Economic data-enabled predictive control using machine learning ⋆

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Abstract: In this paper, we propose a convex data-based economic predictive control method within the framework of data-enabled predictive control (DeePC). Specifically, we use a neural network to transform the system output into a new state space, where the nonlinear economic cost function of the underlying nonlinear system is approximated using a quadratic function expressed by the transformed output in the new state space. Both the neural network parameters and the coefficients of the quadratic function are learned from open-loop data of the system. Additionally, we reconstruct constrained output variables from the transformed output through learning an output reconstruction matrix; this way, the proposed economic DeePC can handle output constraints explicitly. The performance of the proposed method is evaluated via a case study in a simulated chemical process.

Keywords: Data-enabled predictive control; economic model predictive control; learning-based control; nonlinear systems

1. INTRODUCTION

Optimizing process operation performance, such as maximizing economic profits or minimizing economic costs, has been one of the primary objectives of optimal process control. One representative framework to address economic considerations is to rely on a two-layer hierarchical architecture (Marlin et al. (1997)). The upper layer solves steady-state economic optimization. The optimal steady-state values serve as reference points for real-time control in the lower layer to track. Set-point tracking model predictive control (MPC) has been widely adopted to handle the real-time reference tracking tack in the lower layer (Limon et al. (2018); Rawlings (2000)). Meanwhile, the optimal process set-point can vary during process operations. Frequent changes in set-points may compromise the control performance, as set-point tracking MPC requires time to have the process operation reach a new steady-state (Ellis et al. (2014)). Moreover, while steady-state operation is common in the industry, it may not deliver economically optimal results Ellis et al. (2014).

Economic model predictive control (EMPC) offers a promising approach for effectively managing the economic performance of industrial systems and processes (Ellis et al. (2014, 2017)). In EMPC, a general cost function, which represents the economic cost or profit was used as the control objective function. In Diehl et al. (2010), an EMPC method with point-wise terminal constraints was proposed. In Heidarinejad et al. (2012), Lyapunov-based constraints were incorporated in the EMPC framework to guarantee closed-loop stability. In Amrit et al. (2011), the stability of the EMPC framework was guaranteed with terminal cost. In Grüne et al. (2014), the stability of EMPC without terminal conditions was studied. EMPC has also been applied across various industries. In Hovgaard et al. (2013), a nonconvex EMPC design was developed for a commercial refrigeration system and demonstrated a sophisticated response to real-time variations in electricity price. While EMPC addresses the limitations of set-point tracking MPC, it can be hindered by the lack of accurate first-principles models. Exceptions can be found in Albalawi (2023); Han et al. (2024). In Albalawi (2023), Koopman modeling was carried out to construct a linear model based on which an EMPC scheme was developed. In Han et al. (2024), a learning-based input-output Koopman model was established. A convex EMPC problem was formulated by training a quadratic economic cost function.

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Meanwhile, most of the existing EMPC methods require full-state measurements for online control implementation, which has hindered their widespread adoption across industries. This limitation arises because obtaining real-time measurements of all key process state variables can often be impractical Yin et al. (2019, 2018).

The data-enabled predictive control (DeePC) framework offers a promising approach to bypass the need of system modeling Coulson et al. (2019). According to Willems' fundamental lemma (Willems et al. (2005)), linear time-invariant systems can be represented non-parametrically using collected input and output trajectories. In Coulson et al. (2019), data-enabled predictive control (DeePC) was proposed based on Willems' fundamental lemma. In Yang et al. (2023); Zhang et al. (2023), order reduction was performed on the Hankel matrix using singular value decomposition (SVD) to enhance its online computational efficiency. In Shang et al. (2024); Zhang et al. (2025), Willems' fundamental lemma was leveraged to address nonlinear systems. In Xie et al. (2023), a data-driven economic MPC framework for linear systems was proposed to minimize economic cost dynamically. However, incorporating general economic cost functions can lead to non-convex optimization problems, which will reduce the efficiency of online implementation.

In this work, we propose an economic data-enabled predictive control (economic DeePC) framework for nonlinear systems. Inspired by the economic cost function approximation design in Han et al. (2024), the proposed economic DeePC is formulated as an input-output control scheme that handles the nonlinear economic cost function in a convex fashion without requiring a first-principles process model and full-state measurements. In particular, the nonlinear economic cost is approximated by a quadratic function of the transformed output. Key output variables are reconstructed from the transformed output to meet system constraints during online implementation. The appropriate nonlinear mapping that makes the transformed outputs compliant with the Willems' fundamental lemma is represented by a trained neural network.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Notation

$\mathbb{N}_{>0}$ denotes the set of positive integers. $\mathbb{E}$ denotes the expectation. $\{x\}_j^l := \left[x_j^\top, \ldots, x_l^\top\right]^\top$ denotes a sequence that contains vector x from the time instant j to l. $x_{j|k}$ is the state vector for sampling instant j obtained at time instant k. $||x||^2$ represents the square of the Euclidean norm of vector x. A^+ represents the pseudo-inverse of matrix A. diag$(\cdot)$ represents a diagonal matrix. exp$(\cdot)$ denotes the element-wise exponential operator. The superscript 'd' denotes that the corresponding data is collected offline.

2.2 Non-parametric system representation

Consider a discrete linear time-invariant (LTI) system:

$$x_{k+1} = Ax_k + Bu_k \tag{1a}$$

$$y_k = Cx_k + Du_k \tag{1b}$$

where k denotes the sampling instant; $x \in \mathbb{X} \subseteq \mathbb{R}^{n_x}$ denotes the system state vector; $u \in \mathbb{U} \subseteq \mathbb{R}^{n_u}$ is the control input; $y \in \mathbb{Y} \subseteq \mathbb{R}^{n_y}$ is the system output; k denotes the sampling instant; $A \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, $C \in \mathbb{R}^{n_y \times n_x}$, and $D \in \mathbb{R}^{n_y \times n_u}$ are system matrices.

Let $\mathbf{u}_T^d := \{u^d\}_1^T$ and $\mathbf{y}_T^d := \{y^d\}_1^T$ denote the T-step offline collected input and output sequences of system (1), respectively. Next, we introduce the concept of persistent excitation and Willems' fundamental lemma.

Definition 1. (Persistent excitation Willems et al. (2005)). Let $T, L \in \mathbb{N}_{>0}$ and $T \geq L$. A Hankel matrix of depth L is constructed based on the input sequence $\mathbf{u}_T^d$, defined as follows:

$$\mathscr{H}_L(\mathbf{u}_T^d) = \begin{bmatrix} u_1^d & u_2^d & \cdots & u_{T-L+1}^d \\ u_2^d & u_3^d & \cdots & u_{T-L+2}^d \\ \vdots & \vdots & \ddots & \vdots \\ u_L^d & u_{L+1}^d & \cdots & u_T^d \end{bmatrix} \tag{2}$$

The sequence $\mathbf{u}_T^d$ is persistently exciting of order L, if $\mathscr{H}_L(\mathbf{u}_T^d)$ has full row rank.

Lemma 1. (Willems' fundamental lemma Willems et al. (2005)). Consider a controllable LTI system (1). $\mathbf{u}_T^d$ is persistently exciting of order $L+n_x$. According to Willems et al. (2005), any L-step trajectories $\mathbf{u}_L := \{u\}_1^L \in \mathbb{R}^{n_u L}$ and $\mathbf{y}_L := \{y\}_1^L \in \mathbb{R}^{n_y L}$ are the input and output trajectories of system (1) if

$$\begin{bmatrix} \mathscr{H}_L(\mathbf{u}_T^d) \\ \mathscr{H}_L(\mathbf{y}_T^d) \end{bmatrix} g = \begin{bmatrix} \mathbf{u}_L \\ \mathbf{y}_L \end{bmatrix} \tag{3}$$

holds for a column vector $g \in \mathbb{R}^{T-L+1}$.

Lemma 1 (Willems et al. (2005)) provides a non-parametric representation of (1) using historical input and output trajectories $\mathbf{u}_T^d$ and $\mathbf{y}_T^d$.

2.3 Data-enabled predictive control

Data-enabled predictive control (DeePC) is a receding horizon control framework that does not require modeling of the underlying system (Coulson et al. (2019)). Instead, it creates a non-parametric representation of the considered system based only on historical data.

In DeePC, future multi-step ahead predictions are described using partitioned Hankel matrices. Let $T_{ini}, N_p \in \mathbb{N}_{>0}$ and $L = T_{ini} + N_p$, the Hankel matrices $\mathscr{H}_L(\mathbf{u}_T^d)$ and $\mathscr{H}_L(\mathbf{y}_T^d)$ are divided into two segments: the past data and the future data, as follows Coulson et al. (2019):

$$\begin{bmatrix} U_p \\ U_f \end{bmatrix} := \mathscr{H}_L(\mathbf{u}_T^d), \ \begin{bmatrix} Y_p \\ Y_f \end{bmatrix} := \mathscr{H}_L(\mathbf{y}_T^d) \tag{4}$$

In (4), U_p and Y_p contain the past input and output data, which correspond to the first $n_u \times T_{ini}$ rows of $\mathscr{H}_L(\mathbf{u}_T^d)$ and the first $n_y \times T_{ini}$ rows of $\mathscr{H}_L(\mathbf{y}_T^d)$, respectively. Similarly, U_f and Y_f represent the future input and output data corresponding to the last $n_u \times N_p$ rows of $\mathscr{H}_L(\mathbf{u}_T^d)$ and the last $n_y \times N_p$ rows of $\mathscr{H}_L(\mathbf{y}_T^d)$, respectively.

At each time instant k, T_{ini}-step input and output sequences $\mathbf{u}_{ini,k} := \{u\}_{k-T_{ini}}^{k-1}$ and $\mathbf{y}_{ini,k} := \{y\}_{k-T_{ini}}^{k-1}$ are used to initialize the DeePC algorithm. Based on Lemma 1, N_p-step future input and output prediction sequences

$\hat{\mathbf{u}}_k := \{\hat{u}\}_{k|k}^{k+N_p-1|k}$ and $\hat{\mathbf{y}}_k := \{\hat{y}\}_{k|k}^{k+N_p-1|k}$ satisfy (3). The DeePC optimization problem at time instant k can be formulated as follows (Coulson et al. (2019)):

$$\min_{g_k,\hat{\mathbf{u}}_k,\hat{\mathbf{y}}_k} \|\hat{\mathbf{y}}_k - \mathbf{y}_k^r\|_Q^2 + \|\hat{\mathbf{u}}_k - \mathbf{u}_k^r\|_R^2 \tag{5a}$$

$$\text{s.t.} \quad \begin{bmatrix} U_p \\ Y_P \\ U_f \\ Y_f \end{bmatrix} g_k = \begin{bmatrix} \mathbf{u}_{ini,k} \\ \mathbf{y}_{ini,k} \\ \hat{\mathbf{u}}_k \\ \hat{\mathbf{y}}_k \end{bmatrix} \tag{5b}$$

$$\hat{u}_{j|k} \in \mathbb{U}, \quad j = k, \ldots, k+N_p-1 \tag{5c}$$

$$\hat{y}_{j|k} \in \mathbb{Y}, \quad j = k, \ldots, k+N_p-1 \tag{5d}$$

In (5), $\mathbf{u}_k^r := \{u^r\}_k^{k+N_p-1}$ and $\mathbf{y}_k^r := \{y^r\}_k^{k+N_p-1}$ are the input and output references, respectively; $Q \in \mathbb{R}^{n_y N_p \times n_y N_p}$ and $R \in \mathbb{R}^{n_u N_p \times n_u N_p}$ are the weighting matrices. $\hat{\mathbf{u}}_k$ and $\hat{\mathbf{y}}_k$ can be uniquely determined by g_k through $\hat{\mathbf{u}}_k = U_f g_k$ and $\hat{\mathbf{y}}_k = Y_f g_k$ in (5b). The first element $\hat{u}_{k|k}^*$ of the optimal input sequence $\hat{\mathbf{u}}_k^* = \left[\hat{u}_{k|k}^{*\top}, \ldots, \hat{u}_{k+N_p-1|k}^{*\top}\right]^\top$ is used as the system input which is to be applied to system (1) at time instant k (see, e.g., Coulson et al. (2019)).

2.4 Motivation and problem formulation

In this work, we consider general discrete-time nonlinear systems in the following form:

$$x_{k+1} = f(x_k, u_k) \tag{6a}$$

$$y_k = h(x_k) \tag{6b}$$

where $f : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is a nonlinear function that describes the dynamical behaviors of the system; $h : \mathbb{X} \to \mathbb{Y}$ is the output measurement function.

The dependence of the real-time economic operational cost on the system input and output is characterized by a nonlinear economic cost function c_k as follows:

$$c_k = \ell_e(u_k, y_k) \tag{7}$$

where $\ell_e : \mathbb{Y} \times \mathbb{U} \to \mathbb{R}$.

In this work, we aim to leverage the DeePC framework to propose an economic data-enabled predictive control approach, which can be applied to minimize the operational cost while ensuring the satisfaction of hard constraints on system output. As is analogous to the relationship between set-point tracking MPC and EMPC, a potential solution to extend DeePC to economic DeePC is to replace the quadratic stage cost in (5a) with economic cost function ℓ_e; this modification has been implemented in a data-driven economic predictive control design for LTI systems in Xie et al. (2023). However, since the economic cost function ℓ_e in (7) is typically nonlinear, it compromises the convexity of the original DeePC formulation. In that case, solving the resulting non-convex optimization problem can be more computationally expensive than solving the convex optimization problem associated with the original DeePC.

Building on the above considerations, we aim to develop an economic DeePC framework for the nonlinear system in (6) that inherits the convexity of the online optimization problem in the original DeePC.

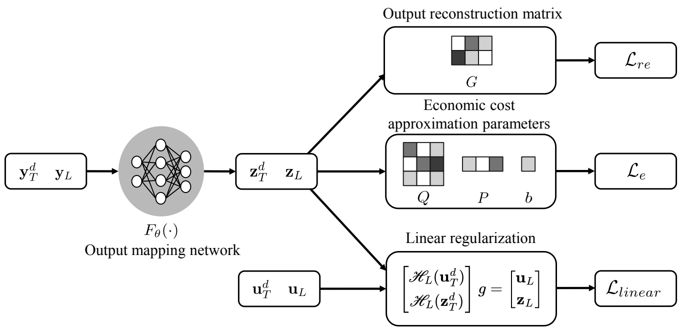

Fig. 1. An illustration of the training of an economic DeePC controller based on the proposed method.

3. ECONOMIC DATA-ENABLED PREDICTIVE CONTROL APPROACH

In this section, we leverage machine learning to propose a convex economic data-enabled predictive control approach, referred to as economic DeePC, designed for economic control of nonlinear systems in (6).

3.1 Training of the proposed economic DeePC

Firstly, we introduce the three key components of the proposed economic DeePC training process and formulate the optimization problem for the training process.

Fig. 1 provides an illustration of the training process for the proposed machine learning-based economic DeePC approach. We elaborate on the three components that contain trainable parameters.

(1) *Neural network for output mapping*: A neural network, $F_\theta(\cdot) : \mathbb{R}^{n_y} \to \mathbb{R}^{n_z}$, is introduced to map the original system output y to vector $z \in \mathbb{R}^{n_z}$ in a new space. The parameters of $F_\theta(\cdot)$, denoted by θ, are trainable. Two open-loop output trajectories of the system, $\mathbf{y}_T^d$ and $\mathbf{y}_L$, are reshaped into matrices $\mathbf{Y}_T^d = [y_1^d, \ldots, y_T^d]^\top \in \mathbb{R}^{T \times n_y}$ and $\mathbf{Y}_L = [y_1, \ldots, y_L]^\top \in \mathbb{R}^{L \times n_y}$ which are then fed into neural network $F_\theta(\cdot)$ to generate their corresponding transformed output sequences, as follows:
$$\mathbf{Z}_T^d = F_\theta(\mathbf{Y}_T^d), \ \mathbf{Z}_L = F_\theta(\mathbf{Y}_L) \tag{8}$$
where $\mathbf{Z}_T^d \in \mathbb{R}^{T \times n_z}$ and $\mathbf{Z}_L \in \mathbb{R}^{L \times n_z}$ are matrices of the transformed outputs. To facilitate the implementation of economic DeePC, we reshape transformed output matrices back into multi-step transformed output trajectories $\mathbf{z}_T^d = \{z^d\}_1^T$ and $\mathbf{z}_L = \{z\}_1^L$.

(2) *Approximation of economic cost in quadratic form*: The economic cost approximation follows the approximation method in the Koopman-based convex EMPC design in Han et al. (2024), where a quadratic function was learned to approximate a non-convex economic state cost for a wastewater treatment process. Specifically, leveraging the approximation method from Han et al. (2024), the transformed vector z in the new space is used to create a quadratic expression to approximate the nonlinear economic cost function ℓ_e in (7). An approximation of the economic cost for sampling instant k, denoted by $\hat{c}_k$, is computed as follows:
$$\hat{c}_k = z_k^\top Q z_k + P z_k + b \tag{9}$$

where $Q \in \mathbb{R}^{n_z \times n_z}$ is a positive-definite matrix; $P \in \mathbb{R}^{1 \times n_z}$ is a real-valued vector; $b \in \mathbb{R}$ is a real-valued scalar. To ensure the positive definiteness of Q, we require that Q takes the form of $Q = \text{diag}\big(\exp(q)\big)$, where $q \in \mathbb{R}^{n_z}$ is a real-valued vector. q, P, and b are trainable parameters.

(3) **Output reconstruction matrix**: The vector of output variables on which hard constraints need to be imposed on is denoted by y_k^c. A trainable output reconstruction matrix $G \in \mathbb{R}^{n_c \times n_z}$ is used to reconstruct y_k^c from the transformed output z_k to ensure that the system constraints are satisfied during system operation. The reconstructed system output can be calculated as follows:

$$\hat{y}_k^c = G z_k \tag{10}$$

where $\hat{y}_k^c$ is an approximation of the output vector y_k^c at sampling instant k.

Next, we present the optimization problem associated with the training process for the proposed economic DeePC method. Given an open-loop data set $\mathcal{D} := \{\mathbf{u}_T^d, \mathbf{u}_L, \mathbf{y}_T^d, \mathbf{y}_L, \mathbf{c}_T^d, \mathbf{c}_L\}$, where $\mathbf{c}_T^d := \{c^d\}_1^T$ and $\mathbf{c}_L := \{c\}_1^L$ are sequences of actual economic costs computed using the corresponding system output data in $\mathbf{y}_T^d$ and $\mathbf{y}_L$ and control input data contained $\mathbf{u}_T^d$ and $\mathbf{u}_L$ based on (7), the objective is to optimize the trainable parameters (θ, q, P, b, and G). Consequently, the optimization problem for the training of the proposed economic DeePC method is formulated as follows:

$$\min_{\theta,q,P,b,G} \mathcal{L} = \min_{\theta,q,P,b,G} \alpha_1 \mathcal{L}_e + \alpha_2 \mathcal{L}_{re} + \alpha_3 \mathcal{L}_{linear} \tag{11}$$

where $\mathcal{L}_e$ denotes the loss for approximating the economic cost; $\mathcal{L}_{re}$ represents the loss for reconstructing the output variables that required to satisfy hard constraints; $\mathcal{L}_{linear}$ represents the loss that quantifies the extent of violation of Willems' fundamental lemma; $\alpha_i, i = 1, 2, 3$, are weights for three different losses.

Specifically, on the right-hand side of (11), $\mathcal{L}_e$ computes the discrepancy between the actual economic cost and the estimated cost, given as follows:

$$\mathcal{L}_e = \mathbb{E}_{\mathcal{D}} \ \left(||\mathbf{c}_T^d - \hat{\mathbf{c}}_T^d||^2 + ||\mathbf{c}_L - \hat{\mathbf{c}}_L||^2\right) \tag{12}$$

where $\hat{\mathbf{c}}_T^d := \{\hat{c}^d\}_1^T$ and $\hat{\mathbf{c}}_L := \{\hat{c}\}_1^L$ denote the sequences of the approximated economic cost obtained based on the transformed output $\mathbf{z}_T^d$ and $\mathbf{z}_L$ following (9), respectively.

$\mathcal{L}_{re}$ in the second term on the right-hand side of (11) accounts for the discrepancy between the ground-truth and the reconstructed system outputs on which hard constraints are imposed; it is with the following form:

$$\mathcal{L}_{re} = \mathbb{E}_{\mathcal{D}} \ \left(\big|\big|\mathbf{y}_T^{c,d} - \hat{\mathbf{y}}_T^{c,d}\big|\big|^2 + \big|\big|\mathbf{y}_L^c - \hat{\mathbf{y}}_L^c\big|\big|^2\right) \tag{13}$$

where $\mathbf{y}_T^{c,d} := \{y^{c,d}\}_0^{T-1}$ and $\mathbf{y}_L^c := \{y^c\}_0^{L-1}$ contain the elements of $\mathbf{y}_T^d$ and $\mathbf{y}_L$ on which hard constraints should be imposed; $\hat{\mathbf{y}}_T^{c,d} := \{\hat{y}^{c,d}\}_0^{T-1}$ and $\hat{\mathbf{y}}_L^c := \{\hat{y}^c\}_0^{L-1}$ represent the reconstructed output calculated based on (10).

$\mathcal{L}_{linear}$ in the third term on the right-hand side of (11) is in the following form:

$$\mathcal{L}_{linear} = \mathbb{E}_{\mathcal{D}} \ ||\mathbf{z}_L - \mathscr{H}_L(\mathbf{z}_T^d) g||^2 \tag{14a}$$

$$= \mathbb{E}_{\mathcal{D}} \ ||\mathbf{z}_L - \mathscr{H}_L(\mathbf{z}_T^d)\mathscr{H}_L(\mathbf{u}_T^d)^{+}\mathbf{u}_L||^2 \tag{14b}$$

This term is to guide the training of $F_\theta(\cdot)$ in a way that the transformed outputs, which are generated by $F_\theta(\cdot)$, conform to the Willems' fundamental lemma.

3.2 Economic data-enabled predictive control

The economic cost approximation follows the approximation method in the Koopman-based convex EMPC design in Han et al. (2024), where a quadratic function was learned to approximate a non-convex economic state cost for a wastewater treatment process. Specifically, leveraging the economic cost approximation design from Han et al. (2024), the transformed vector z in the new space is used to create a quadratic expression to approximate the nonlinear economic cost function ℓ_e in (7).

Once the training is completed, the optimal parameters θ^*, Q^*, P^*, b^*, and G^* are used to formulate the economic DeePC design. Inspired by Coulson et al. (2019), the online optimization problem for the proposed economic DeePC is formulated as follows:

$$\min_{g_k} \sum_{j=k}^{k+N_p-1} \beta(z_{j|k}^\top Q^* z_{j|k} + P^* z_{j|k} + b^*) + (\Delta \hat{u}_{j|k}^\top R \Delta \hat{u}_{j|k}) \tag{15a}$$

$$\text{s.t.} \begin{bmatrix} U_p \\ Z_p \\ U_f \\ Z_f \end{bmatrix} g_k = \begin{bmatrix} \mathbf{u}_{ini,k} \\ \mathbf{z}_{ini,k} \\ \hat{\mathbf{u}}_k \\ \hat{\mathbf{z}}_k \end{bmatrix} \tag{15b}$$

$$\hat{u}_{j|k} \in \mathbb{U}, \ j = k, \ldots, k + N_p - 1 \tag{15c}$$

$$G^* \hat{z}_{j|k} \in \mathbb{Y}_c, \ j = k, \ldots, k + N_p - 1 \tag{15d}$$

where N_p represents the prediction horizon; $R \in \mathbb{R}^{n_u \times n_u}$ is a positive-definite matrix; $\beta \in \mathbb{R}$ is a positive scalar; $\hat{\mathbf{z}}_k := \{\hat{z}\}_{k|k}^{k+N_p-1|k}$ denotes the predicted transformed output sequence; $\mathbb{Y}_c$ is the output space of the output vector $\hat{y}_k^c$ on which hard constraints need to be imposed; $\Delta \hat{u}_{j|k} = \hat{u}_{j|k} - \hat{u}_{j-1|k}$ is the rate of change in the control input; Z_p and Z_f are the partitions of the Hankel matrix constructed by the transformed offline collected output, that is, $[Z_p^\top, Z_f^\top]^\top := \mathscr{H}_{T_{ini}+N_p}(\mathbf{z}_T^d)$; $\mathbf{z}_{ini,k}$ and $\mathbf{z}_T^d$ are the trajectories of the initial transformed output and the offline transformed output generated using $\mathbf{y}_{ini,k}$ and $\mathbf{y}_T^d$ according to (8).

The optimal control input sequence $\hat{\mathbf{u}}_k^*$ can be obtained based on the optimized DeePC operator g_k^* as follows:

$$\hat{\mathbf{u}}_k^* = U_f g_k^* \tag{16}$$

where $\hat{\mathbf{u}}_k^* = \left[\hat{u}_{k|k}^{*\top}, \ldots, \hat{u}_{k+N_p-1|k}^{*\top}\right]^\top$. The first element $\hat{u}_{k|k}^*$ of this optimal control sequence is applied to system (6) for real-time control.

3.3 Reduced-order economic DeePC

The dimension of g_k in (15) is dependent on the size of the Hankel matrix, which is typically large. To reduce the computation time for solving the optimization problem (15) during online implementation, we incorporate the singular value decomposition (SVD)-based dimension reduction method in Zhang et al. (2023) into the proposed economic DeePC, to achieve faster online control implementation.

A reduced-order matrix can be established to approximate the Hankel matrix and form a corresponding reduced-ordered DeePC operator $\bar{g}_k$. Based on SVD (Wall et al. (2003)), the Hankel matrix can be represented as follows:

$$\begin{bmatrix} \mathscr{H}_{T_{ini}+N_p}(\mathbf{u}_T^d) \\ \mathscr{H}_{T_{ini}+N_p}(\mathbf{z}_T^d) \end{bmatrix} = [W_1, W_2] \begin{bmatrix} \Sigma_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} [V_1, V_2]^\top \quad (17)$$

where $\Sigma_1 \in \mathbb{R}^{n_r \times n_r}$ is the diagonal matrix of non-zero singular values; $W := [W_1, W_2]$ and $V := [V_1, V_2]$ are singular vector matrices satisfying $WW^\top = W^\top W = \mathbf{I}_{(n_u+n_y)L}$ and $VV^\top = V^\top V = \mathbf{I}_{T-T_{ini}-N_p+1}$. The reduced-order approximation $\bar{\mathscr{H}}_{T_{ini}+N_p}$ can be established as follows (Zhang et al. (2023)):

$$\bar{\mathscr{H}}_{T_{ini}+N_p} = W_1 \Sigma_1 \quad (18)$$

where $\bar{\mathscr{H}}_{T_{ini}+N_p} \in \mathbb{R}^{(n_u+n_y)(T_{ini}+N_p)\times n_r}$ is the reduced-order Hankel matrix. The number of columns of the Hankel matrix and the dimension of $\bar{g}_k$ is reduced from $T - T_{ini} - N_p + 1$ to n_r.

The optimization problem of the proposed reduced-order economic DeePC can be formulated as follows:

$$\min_{\bar{g}_k} \sum_{j=k}^{k+N_p-1} \beta(z_{j|k}^\top Q^* z_{j|k} + P^* z_{j|k} + b^*) + (\Delta\hat{u}_{j|k}^\top R \Delta\hat{u}_{j|k}) \quad (19a)$$

$$\text{s.t. } \bar{\mathscr{H}}_{T_{ini}+N_p}\bar{g}_k = \begin{bmatrix} \mathbf{u}_{ini,k} \\ \mathbf{z}_{ini,k} \\ \hat{\mathbf{u}}_k \\ \hat{\mathbf{z}}_k \end{bmatrix} \quad (19b)$$

$$\hat{u}_{j|k} \in \mathbb{U},\ j = k, \ldots, k+N_p-1 \quad (19c)$$

$$G^* \hat{z}_{j|k} \in \mathbb{Y}_c,\ j = k, \ldots, k+N_p-1 \quad (19d)$$

4. SIMULATION RESULTS

4.1 Process description and operation objective

The studied process comprises two continuous-stirred tank reactors connected in series. In this process, two irreversible second-order reactions that convert reactant A to the desired product B take place simultaneously. A schematic of the process and the detailed process descriptions are referred to Wu et al. (2020).

The state variables include the concentration of material A and the temperature in each of the reactors. The state vector $x = [C_{\mathrm{A}1}, T_1, C_{\mathrm{A}2}, T_2]^\top$, where $C_{\mathrm{A}i}$ represents the concentration of material A in the ith reactor, T_i is the temperature in the ith reactor, $i = 1, 2$. The input vector includes the concentration of reactant A in the feed inlets to the two reactors $C_{\mathrm{A}10}$ and $C_{\mathrm{A}20}$, and the heating input rates for the two reactors Q_1 and Q_2, that is, $u = [C_{\mathrm{A}10}, Q_1, C_{\mathrm{A}20}, Q_2]^\top$. For this process, we consider that the output vector is the same as the state vector.

The economic profit, as is adopted from Wu et al. (2020), is as follows:

$$\ell_e(u_k, y_k) = k_0 e^{\frac{-E}{RT_1}} C_{\mathrm{A}1}^2 + k_0 e^{\frac{-E}{RT_2}} C_{\mathrm{A}2}^2 \quad (20)$$

4.2 Simulation settings

Control methods and parameters We consider two data-driven economic predictive control methods: the economic DeePC proposed in this work, and the learning-based Koopman EMPC in Han et al. (2024).

Table 1. Hyperparameters for training.

Parameters	Values
Dimension of hidden states	128
Number of hidden layers	2
Activate function	ReLU
Training epoch	100
Batch size	100

The following parameters are used for the proposed method: $T = 10^3$, $T_{ini} = 2$, $N_p = 5$, and $n_z = 10$. The hyperparameters for training the neural network in the economic DeePC framework are provided in Table 1. Additionally, in this chemical process, we aim to maximize profit instead of minimize operating costs. Therefore, matrix Q is set to be negative definite in the training phase to facilitate the formulation of a convex optimization problem, that is, $Q = \mathrm{diag}(-\exp(q))$.

For the learning-based Koopman EMPC proposed in Han et al. (2024), the Koopman model is trained over 100 epochs. The dimension of the lifted state is set to 10. Other hyperparameters follow the configuration outlined in Han et al. (2024). The control objective function is the same as that of the economic DeePC.

Data generation First, open-loop simulations are conducted to generate data for constructing the Hankel matrices and training the neural networks. The sampling period is 0.025 hours. The control inputs are bounded by $1.5\ \mathrm{kmol \cdot m^{-3}} \leq C_{\mathrm{A}i0} \leq 6.5\ \mathrm{kmol \cdot m^{-3}}$ and $-10^4\ \mathrm{kJ \cdot h^{-1}} \leq Q_i \leq 10^5\ \mathrm{kJ \cdot h^{-1}}$, $i = 1, 2$. Bounded random disturbances are added to the process. Specifically, stochastic disturbances added to mass fractions are generated following Gaussian distribution $\mathcal{N}(0, 0.01)$ and then made bounded within a range of $[-1, 1]$. Stochastic disturbances added to temperatures are generated following Gaussian distribution $\mathcal{N}(0, 1)$ and then made bounded within a range of $[-50, 50]$.

In training, we consider two cases to evaluate the control methods: Case 1 with 2×10^3 data samples and Case 2 with 10^4 samples. Datasets are divided into training, validation, and test data in a ratio of 7:2:1. In both cases, economic DeePC uses 10^3 samples to construct the Hankel matrix and uses the remaining data for training. The learning-based Koopman EMPC utilizes all the samples to train the neural network. Trainable parameters in both method are optimized using Adam (Kingma et al. (2014)).

4.3 Control performance

Fig. 2 presents the trajectories of the averaged economic profits obtained based on the two methods with different data sizes. The results are generated based on 20 repeated simulations, respectively. As shown in Fig. 2(a), when 2×10^3 samples are used, the proposed control method provides better economic performance as compared to the learning-based Koopman EMPC in Han et al. (2024). When 10^4 samples are used, the two methods provide comparable performance, with the learning-based Koop-

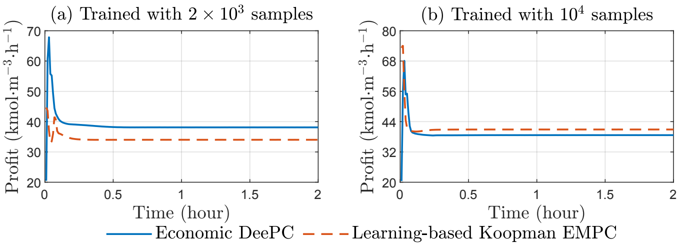

Fig. 2. The economic profits for the closed-loop chemical process under the proposed economic DeePC and learning-based Koopman EMPC in Han et al. (2024).

Table 2. The results of average economic profit (with unit kmol·m^{-3}·h^{-1}).

	2×10^3 samples	10^4 samples
Economic DeePC	38.7502	39.0457
Learning-based Koopman EMPC (Han et al. (2024))	34.2901	41.2660

man EMPC (Han et al. (2024)) providing a slightly higher profit, as shown in Fig. 2(b). The average economic profits for both methods under two different data sizes are shown in Table 2. Based on the simulation results, the proposed economic DeePC method shows better data efficiency.

5. CONCLUSION

We proposed a convex economic data-enabled predictive control method. A neural network was utilized to map the original system output to a transformed output vector, which was used to build a quadratic approximation of the nonlinear economic cost function. The output variables were reconstructed from the transformed output using a trainable output reconstruction matrix. With the reconstructed outputs, hard constraints on the system output were explicitly addressed. The trainable parameters, including the neural network parameters, the coefficients in the quadratic approximation of the economic cost, and the constrained output reconstruction matrix, were learned using open-loop system data. A simulated chemical process was used to illustrate the proposed method.

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