Performance Change Recovery in Soft-Sensor Control Loops

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Abstract: In most industrial settings, soft sensors are used to measure variables that are challenging to estimate. However, due to various factors, the estimated performance may vary over time, causing it to fail in estimating key variables quickly and accurately enough, which could result in financial losses and security risks. The variation in the predictive performance of a soft sensor is referred to as the performance drift of the soft sensor. These changes occur due to the differences between the current characteristics of the process or plant and the soft sensor. This discrepancy is also defined as plant-model mismatch (PMM). Therefore, once the soft sensor is designed, a way to recover the performance change is required. This paper proposes a method that reduces the impact of PMM without updating the soft sensor itself. This method reconstructs the closed-loop controller based on the performance change index (PCI) for the soft sensor. Then, the online identification of the fault model is studied. Finally, the modification rules for the controller are given using the Youla-Kučera parameterisation. This approach is tested on a three-tank system, where it is shown that the performance changes of the soft sensor caused by PMM are recovered.

Keywords: **Soft sensor, Performance-change recovery, Plant-model mismatch, Process control**

1. INTRODUCTION

In the current industrial environment, soft sensors are typically used to estimate critical variables that are challenging to measure directly. Depending on the modelling method, soft sensors can be categorised into three groups: model-based, data-driven, and hybrid models. The first group, model-based or white-box soft sensors, is based on a first-principles model (FPM) (Fortuna, Graziani, Rizzo, & Xibilia, 2007). As well, Kalman filters (Mangold, 2012) (Ahmad, Ayub, Kano, & Cheema, 2020) and adaptive observers (Bastin & Dochain, 1990) have been used. Since an FPM calculates the target variables based on the physical and chemical knowledge of the real process, it does not consider the effect of disturbances on the system. Thus, the second group, data-drive or black-box models have been developed. The models are developed using many different methods including principle component regression (PCR) (Krdlec, Gabrys, & Strandt, 2009) and acritical neural networks (ANN) (Yan, Tang, & Lin, 2017). Since this soft sensor is developed without explicit process knowledge, performance can be improved by including some process knowledge. Adding this information creates the third group of soft sensors: hybrid or grey-box models (Fortuna, Graziani, Rizzo, & Xibilia, 2007). Despite the accuracy involved in initially modelling the process, over time the performance of the model will deteriorate due to changes in the properties of the raw materials, in the external environment, and in the catalyst activity. To optimise the performance, online adaptive methods have been developed, where the model is regularly updated using newly collected data samples, for example, moving-window-kernel principal component analysis (Liu, Kruger, Littler, & Wang, 2009). Some progress has also been made in using a just-in-time learning (JITL) strategy, which builds local models based on multiple nearest neighbors from test data to perform adaptive prediction (Ge & Song, 2010). Although these methods have reported good results, there are still some problems with the practical application of these soft sensors. First, if the soft sensor is updated with any bad data, the performance will inevitably decrease. Currently, it can be difficult to accurately detect all the bad data. Second, model updates typically use a narrower range of data. Thus, if the soft sensor is adaptively updated, some model parameters will change significantly, which means that the model may need to be re-evaluated.

Plant-model mismatch (PMM) is the generic name for the discrepancy between the expected and actual behaviour. It would be advantageous if the performance can be maintained despite PMM without requiring an update of the soft sensor. Recently, a performance-change index (PCI) (Zhai & Shardt, 2024) was proposed to detect PMM. Therefore, if this index is combined with a compensation signal from outside to offset the residual signal caused by PMM, then the performance can be recovered without changing the soft sensor.

Thus, this paper proposes such a system and examines the requirements to attain this objective. As well, the effectiveness of the proposed method is tested on a three-tank system.

2. BACKGROUND AND PROBLEM FORMULATION

2.1. Background

This article focuses on the white-box soft sensor. We can consider the controlled white-box soft sensor system shown in Figure 1, where $G(z) = (A_G, B_G, C_G, D_G)$ represents the plant (or real process), $S(z) = (A, B, C, D)$ is the assumed process model (or the soft sensor), $K(z)$ represents the controller, $G_d(z)$ is the disturbance transfer function, $v(z) \in R^n$ is the reference signal, $u(z) \in R^m$ is the input signal, $y(z) \in R^n$ is the (true) plant output, $\theta(z) \in R^n$ is the soft-sensor output, and $d(z) \in R^n$ is the disturbance, assumed to be white, Gaussian noise.

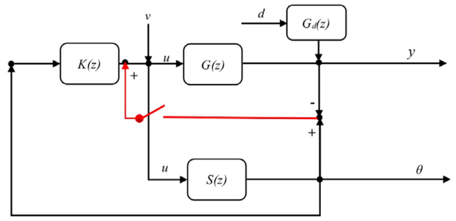

Figure 1: Closed-loop structure with soft sensors

Assuming that there is no PMM in the soft sensor, the switch in Figure 1 is disconnected. Therefore, we have

$$v(z) = u(z) - K(z)\theta(z) \tag{1}$$

$$\theta(z) = S(z)u(z) \tag{2}$$

The control signal is

$$u = K(z)\theta(z) + v \tag{3}$$

Lemma 1 (Zhou & Doyle, 1998) The left coprime factorisation (LCF) and right coprime factorisation (RCF) for the system $S(z)$ and the controller *K(z)* shown in Figure 1 are given by

$$S(z) = N(z)M(z)^{-1} = \hat{M}^{-1}(z)\hat{N}(z) \tag{4}$$

$$K(z) = U(z)V(z)^{-1} = \hat{V}^{-1}(z)\hat{U}(z) \tag{5}$$

where $\hat{M}(z), \hat{N}(z)$ are in the RH_∞ space with an equal number of rows, if there are two other matrices $\hat{X}(z)$ and $\hat{Y}(z)$ in the same space that satisfy:

$$[\hat{M}(z) \quad \hat{N}(z)]\begin{bmatrix}\hat{X}(z)\\ \hat{Y}(z)\end{bmatrix} = I \tag{6}$$

Similarly, *M(z), N(z)* are in the RH_∞ space with an equal number of columns, if there are other matrices *X(z)* and *Y(z)* in the same space that satisfy:

$$[X(z) \quad Y(z)]\begin{bmatrix}M(z)\\ N(z)\end{bmatrix} = I \tag{7}$$

Definition 1 (Vinnicombe, 2000) Consider the model *S(z)* in Figure 1 and a controller that can stabilise this model. Then all controllers that stabilise this model can be parameterised as

$$K = U(z)V(z)^{-1} = \hat{V}^{-1}(z)\hat{U}(z)$$
$$= -(\hat{Y}_0 + M_0 Q)(\hat{X}_0 - N_0 Q)^{-1}$$
$$= -(X_0 - Q\hat{N}_0)^{-1}(Y_0 + Q\hat{M}_0) \tag{8}$$

where *Q(z)* is the Youla-Kučera parameterisation matrix and *M, N, X, Y,* $\hat{M}$, $\hat{N}$, $\hat{X}$, $\hat{Y}$ are the transfer matrices satisfying Bézout's identity (Tay & Mareels, 1998):

$$\begin{bmatrix} X(z) & Y(z)\\ -\hat{N}(z) & \hat{M}(z)\end{bmatrix}\begin{bmatrix} M(z) & -\hat{Y}(z)\\ N(z) & \hat{X}(z)\end{bmatrix} = \begin{bmatrix} I & 0\\ 0 & I\end{bmatrix} \tag{9}$$

$$M(z) = \begin{bmatrix} A+BF & B\\ F & I\end{bmatrix}, \qquad N(z) = \begin{bmatrix} A+BF & B\\ C+DF & D\end{bmatrix}$$

$$\hat{X}(z) = \begin{bmatrix} A+BF & B\\ F & I\end{bmatrix}, \qquad \hat{Y}(z) = \begin{bmatrix} A+BF & -L\\ F & 0\end{bmatrix} \tag{10}$$

$$\hat{M}(z) = \begin{bmatrix} A-LC & L\\ -C & I\end{bmatrix}, \qquad \hat{N} = \begin{bmatrix} A-LC & B-LD\\ C & D\end{bmatrix}$$

$$X(z) = \begin{bmatrix} A-LC & -B+LD\\ L & I\end{bmatrix}, Y(z) = \begin{bmatrix} A-LC & -L\\ F & 0\end{bmatrix}$$

Here, $\mathcal{F}$ and $\mathcal{L}$ satisfy, respectively, $\mathcal{A}+\mathcal{BF}$ and $\mathcal{A}-\mathcal{LC}$ stability.

If PMM occurs in a soft sensor, the output $\theta(k)$ will shift. Therefore, a state observer and the observer-based residual generator (Ding, 2013) are given as

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L\left(\theta(k) - \hat{\theta}(k)\right) \tag{11}$$

$$\hat{\theta}(k) = C\hat{x}(k) + Du(k), r = \theta(k) - \hat{\theta}(k) \tag{12}$$

and the residual generator is

$$r = \hat{M}(z)\theta(z) - \hat{N}(z)u(z) \tag{13}$$

where $\hat{x}(k)$ is the state estimate and $\hat{\theta}(k)$ is the estimated value of $\theta(k)$.

Substituting Equation (12) into Equation (11) gives

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L\big(\theta(k) - C\hat{x}(k) - Du(k)\big)$$
$$= (A-LC)\hat{x}(k) + (B-LD)u(k) + L\theta(z) \tag{14}$$

Taking the *z*-transformation of Equation (14) gives

$$z\hat{x}(z) = (A-LC)\hat{x}(z) + (B-LD)u(z) + L\theta(z)$$

$$z\hat{x}(z) - (A-LC)\hat{x}(z) = (B-LD)u(z) + L\theta(z)$$

$$[zI\text{-}(A-LC)] \ \hat{x}(z) = (B-LD)u(z) + L\theta(z)$$

$$\hat{x}(z) = [zI-(A-LC)]^{-1}(B-LD)u(z) + [zI-(A-LC)]^{-1}L\theta(z) \tag{15}$$

Taking the *z*-transformation of Equation (12) gives

$$\hat{\theta}(z) = C\hat{x}(z) + Du(z)$$

Considering $\hat{x}(z)$ in Equation (15):

$$\hat{\theta}(z) = C[zI-(A-LC)]^{-1}(B-LD)u(z) + C[zI-(A-LC)]^{-1}L\theta(z) + Du(z)$$

$$r = \theta(z) - \hat{\theta}(z)$$
$$= \theta(z) - C[zI-(A-LC)]^{-1}(B-LD)u(z) - C[zI\text{-}(A-LC)]^{-1}L\theta(z) - Du(z)$$
$$= [I - C[zI-(A-LC)]^{-1}L]\theta(z) - [D + C[zI-(A-LC)]^{-1}(B-LD)]u(z) \tag{16}$$

From Equation (16), we get the residual generator (13). Here, $\hat{M}$, $\hat{N}$ are the transfer-function matrices:

$$\hat{M}(z) = [I - C[zI-(A-LC)]^{-1}L]$$

$$\hat{N}(z) = [D + C[zI-(A-LC)]^{-1}(B-LD)]$$

Substituting Equation (8) into Equation (3) gives

$$X_0 u + Y_0\theta = Q\hat{N}_0 u - Q\hat{M}_0\,\theta + \bar{v} \tag{17}$$

with $\bar{v} = (X_0 - Q\hat{N}_0)v$

Comparing Equations (17) and (13), shows that

$$X_0 u = -Qr - Y_0\theta + \bar{v} \tag{18}$$

With transfer matrices $X(z)$ and $Y(z)$ in Equation (10), we have

$$u(z) = -Q(z)r(z) + F[zI - A_L]^{-1}[L\theta(z) + B_L u(z)] + \bar{v} \tag{19}$$

where $A_L = A - LC, B_L = B - LD$.

Comparing Equations (19) and (15), a new control rule can be given as

$$u(z) = F\hat{x}(z) - Q(z)r(z) + \bar{v}(z) \tag{20}$$

At this point, the control block diagram becomes that shown in Figure 2. The switch is finally closed to verify the effect of accuracy of PMM on the effectiveness of the proposed method.

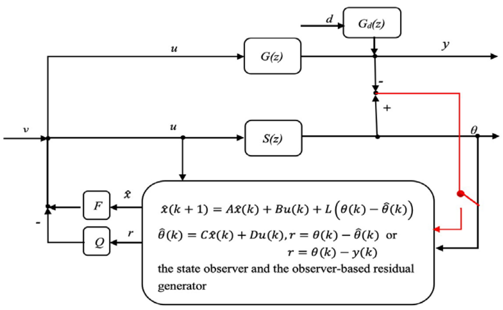

Figure 2: Closed-loop structure with soft sensors

In Figure 2, when the performance of the soft sensor degrades to an unacceptable level, the generated residual signal $r(z)$ can activate the optimal Q to update the control module online and generate a compensation signal to offset the error signal of the system caused by PMM.

2.2 Problem formulation

Let

$$S_P = \hat{M}^{-1}\hat{N} = \left(\hat{M}_0 + \Delta_{\hat{M}}\right)^{-1}\left(\hat{N}_0 + \Delta_{\hat{N}}\right) \tag{21}$$

where $\hat{M}_0, \hat{N}_0 \in RH_\infty$ represent the LCF of the soft sensor when there are not faults and $\Delta_{\hat{M}}, \Delta_{\hat{N}} \in RH_\infty$ represent the model uncertainties (Vinnicombe, 2000).

The main goal of the paper is to find the parameter Q to provide the compensation signal. Specifically, the Youla-Kučera parameterization is first studied, which lays the foundation for this paper. Then, based on the realisation form of the PMM and online identification of fault model, a performance change recovery method is developed. Finally, a three-tank system was used to verify the effectiveness of the proposed method.

Let us assume that the soft sensor can be modelled as a linear and time-invariant system. In addition, a controller and an observer can be designed to satisfy the performance and stability requirements in the soft-sensor control loop. Finally, assume that the reference signal $v(z)$ is continuous exciting.

3. PERFORMANCE CHANGE RECOVERY

3.1 Calculation of the Youla-Kučera parameter Q

The performance-change index (PCI) (Zhai & Shardt, 2024) can detect the performance change of a soft sensor:

$$\text{PCI} = \left\| \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 & -U \\ N_0 & V \end{bmatrix} \right\|_\infty \tag{22}$$

where $\Delta_{\hat{M}}, \Delta_{\hat{N}} \in RH_\infty$ are the PMM of the soft sensor.

Substituting Equation (8) into Equation (22):

$$\text{PCI} = \left\| \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 & -\hat{Y}_0 - M_0 Q \\ N_0 & \hat{X}_0 - N_0 Q \end{bmatrix} \right\|_\infty \tag{23}$$

where the Youla-Kučera parameter Q is an unknown matrix. It obviously can negate the effect of PMM. If we can find the optimal Q^*, it will generate an optimal compensation signal to offset the influence of PMM, which will restore the PCI to its original state:

$$\text{PCI}^* = \left\| \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} M_0 & -\hat{Y}_0 - M_0 Q^* \\ N_0 & \hat{X}_0 - N_0 Q^* \end{bmatrix} \right\|_\infty \tag{24}$$

$P_1 = [-\Delta_{\hat{N}} \quad \Delta_{\hat{M}}]\begin{bmatrix} M_0 \\ N_0 \end{bmatrix}$; $P_2 = [-\Delta_{\hat{N}} \quad \Delta_{\hat{M}}]\begin{bmatrix} -\hat{Y}_0 - M_0 Q^* \\ \hat{X}_0 - N_0 Q^* \end{bmatrix}$

If P_2 satisfies the following stability condition (Georgiou & Smith, 1990):

$$\|[-\Delta_{\hat{N}} \quad \Delta_{\hat{M}}]\|_\infty \left\| \begin{bmatrix} -\hat{Y}_0 - M_0 Q \\ \hat{X}_0 - N_0 Q \end{bmatrix} \right\|_\infty < 1 \tag{25}$$

then, the soft sensor will be internally stable. Therefore, the optimal parameter Q needs to be found such that the above condition can be met:

$$Q^* = arg \inf_{Q \in RH_\infty} \left\| \begin{bmatrix} -\Delta_{\hat{N}} & \Delta_{\hat{M}} \end{bmatrix} \begin{bmatrix} -\hat{Y}_0 - M_0 Q \\ \hat{X}_0 - N_0 Q \end{bmatrix} \right\|_\infty \tag{26}$$

However, P_2 contains the unknown PMM. Therefore, identifying the PMM is a prerequisite for calculating Q^*. To solve this, we will consider the realisation form of PMM.

3.2 The realisation form of PMM

The stable kernel representation (SKR) of the model is not unique. Assume that $[-\hat{N} \quad \hat{M}]$ and $[-\hat{N}_s \quad \hat{M}_s]$ are two different SKRs of the faulty soft sensor. There exists $R(Z) \in RH_\infty$ (Ding, 2013) such that

$$[-\hat{N}_s \quad \hat{M}_s] = R[-\hat{N} \quad \hat{M}], R^{-1}(Z) \in RH_\infty$$

$$[-\Delta_{\hat{N}} - \hat{N}_0 \quad \Delta_{\hat{M}} + \hat{M}_0] = R[-\hat{N} \quad \hat{M}]$$

where $[\hat{N}_0 \quad -\hat{M}_0]$ is the SKR of the fault-free soft sensor and $[\hat{N} \quad -\hat{M}]$ is the SKR of the faulty soft sensor.

The PMM can be written as

$$[-\Delta_{\hat{N}} \quad \Delta_{\hat{M}}] = [-R\hat{N} + \hat{N}_0 \quad R\hat{M} - \hat{M}_0] \tag{27}$$

where $R(z)$ can be any transfer function.

Thus, PMM can be described as

$$\|[-\Delta_{\hat{N}} \quad \Delta_{\hat{M}}]\|_\infty \geq \inf_{R \in RH_\infty} \|[-R\hat{N} + \hat{N}_0 \quad R\hat{M} - \hat{M}_0]\|_\infty \tag{28}$$

According to the stability condition (Georgiou & Smith, 1990), the effect of PMM on model stability has a unique expression:

$$[-\Delta_{\hat{N}} \quad \Delta_{\hat{M}}] = [\hat{N}_0 \quad -\hat{M}_0] - R^*[\hat{N} \quad -\hat{M}] \tag{29}$$

$$R^* = \arg \inf_{R \in RH_\infty} \|[\hat{N}_0 \quad -\hat{M}_0] - R^*[\hat{N} \quad -\hat{M}]\|_\infty \tag{30}$$

$\begin{bmatrix}\hat{N} & -\hat{M}\end{bmatrix}$ is the SKR of the faulty soft sensor, which is unknown. To calculate the PMM, The SKR should first be identified.

Thus, based on input/output data, the SKR data-driven implementation is examined. To this end, some necessary notation is introduced (Jiang, An, Huo, & Yin, 2018):

$$w_{s,k} = \begin{bmatrix} w_k \\ \vdots \\ w_{k+s-1} \end{bmatrix};\ W_{k,s} = \begin{bmatrix} w_{s,k} & \cdots & w_{s,k+N-1} \end{bmatrix} \tag{31}$$

where w_k is the sampled data at the time K and s is the truncation length of the signal sequence w_k. S and K are sufficiently large positive integers.

According to Equation (31), construct $u_{s,k}$, $\theta_{s,k}$ and the Hankel matrices $U_{k,s}$, $\theta_{k,s}$:

$$u_{s,k} = \begin{bmatrix} u_k \\ \vdots \\ u_{k+s-1} \end{bmatrix};\ U_{k,s} = \begin{bmatrix} u_{s,k} & \cdots & u_{s,k+N-1} \end{bmatrix} \tag{32}$$

$$\theta_{s,k} = \begin{bmatrix} \theta_k \\ \vdots \\ \theta_{k+s-1} \end{bmatrix};\ \theta_{k,s} = \begin{bmatrix} \theta_{s,k} & \cdots & \theta_{s,k+N-1} \end{bmatrix} \tag{33}$$

There exists a matrix with row full rank $\kappa_{d,s}$ satisfying the following relationship (Ding, 2014):

$$\kappa_{d,s}\begin{bmatrix} u_{s,k} \\ \theta_{s,k} \end{bmatrix} = \begin{bmatrix} \kappa_{u,s} & \kappa_{\theta,s} \end{bmatrix}\begin{bmatrix} u_{s,k} \\ \theta_{s,k} \end{bmatrix} = 0 \tag{34}$$

This $\kappa_{d,s}$ is called the data-driven realization of SKR.

According to the residual generator (13), we have

$$r = \hat{M}(z)\theta(z) - \hat{N}(z)u(z) = \begin{bmatrix} -\hat{N} & \hat{M} \end{bmatrix}\begin{bmatrix} u \\ \theta \end{bmatrix} \tag{35}$$

Based on Equations (34) and (36),

$$\kappa_{d,s} = \begin{bmatrix} -\hat{N} & \hat{M} \end{bmatrix} \tag{36}$$

They just express it differently.

On this basis, define the Hankel matrix as:

$$Z_{p,N} = \begin{bmatrix} U_{p,N} \\ \theta_{p,N} \end{bmatrix} = \begin{bmatrix} U_{k-sp-1,s} \\ \theta_{k-sp-1,s} \end{bmatrix} \tag{37}$$

where SP is the forward-truncated signal length, indicating the past data.

For $\kappa_{d,s}$ given by Algorithm 1 (Ding, 2014), the Luenberger diagnostic observer is

$$\begin{aligned} x_z(k+1) &= A_z x_z(k) + B_z u(k) + L_z\theta(k) \\ \hat{\theta}(k) &= \bar{C}_z z(k) + \bar{D}_z u(k) + \bar{G}_z\theta(k) \\ r_0(k) &= G\theta(k) - C_z x_z(k) - D_z u(k) \end{aligned} \tag{38}$$

where

$$A_z = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 1 & 0 \end{bmatrix}, L_z = \begin{bmatrix} \alpha_{s,0} \\ \vdots \\ \alpha_{s,s-1} \end{bmatrix}, G = \alpha_{s,s}$$

$$B_z = TB - L_z D, D_z = GD, C_z = \begin{bmatrix} 0 & 0 & \dots & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} \alpha_{s,1} & \alpha_{s,2} & \dots & \alpha_{s,s-1} & \alpha_{s,s} \\ \alpha_{s,2} & \alpha_{s,3} & \dots & \alpha_{s,s} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ \alpha_{s,s} & 0 & \dots & \dots & 0 \end{bmatrix}\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-1} \end{bmatrix}$$

The parity vector α_s is a row of $\kappa_{y,s}$ in Equation (34).

Algorithm 1: Obtaining the SKR

1. Collect input and output data and build $\theta_{k,s}$, $U_{k,s}$ and $Z_{p,N}$.
2. Perform a LQ decomposition

$$\begin{bmatrix} Z_{p,N} \\ U_{k,s} \\ \theta_{k,s} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

3. Perform SVD

$$\begin{bmatrix} L_{21} & L_{22} \\ L_{31} & L_{32} \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \end{bmatrix}\begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_1 \end{bmatrix}\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$$

4. $\kappa_{d,s} = \begin{bmatrix} \kappa_{u,s} & \kappa_{\theta,s} \end{bmatrix} = U_2^T$

Using a Luenberger diagnostic observer (38), an equivalent residual generator can be constructed by

$$r(k) = \theta(k) - C_f x_z(k) - D_f u(k) \tag{39}$$

where $C_f = G^{-1}C_z, G_f = G^{-1}D_z$

It can be seen from Equation (35) that $\hat{M}$ is the transfer function between the signal $r(z)$ and $\theta(z)$ and $\hat{N}$ is the transfer function between the signal $r(z)$ and $u(z)$.

Combine Equations (38) and (39), the transfer function for SKR can be written as

$$\hat{M} = (A_z, L_z, -C_f, I);\ \hat{N} = (A_z, B_z, C_f, D_f) \tag{40}$$

Since the SKR of the faulty soft sensor can be recognised by Equation (40), the PMM can also be calculated by the Equation (29). The required R^* can be found by Equation (30). Q^* can be obtained using Equation (26). Thus, the new control signal $u(z)$ in Equation (20) is obtained.

Algorithm 2 summarises the computation of the updated control signal $u(z)$.

Algorithm 2: Computing the updated control signal

1. If the PCI detects that the performance has changed to an unacceptable level, calculate $\hat{M}$ and $\hat{N}$ using Equation (40).
2. Compute R^* using Equation (30).
3. Obtain Q^* using Equation (26).
4. Realise the updated control signal $u(z)$ in Equation (20) to restore the performance of the soft sensor.

4. SIMULATION RESULTS

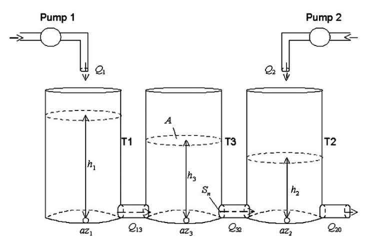

Figure 3: The structure of the three-tank system

The three-tank system, shown in Figure 3, is a typical nonlinear process that consists of three tanks T1, T2, and T3 with the same cross-sectional area A. The water level of T1 is the measurable state vector, while pump 1 is a continuous input to T1 with a mass flow rate Q_1. Linearising the model about the setting position $h_1 = 45$ cm and $h_2 = 15$ cm to obtain the linear state-space model that can work as the soft sensor:

$$A = \begin{bmatrix} -0.0085 & 0 & 0.0085 \\ 0 & -0.00195 & 0.0084 \\ 0.0085 & 0.0084 & -0.0169 \end{bmatrix}; \; B = \begin{bmatrix} 0.0065 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [1 \quad 0 \quad 0]; \qquad D = 0$$

The controller is given by Equation (8) with $Q = 0$ and

$$F = [5.0231 \quad 14.7418 \quad 7.8584];$$

$$L = [0.0926 \quad 0.8163 \quad 0.3416]^T;$$

Assume that the simulation time is 14,000 s and the disturbance variable follows a Gaussian distribution with mean 0 and variance 1. The sampling time is 1 s. The threshold $J_{th} = 1.15$ (Zhai & Shardt, 2024). PMM is simulated from the 3000$^{\text{th}}$ sample, resulting in a change of approximately 25% in the $S(z)$ matrix, that is,

$$A_p = \Delta \times A, \Delta = \begin{cases} 1, & i \le 3000 \\ \dfrac{i - 3000}{4000} \times 0.25, & 3000 < i \le 7000 \end{cases}$$

To compare the effect of the PMM accuracy between the real model and the assumed process model on performance recovery, we can change how the PMM $r(z)$ is calculated. Therefore, the control signal remains the same:

$$u(k) = F\hat{x}(k) - Q(z)r(z) + \bar{v}(z) \qquad (41)$$

and different PMM calculation methods are simulated separately:

Case 1: $r(k) = \theta(k) - \hat{\theta}(k)$;

Case 2: $r(k) = \theta(k) - y(k)$

where $y(k)$ is the output of the real model, including external disturbance. $\hat{\theta}(k)$ is the output of the state observer.

As shown in Figure 2, we can compare the effects of the two residual generation methods on the performance-change recovery algorithm by closing the switch.

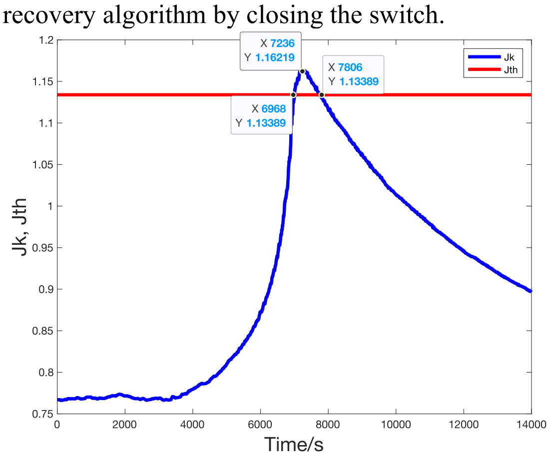

Figure 4: Performance change for Case 1

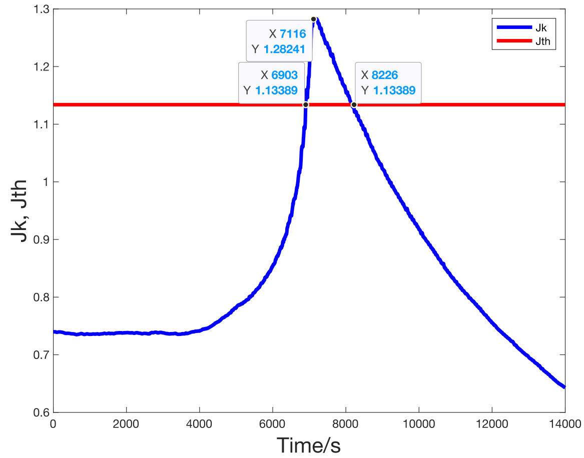

Figure 5: Performance change for Case 2

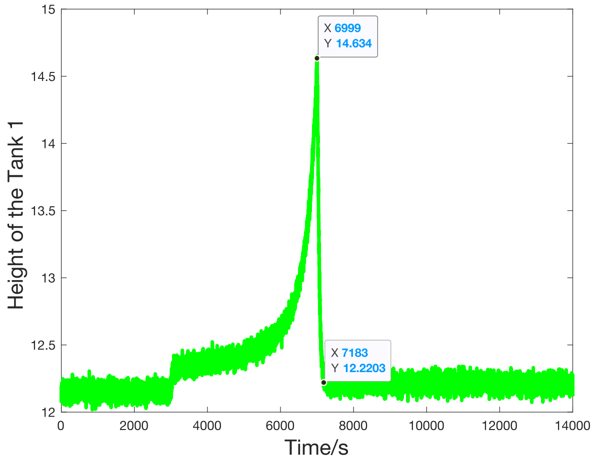

Figure 6: Output change for Case 1

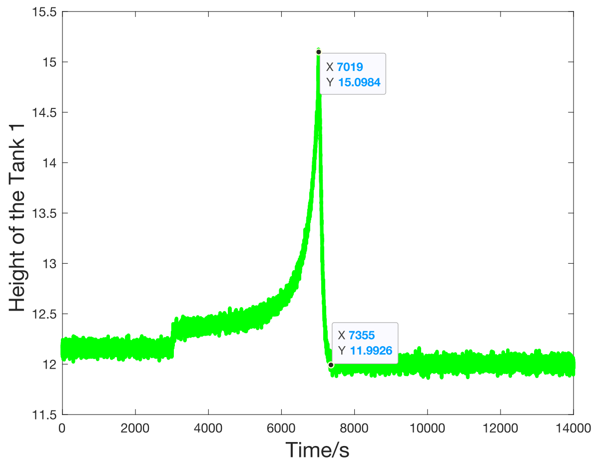

Figure 7: Output change for Case 2

Figure 4 shows the simulation results for the case where the residual generator is used. The blue line represents the performance evaluation of the soft sensor, while the red line represents the threshold. It can be seen that the performance changes to an unacceptable level around 7,000 s. At this point, the parameter Q begins to be calculated, and at 7,230 s, the optimal Q is obtained for continuously sending a compensation signal. At about 7,800 s, the performance is restored to normal levels.

Comparing Figure 4 and Figure 5 shows that Case 1 is faster than Case 2 to restore the changed performance, which saves about 400 s. Since in Case 2, we calculate the residual using the real output $y(z)$, but the real output contains disturbance $d(z)$, which affects the accuracy of the residual calculation and therefore affects the accuracy of the optimal Q calculation and compensation effect of the new control signal. Figure 6 and Figure 7 show the change of the output level in the different cases. Figure 6 indicates that the liquid level rises to the maximum value at 7,000 s, then begins to fall under the influence of compensation signals and returns to the normal height at 7,180 s. A comparison of Figure 6 and Figure 7 shows that the liquid level in Case 1 returns to the correct height faster.

5. CONCLUSIONS

This paper proposes a performance-change recovery algorithm for linear soft sensors based on the Youla-Kučera parameter Q. By identifying the SKR of the unknown model, the Youla-Kučera parameter Q can be calculated, which updates the control signal, releases the compensation signal, and cancels the effect of PMM. A three-tank model is used to construct a soft sensor to verify the effectiveness of the proposed method and compared the effects of different residual generation methods on performance change recovery. In the future, we will work on how to extend the proposed approach to nonlinear system, how to distinguish and isolate the effects of different kinds of PMMs on the performance change of the soft sensor and study other control performance change recovery methods. For example, the control method of system tracking performance, weighted fusion of control performance indicators and system performance indicators according to the expected control effect and use this as the optimization object to improve the control performance of the closed-loop system to restore system performance.

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