach between the model predictions and measurements is used to find the real values of the parameters of the plant model. The optimization problem is formulated as:

$$\min_{\boldsymbol{p}} \quad \frac{1}{2}\sum_{j=1}^{M} \|\boldsymbol{x}(t_j) - \boldsymbol{x}_j^\mathrm{m}\|_2^2 \tag{12a}$$

$$\text{s.t.} \quad \dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}), \quad \forall t \in [0, t_\mathrm{M}], \tag{12b}$$

$$\boldsymbol{x}(0) = \boldsymbol{x}_\mathrm{init}, \tag{12c}$$

where $\{t_1, \dots, t_M\}$ are the measurement time instants of M measurements, $\boldsymbol{x}$ is the estimated state of the plant at the time of measurement, $\boldsymbol{p}$ are the parameters of the model, $\boldsymbol{f}(\cdot)$ is the model of the plant in (1)–(10), $\boldsymbol{x}^\mathrm{m}$ are the measurements of the plant, $\boldsymbol{x}_\mathrm{init}$ is the initial state of the plant, $\boldsymbol{u}$ is the vector of control inputs, t_M is the time of the last measurement, respectively.

3.3 Plant Growth Control

The optimal control problem is formulated as:

$$J^\star(\boldsymbol{p}, \boldsymbol{x}_0) = \min_{\boldsymbol{u}(t), t_\mathrm{f}} \quad t_\mathrm{f} + \int_0^{t_\mathrm{f}} \boldsymbol{u}(t)^\intercal \boldsymbol{Q}_\mathrm{u} \boldsymbol{u}(t)\ \mathrm{d}t \tag{13a}$$

$$\text{s.t.} \quad \dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{p}), \tag{13b}$$

$$\boldsymbol{u}_\mathrm{min} \le \boldsymbol{u}(t) \le \boldsymbol{u}_\mathrm{max}, \tag{13c}$$

$$1000 \le x_1(t_\mathrm{f}) + x_2(t_\mathrm{f}), \tag{13d}$$

$$\boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{13e}$$

where t_f is the final time for harvesting, $\boldsymbol{x}_0$ is the current measured state of the plant, $\boldsymbol{Q}_\mathrm{u}$ is the weight matrix for the control input, $\boldsymbol{u}_\mathrm{min}$ and $\boldsymbol{u}_\mathrm{max}$ are the lower and upper constraints of the control input, respectively. The optimal control of the lettuce growth is based on the minimization of the time and energy needed for the plant to grow at least to a minimal dry weight, which is 1000 grams. The energy minimization is expressed through weighted 2-norm of inputs.

4. WORKFLOW

The presented workflow is designed to be used for the optimal control of the batch (or even continuous) systems under parametric uncertainty with scarce measurements. We consider the processes under parametric uncertainties in every batch, due to the composition of input, in our case, different plants, variety of plants or the different quality of seeds. By scarce measurements, we mean that the measurements are expensive and time-consuming to conduct. When estimating the parameters using scarce measurements, it is essential to schedule them effectively. We use sensitivity analysis to plan when to make the measurements. In the presented experiment, we assume to have access only to 3 measurements during a 50 days period. The proposed workflow is summarized in Algorithm 1.

In Algorithm 1, $\boldsymbol{T}$ is the vector of times of the absolute maximum of the sensitivity w.r.t. the j-th parameter. The $t^\star$ is then the earliest time instant in $\boldsymbol{T}$, i.e., the closest measurement point. Variables τ and $t_\mathrm{f}^\star$ denote elapsed and remaining growth time, respectively. Vector $\boldsymbol{S}_{\boldsymbol{p}_j}(t)$ represents the state sensitivities w.r.t. the j-th parameter. Initial parameter guess is set based on values in Table 1. During each while-loop body execution, only the parameters $\boldsymbol{p}$ differ, which are estimated from the measurements $\boldsymbol{x}^\mathrm{m}$. The parametric sensitivities alter as the parameters are adapted. To prevent taking measurements most sensitive to some parameter repeatedly, we exclude the parameter from the sensitivity analysis once the corresponding measurement is planned.

The regularization term $\boldsymbol{Q}_\mathrm{u}$ is chosen to balance the contributions of time and control inputs to the objective function in (13). Further the optimization problem is discretized with piecewise constant control profile with the number of elements N. The specific parameters are:

$$R = 2 \times 10^{-4}, \tag{14a}$$

$$\boldsymbol{Q}_\mathrm{u} = \mathrm{diag}\left[\frac{1}{20}, \frac{1}{100}, \frac{1}{400}\right] R, \tag{14b}$$

$$N = 100, \tag{14c}$$

$$\boldsymbol{u}_\mathrm{min} = (10, 0, 400)^\intercal, \tag{14d}$$

$$\boldsymbol{u}_\mathrm{max} = (40, 200, 800)^\intercal, \tag{14e}$$

and the problem is solved with the Python version 3.11.9 using CasADi (Andersson et al., 2019) library version

Algorithm 1 Adaptive Lettuce Growth Optimization Workflow

Initialize: Guess $\boldsymbol{p}$. $\mathcal{J} := \{1, \dots, N_p\}$. $\tau := 0$.
Get $\boldsymbol{x}_0 \leftarrow$ measure initial state $\boldsymbol{x}_{\text{init}}$
Get $\boldsymbol{u}^\star, t_{\text{f}}^\star \leftarrow$ solve problem (13) for $\boldsymbol{x}_0, \boldsymbol{p}$
Get $\boldsymbol{S}(t) \leftarrow$ solve Eq. (11) $\forall t \in [0, t_{\text{f}}^\star]$ for $\boldsymbol{x}_0, \boldsymbol{p}, \boldsymbol{u}^\star$
for $j \in \mathcal{J}$ **do**
    $\boldsymbol{T}_j := \arg\max_t \|\boldsymbol{S}_{\boldsymbol{p}_j}(t)\|_\infty$
end for
Get $t^\star := \min_j \ \boldsymbol{T}_j$.
while $t_{\text{f}}^\star \geq t_{\text{s}}$ **do**
    if $\tau = t^\star$ **then**
        $\boldsymbol{x}_0 \leftarrow$ measure current state
        Push $\boldsymbol{x}_0$ to $\boldsymbol{x}^{\text{m}}(t^\star)$.
        $\boldsymbol{p} \leftarrow$ solve problem (12) with $\boldsymbol{x}^{\text{m}}(t^\star)$
        Get $\boldsymbol{u}^\star, t_{\text{f}}^\star \leftarrow$ solve problem (13) for $\boldsymbol{x}_0, \boldsymbol{p}$
        Get $\boldsymbol{S}(t) \leftarrow$ solve Eq. (11) $\begin{cases} \forall t \in [0, \tau + t_{\text{f}}^\star] \\ \text{for } \boldsymbol{x}_{\text{init}}, \boldsymbol{p}, \boldsymbol{u}^\star \end{cases}$
        for $j \in \mathcal{J}$ **do**
            $\boldsymbol{T}_j := \arg\max_t \|\boldsymbol{S}_{\boldsymbol{p}_j}(t)\|_\infty$
        end for
        Get $j^\star := \arg\min_j \ \boldsymbol{T}_j$.
        Assign $t^\star := \boldsymbol{T}_{j^\star}$. $\mathcal{J} := \mathcal{J} \backslash \{j^\star\}$.
    end if
    Apply $\boldsymbol{u}^\star(t)$ to the plant $\forall t \in [0, t_{\text{s}}]$.
    $\tau := \tau + t_{\text{s}}$
    $t_{\text{f}}^\star := t_{\text{f}}^\star - t_{\text{s}}$
end while
Terminate: Harvest

3.6.5 using Opti stack — a collection of CasADi helper classes and is applied to the simulation every $t_{\text{s}} = 1\,\text{h}$, representing the sampling time.

Reflecting the analysis in Van Henten (1994) and Van Henten and Van Straten (1994), the vector of adapted parameters $\boldsymbol{p}$ is defined as

$$\boldsymbol{p} = (c_{\text{g,max}}, c_{\text{e}}c_{\text{a}}, c_{\epsilon}, c_{\text{y}})^{\mathsf{T}}\,, \tag{15}$$

where the product of the light extinction coefficient and the structural leaf area ratio $c_{\text{e}}c_{\text{a}}$ is considered as a single parameter since, based on the formulation in (4), their individual values are not estimable. The parameter estimation problem (12) is solved with SciPy (Virtanen and SciPy 1.0 Contributors, 2020) library version 1.11.2 using the function `optimize.least_squares`.

5. RESULTS

For the purpose of comparison, we denote the variables obtained during the simulation using the proposed workflow with a double prime ($''$) and the results obtained from the simulation without any modifications to the model (non-adaptive approach) during control with a single prime ($'$).

In the example simulations, the trajectories of the states and control inputs are depicted in Figure 1 for the proposed workflow and for the non-adaptive approach. The starting nominal initial states are $\boldsymbol{x}(0) = (0.72, 2.70)$ and the parameters $\boldsymbol{p}$ from Table 1 are lowered by 15%. After obtaining the $\boldsymbol{u}^\star$ and performing a sensitivity analysis, the first measurement is scheduled at $t = 6.125\,\text{d}$. At this point, the earliest maximum sensitivity is reached for the parameter $c_{\text{e}}c_{\text{a}}$. From this measurement, 2 parameters

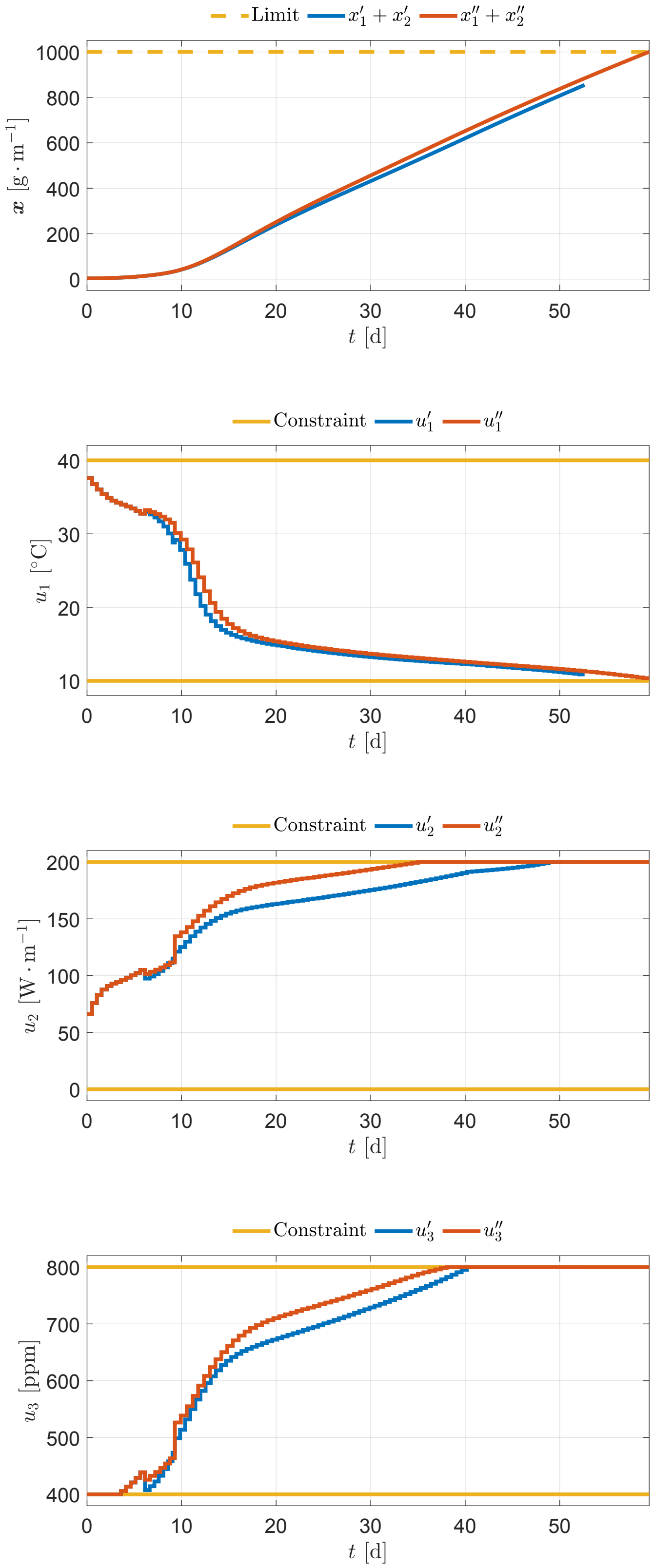

Fig. 1. Trajectories of controlled and manipulated variables for a representative scenario of initial parameters and model perturbations in simulation.

Table 2. The statistics of the simulation results: variables with a double prime ($''$) represent the values obtained by applying proposed workflow, while variables with a single prime ($'$) represent the values reached with non-adaptive control.

Variables	Units	Mean	St.Dev.
$x_1'' + x_2''$	$\mathrm{g \cdot m^{-2}}$	1.000×10^3	0.344
$x_1' + x_2'$	$\mathrm{g \cdot m^{-2}}$	0.996×10^3	67.275
t''_{end}	d	5.196×10^1	2.801
t'_{end}	d	5.146×10^1	0.319
$\sum \left(u_1'' - u_{1\,\mathrm{min}}\right)$	°C	8.836×10^3	519.7
$\sum \left(u_1' - u_{1\,\mathrm{min}}\right)$	°C	8.795×10^3	260.2
$\sum \left(u_2'' - u_{2\,\mathrm{min}}\right)$	$\mathrm{W \cdot m^{-2}}$	2.013×10^5	17.695×10^3
$\sum \left(u_2' - u_{2\,\mathrm{min}}\right)$	$\mathrm{W \cdot m^{-2}}$	1.991×10^5	1.222×10^3
$\sum \left(u_3'' - u_{3\,\mathrm{min}}\right)$	ppm	3.357×10^5	37.969×10^3
$\sum \left(u_3' - u_{3\,\mathrm{min}}\right)$	ppm	3.323×10^5	1.945×10^3

are estimated: $c_{\mathrm{e}}c_{\mathrm{a}}$ and $c_{\mathrm{g,max}}$, as they have the highest absolute sensitivity at the time. Further, the plant states are measured and the selected parameters are estimated. Then, the control input $\boldsymbol{u}^\star$ is recomputed based on the measurement and the updated parameters. After this, the sensitivity analysis is performed again with the updated model. The time of the next measurement is planned at $t = 9.292\,\mathrm{d}$. The recomputation of sensitivity analysis is important because the same measurement, without recomputing the analysis for the updated model, would be planned on the $t = 6.625\,\mathrm{d}$. Upon gathering the second measurement, the remaining two parameters are estimable. A precise values of parameters are obtained as no process noise is present. At this point, the $\boldsymbol{u}^\star$ is computed for the updated model and latest measurement. It is being applied for system control to the desired minimal weight of the dry weight.

The trajectories of the control input are changed at the time of measuring the plant states and predicting the parameters. The non-adaptive approach only changes based on the measurement undergone at the same time as the measurements based on sensitivity analysis without changing the parameters of a model. The measurement approach for the non-adaptive approach is performed such that both approaches provide the same information. The measurement times, based on sensitivity analysis, are $t = 6.125\,\mathrm{d}$ and $t = 9.292\,\mathrm{d}$. After these two experiments, the four parameters are exactly estimated, and the plant grows to the required dry weight. It is important to mention that this is achieved due to no model-plant mismatch and no consideration of the measurement noise during the experiments. The non-adaptive approach is not able to reach the desired dry weight and the plant is harvested too early, after $52.625\,\mathrm{d}$. The dry weight reached with the non-adaptive approach is more than 14% less than the desired value; in this case, the product would not be marketable. The workflow approach is able to reach the desired dry weight and is harvested after $59.458\,\mathrm{d}$.

For better assessment of the proposed workflow, we carried out a $1,000$ example simulations. Each simulation started with randomly generated states and parameters. The random initial states and parameters are within $\pm 15\%$ of the nominal initial states $\boldsymbol{x}(0) = (0.72, 2.70)$ and the initial selected parameters $\boldsymbol{p}$ are taken from Table 1. The results of the simulations are summarized in Table 2.

The proposed workflow in Section 4 is applied and x'', u'' and t'' are obtained. For the non-adaptive approach, the simulation is computed based on $\boldsymbol{x}(0)$ and following measurements, without any further changes to the model or parameter estimation and x', u' and t' are obtained. The results are provided in Table 2. The most important improvement is the final value of the sum of structural and nonstructural dry weight, $x_1 + x_2$. It is the most important indicator since it represents the final dry weight of the plant and the criteria of the product quality. The non-adaptive approach is often unable to reach the minimal desired value of $1\,\mathrm{kg \cdot m^{-2}}$ of dry weight or highly overshoots this value, while the energy is wasted for the plant growth more than necessary. By applying the proposed workflow, it is possible to consistently reach the minimal desired value of the final dry weight and to lower its standard deviation more than 200 times. However, this crucial improvement comes with little cost in terms of time and use of control inputs. The inputs fluctuated significantly during the simulations, yet the mean values over $1,000$ simulations are essential. The time to grow the plants to desired weight increased by 0.97%. The usage of control input u_1 increased by 0.47%, the usage of control input u_2 increased by 1.10%, and the usage of control input u_3 increased by 1.02%. These values are essential for the economic aspects of the plant cultivation as well as the final dry weight. The reported increase is not significant.

6. CONCLUSION

In this paper, a workflow for the optimal control of lettuce growth, addressing the challenges posed by parameter uncertainty and limited measurement availability, is proposed. The sensitivity analysis allowed for guiding the timing of measurements and improving the parameter estimation process. By integrating these approaches into an optimal control framework, we achieved consistent growth of lettuce to a target dry weight while optimizing energy use. The workflow demonstrated significant improvements in terms of growth accuracy and negligible increase in necessary time and control inputs over $1,000$ simulations. The presented approach can be further tested in a controlled environment in Smart Eco Greenhouse VESNA (Oravec et al., 2023). We also see the potential of the proposed workflow to be used in other processes with scarce measurement and not be limited to the plant grow control.

ACKNOWLEDGEMENTS

This paper is funded by the European Union's Horizon Europe under grant no. 101079342 (Fostering Opportunities Towards Slovak Excellence in Advanced Control for Smart Industries). PV and JV thank for financial contribution from the STU Grant Scheme for Support of Young Researchers. RP acknowledges the financial support by the European Commission under the grant scheme NextGenerationEU project no. 09I03-03-V04-00530.

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