Generic model control of diffusion-reaction systems

A. Maidi * **R. Paulen** ** **J.-P. Corriou** ***

* *Laboratoire de Conception et Conduite des Systèmes de Production, Université Mouloud MAMMERI, 15 000 Tizi-Ouzou, Algérie (e-mail: ahmed.maidi@ ummto.dz)*
** *Faculty of Chemical and and Food Technology, Slovak University of Technology in Bratislava, Bratislava, Slovakia (e-mail: radoslav.paulen@stuba.sk)*
*** *Reaction and Process Engineering Laboratory, UMR 7274-CNRS, Lorraine University, ENSIC 1, rue Grandville, BP 20451, 54001 Nancy Cedex, France (e-mail: jean-pierre.corriou@univ-lorraine.fr)*

Abstract: Late lumping controller design for distributed parameter systems presents a significant challenge. This paper extends the generic model control technique to diffusion-reaction systems governed by semilinear parabolic equations, focusing on controlling an output defined as a spatially weighted average of the system state. Both distributed and boundary control approaches are thoroughly investigated. While the design for distributed control is straightforward, boundary control necessitates the use of the extended operator concept. This concept allows to convert the boundary control problem into a pointwise one, a particular case of distributed control, which simplifies the controller design. The effectiveness of the proposed controllers in tracking and disturbance rejection is validated through numerical simulations using a case of a heated rod.

Keywords: distributed parameter system, diffusion-reaction system, partial differential equation, generic model control, distributed control, boundary control, extended operator, output tracking.

1. INTRODUCTION

The early lumping approach is a straightforward method adopted for control design of distributed parameter systems (DPSs) described by partial differential equations (PDEs). Its principle consists of approximating the distributed parameter system (DPS) by a lumped parameter system (LPS), achieved either by approximating the PDEs or their solutions. A review of the different methods used for DPS reduction or approximation can be found in Li and Qi (2010). The objective behind approximation (reduction) is to exploit the well-established and rich control theory for lumped parameter systems (LPSs).

Although this approach has been widely applied, many studies have pointed out notable limitations. It is well known that with the early lumping approach, fundamental control properties of the DPS can be masked, leading to erroneous conclusions regarding observability, controllability, and stability (Ray, 1989; Christofides and Daoutidis, 1996; Christofides, 2001b,a). For instance, Singh and Hahn (2007) examined the influence of approximation on observability, while Ray (1989) focused on the influence on both controllability and observability. Singh and Hahn (2007) studied the effect of the approximation of the PDEs using finite difference schemes, while Ray (1989) focused on this effect in the case of the approximation of the solution using spatial basis functions. Additionally, the approximation process induces performance degradation due to the spillover phenomenon, i.e., the contribution of neglected modes in the closed-loop system (Christofides, 2001b; Morris, 2020).

A recommended alternative to the early lumping approach is the late lumping approach. In this method, the analysis and controller design are carried out using the PDE model directly, without any prior reduction or approximation (Christofides, 2001b), which allows to preserve the fundamental control properties of the DPS (controllability, observability, and stability). Numerous studies have demonstrated the superiority of the late lumping approach over the early lumping method; a comprehensive review can be found in (Christofides, 2001b) and Meurer (2013). Nevertheless, manipulating the PDE model during the controller design process is a complex task, requiring sophisticated mathematical tools from functional analysis, which limits and constrains the use of this approach.

The remarkable advantages of the late lumping approach deserve more research attention, motivating the community to develop simple and effective controller design methods for DPS. For example, Christofides and Daoutidis (1996) extended the input-output linearization technique to hyperbolic systems, while Maidi et al. (2024) extended the zeroing dynamics approach for the velocity control of a single fluid flow DPS. In both approaches, the controller design is limited to evaluating the partial derivatives of the DPS dependent variables (the output or the state), a task that can be accomplished without significant difficulties.

Generic model control (GMC) is an interesting and simple design controller method (Lee and Sullivan, 1988b,a; Lee, 1993). It has been successfully applied to LPSs (Cott and Macchietto, 1989; Aziz et al., 2000; Cong et al., 2015; Zangina et al., 2021; Hamid and Liu, 2024). To the best of our knowledge, no study has focused on GMC control of DPS following the late lumping approach.

The aim of this work is to extend the application of GMC to DPS described by PDEs, following the late lumping aproach, focusing on a particular class of linear diffusion-reaction systems modeled by semilinear parabolic equations. Both distributed and boundary control problems are addressed. In the distributed control case, where the control acts in the spatial domain, the controller design is straightforward. However, in the boundary control case, where the control acts at the boundary, the situation is more complex. To overcome the difficulty and develop a GMC controller, we propose to convert the boundary control problem into a pointwise control one using the notion of an extended operator (Maidi and Corriou, 2011; Stafford and Dowrick, 1977). The resulting pointwise model, which is a particular case of distributed control, is then used to easily develop a controller for the original boundary control problem. To support these findings and demonstrate the tracking and disturbance rejection capabilities of the developed GMC controllers, we apply the method to a heated rod system through numerical simulation.

The remainder of the paper is structured as follows. In Section 2, the distributed and boundary control problems of the diffusion-reaction system are formulated. In Section 3, distributed and boundary GMC controllers are developed within the framework of late lumping approach. Section 4 illustrates the performance of the developed GMC controllers in the case of a heated rod. Finally, Section 5 concludes the paper.

2. CONTROL PROBLEM STATEMENT

Consider the reaction-diffusion system described by the following semilinear parabolic equation

$$\frac{\partial x(t,\, z)}{\partial t} = \alpha\, \frac{\partial^2 x(t,\, z)}{\partial z^2} + r(x(t,\, z)) + \beta\, b(z)\, u(t) \quad (1)$$

$$x(t,\, 0) = (1-\beta)\, u(t) + \beta\, x_0(t) \quad (2)$$

$$x(t,\, l) = x_l(t) \quad (3)$$

$$x(0,\, z) = \varphi(z) \quad (4)$$

where $z \in \Omega$ and t are the spatial and time variables, respectively, and $\Omega = [0,\, l]$ is the spatial domain. $x \in L^2(\Omega)$ and u are the state and the control, respectively. The function $b \in L^2(z)$ characterizes the spatial distribution of the control u in the spatial domain Ω (or the geometrical structure of the actuator). x_0 and x_l are functions that represent the values of the state at the boundaries $z = 0$ and $z = l$, respectively. The function $\varphi \in L^2(\Omega)$ is a smooth initial spatial profile and α is the diffusion coefficient. The nonlinear function r represents the reaction term. The parameter β, which can take values of either 0 or 1, is used to define the type of actuation. Specifically, when $\beta = 0$, the control input u is applied at the left-hand boundary ($z = 0$), representing boundary control (Fig. 1). Conversely, when $\beta = 1$, the control input u acts throughout the entire spatial domain Ω, indicating distributed control (Fig. 1).

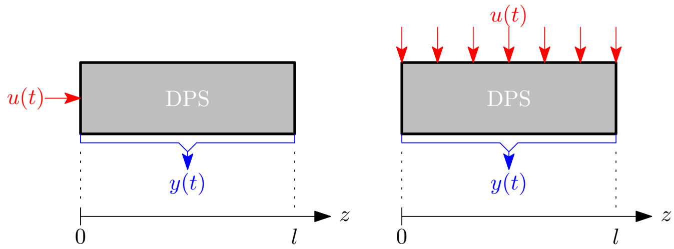

Fig. 1. Types of actuation: Boundary control (left). Distributed control (right).

The space $L^2(\Omega)$ consists of square-integrable functions, endowed with the following scalar product:

$$\langle f(z), g(z) \rangle = \int_0^l f(z) g(z)\, dz \quad (5)$$

This work addresses the control of the output variable y, defined as the spatial weighted average of the DPS state x. Specifically,

$$y(t) = \langle c(z),\, x(t,\, z) \rangle \quad (6)$$

The objective consists in designing a controller that forces the output y to track asymptotically the desired reference y^d. The smooth shaping function c characterizes the geometrical structure of the sensor.

Remark 1. The distributed and boundary control problems (1)–(4) are formulated with Dirichlet boundary conditions. However, other types of boundary conditions, such as Neumann and Robin conditions with boundary control acting at $z = l$, can also be considered, and the development provided below remains applicable.

3. GENERIC MODEL CONTROL DESIGN

The GMC method is used in this work to solve the distributed and boundary control problems following the late lumping approach, i.e., using the partial differential equation (PDE) model.

3.1 Overview of Generic Model Control

To simplify the presentation, consider the case of a first-order LPS given by the following state-space model:

$$\dot{x}(t) = f(x(t)) + g(x(t))\, u(t) \quad (7)$$

$$y(t) = x(t) \quad (8)$$

with $g(x(t)) \neq 0, \forall x(t)$.

Denoting the tracking error by $e = y^d - y$, the GMC controller design involves solving the following equation

$$\dot{y}(t) = k_1\, e(t) + k_2 \int_0^t e(\zeta)\, d\zeta \quad (9)$$

with respect to u (Lee and Sullivan, 1988b,a; Lee, 1993). k_1 and k_2 are the tuning parameters of the controller.

Thus, using (8) and (7), it follows that

$$\dot{x}(t) = k_1\, e(t) + k_2 \int_0^t e(\zeta)\, d\zeta \quad (10)$$

$$f(x(t)) + g(x(t))\, u(t) = k_1\, e(t) + k_2 \int_0^t e(\zeta)\, d\zeta \quad (11)$$

Since $g(x) \neq 0$, $\forall x$, the solution of (11) with respect to u yields the following GMC controller:

$$u(t) = g^{-1}(x) \left[-f(x(t)) + k_1\, e(t) + k_2 \int_0^t e(\zeta)\, d\zeta \right] \quad (12)$$

By differentiating (9) with respect to time, we obtain

$$\ddot{y}(t) + k_1\, \dot{y}(t) + k_2\, y(t) = k_1\, \dot{y}^d(t) + k_2\, y^d(t) \quad (13)$$

The controller parameters k_1 and k_2 are tuned such that the linear closed-loop system (13) is stable using, for instance, the pole placement technique. Since y^d converges to a constant value y^d_∞, i.e., $\lim_{t\to\infty} y^d = y^d_\infty$, it follows from (13) that:

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} y^d(t) = y^d_\infty \quad (14)$$

consequently, the output y will asymptotically track the desired reference y^d, achieving steady-state convergence.

In the following, the GMC technique is extended to diffusion-reaction systems.

3.2 Distributed control design

Assumption 1. The shaping functions c and b are not orthogonal, i.e.,

$$\langle c(z),\, b(z)\rangle \neq 0 \quad (15)$$

For the distributed control problem (case $\beta = 1$), the time differentiation of the controlled output yields:

$$\begin{aligned}\dot{y}(t) &= \left\langle c(z),\, \frac{\partial x(t,\, z)}{\partial t} \right\rangle \\ &= \left\langle c(z),\, \alpha \frac{\partial^2 x(t,\, z)}{\partial z^2} + r(x(t,\, z)) \right\rangle + \langle c(z),\, b(z)\rangle\, u(t)\end{aligned} \quad (16)$$

and the use of the GMC formula (9) leads to the following algebraic equation:

$$\left\langle c(z),\, \alpha \frac{\partial^2 x(t,\, z)}{\partial z^2} + r(x(t,\, z)) \right\rangle + \langle c(z),\, b(z)\rangle\, u(t) = k_1\, e(t) + k_2 \int_0^t e(\zeta)\, d\zeta \quad (17)$$

Taking into account Assumption 1, the solution of (17), with respect to the control u, yields the following distributed GMC controller:

$$u(t) = \frac{-\left\langle c(z),\, \alpha \frac{\partial^2 x}{\partial z^2} + r(x) \right\rangle + k_1\, e(t) + k_2 \int_0^t e(\zeta)\, d\zeta}{\langle c(z),\, b(z)\rangle} \quad (18)$$

3.3 Boundary control design

Assumption 2. The function c is such that $c(0) \neq 0$.

For boundary control (case $\beta = 0$), GMC cannot be directly designed, as the control acts at the boundary rather than within the spatial domain. In this case, the first-time derivative of y does not involve the control u. To overcome this challenge and extend GMC to the boundary control case, the notion of an extended operator is used to convert the boundary control into a pointwise control (Fig. 2). The pointwise control form can be obtained using the notion of the self-adjoint operator (Stafford and Dowrick, 1977) or the Laplace transform in spatial domain (Maidi and Corriou, 2011). In the following, the Laplace transform approach is used, and for brevity the reaction term r is assumed equal to zero, i.e., $r = 0$. Indeed, in the case $r \neq 0$, a linearization step around a uniform spatial profile is necessary. In both cases, the Laplace transform approach leads to the same pointwise control problem form. More details about the use of the Laplace transform approach for a left-hand boundary condition ($z = 0$) can be found in Maidi and Corriou (2011) and for a right-hand boundary condition ($z = l$) in Hamdadou et al. (2019).

The Laplace transform in the spatial domain of the PDE (1), with $\beta = 0$, yields

$$\frac{\partial X(t,\, s)}{\partial t} = \alpha\, s^2\, X(t,\, s) - \alpha\, s\, x(t,\, 0) - \alpha\, \left.\frac{\partial x(t,\, z)}{\partial z}\right|_{z=0} \quad (19)$$

Substituting (2), with $\beta = 0$, into (19), the following equation results

$$\frac{\partial X(t,\, s)}{\partial t} = \alpha\, s^2\, X(t,\, s) - \alpha\, s\, u(t) - \alpha\, \left.\frac{\partial x(t,\, z)}{\partial z}\right|_{z=0} \quad (20)$$

which can be rewritten under the following form

$$\frac{\partial X(t,\, s)}{\partial t} = \alpha\, s^2\, X(t,\, s) - \alpha\, s\, x(t,\, 0) - \alpha\, \left.\frac{\partial x(t,\, z)}{\partial z}\right|_{z=0} - \alpha\, s\, u(t) \quad (21)$$

with the following boundary condition

$$x(t,\, 0) = 0 \quad (22)$$

Now, applying the inverse Laplace transform in spatial domain of (21) gives

$$\frac{\partial x(t,\, z)}{\partial t} = \alpha \frac{\partial^2 x(t,\, z)}{\partial z^2} - \alpha\, \dot{\delta}(z)\, u(t) \quad (23)$$

with the boundary conditions (22) and (3).

Therefore, for $r \neq 0$ the certainty equivalence pointwise control form for the diffusion-reaction system (1)–(3) is given by

$$\frac{\partial x(t,\, z)}{\partial t} = \alpha \frac{\partial^2 x(t,\, z)}{\partial z^2} + r(x(t,\, z)) - \alpha\, \dot{\delta}(z)\, u(t) \quad (24)$$

$$x(t,\, 0) = 0 \quad (25)$$

$$x(t,\, l) = x_l(t) \quad (26)$$

$$x(0,\, z) = \varphi(z) \quad (27)$$

Notice that the obtained pointwise control problem (24)–(27) is a specific case of a diffusion-reaction system with distributed control, where

$$b(z) = -\alpha\, \dot{\delta}(z) \quad (28)$$

Therefore,

$$\langle c(z),\, b(z)\rangle = \langle c(z),\, -\alpha\, \dot{\delta}(z)\rangle \quad (29)$$

$$= -\alpha \int_0^l c(z)\, \dot{\delta}(z)\, dz \quad (30)$$

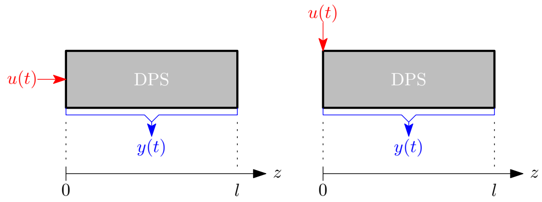

Fig. 2. Transformation from boundary control to pointwise control.

and by integration by parts, we obtain

$$\langle c(z),\, -\alpha\, \dot{\delta}(z)\rangle = \alpha \int_0^l \dot{c}(z)\, \delta(z)\, dz \tag{31}$$

and using the following Dirac delta function property

$$\int_0^l \dot{c}(z)\, \delta(z)\, dz = c(0) \tag{32}$$

it follows that

$$\langle c(z),\, -\alpha\, \dot{\delta}(z)\rangle = \alpha\, c(0) \tag{33}$$

Taking into account Assumption 2, the same approach presented in Subsection 3.2 yields the following GMC boundary controller

$$u(t) = \frac{-\alpha \left\langle c(z),\, \dfrac{\partial^2 x}{\partial z^2} + r(x) \right\rangle + k_1\, e(t) + k_2 \int_0^t e(\zeta)\, d\zeta}{\alpha\, \dot{c}(0)} \tag{34}$$

which incorporates integral action that helps to mitigate the impact of uncertainties, unmodeled dynamics, and disturbance rejection.

4. APPLICATION EXAMPLE

Consider the heat flow in a thin metal rod with a linear source term (Farlow, 1993). The temperature of the rod can be controlled by manipulating a heat flux q. The controlled output T_m is defined as the spatially weighted average temperature of the rod. The initial temperature profile is assumed uniform. In the sequel, the abscissa z and the time t of the rod model are dimensionless.

4.1 Distributed control

The heat flux q is assumed to be distributed within the spatial domain through the shaping function $b(z) = 10\, z\, (1 - z)$. The left and right ends of the rod are assumed to follow the a constant and a variable temperature profiles, respectively, leading to Dirichlet-type boundary conditions. The abscissa z is dimensionless, $z \in \Omega = [0, 1]$. The model of the rod temperature is given as

$$\frac{\partial T(t,\, z)}{\partial t} = \frac{\partial^2 T(t,\, z)}{\partial z^2} + 2\, T(t,\, z) + 10\, z\, (1 - z)\, q(t) \tag{35}$$

$$T(t,\, 0) = 25, \quad T(t,\, 1) = T_1(t) \tag{36}$$

$$T(0,\, z) = 25, \quad T_m(t) = \int_0^1 (1 - z)\, T(t,\, z)\, dt \tag{37}$$

Using (18) the distributed GMC controller results as

$$q(t) = \frac{25}{3} \left[-\left\langle 1 - z,\, \frac{\partial^2 T}{\partial z^2} - 2\, T \right\rangle + k_1\, \left(T_m^d(t) - T_m(t)\right) + k_2 \int_0^t \left(T_m^d(\zeta) - T_m(\zeta)\right)\, d\zeta \right] \tag{38}$$

4.2 Boundary control

The heat flux q is now applied at the left-hand end of the rod, while the right-hand end is assumed to follow a variable temperature profile. The model of the rod temperature is given as

$$\frac{\partial T(t,\, z)}{\partial t} = \frac{\partial^2 x(t,\, z)}{\partial z^2} + 2\, T(t,\, z) \tag{39}$$

$$T(t,\, 0) = q(t), \quad T(t,\, 1) = T_1(t) \tag{40}$$

$$T(0,\, z) = 25, \quad T_m(t) = \int_0^1 (1 - z)\, T(t,\, z)\, dt \tag{41}$$

Using (34), the following boundary GMC controller results:

$$q(t) = -\left\langle 1 - z,\, \frac{\partial^2 T}{\partial z^2} - 2\, T \right\rangle + k_1\, \left(T_m^d(\zeta) - T_m(\zeta)\right) + k_2 \int_0^t \left(T_m^d(\zeta) - T_m(\zeta)\right)\, d\zeta \tag{42}$$

4.3 Simulation results

The output tracking and disturbance rejection performance are evaluated via numerical simulation. The method of lines (Vande Wouwer et al., 2004), based on the finite difference, is used to simulate the closed-loop system. To accurately assess the GMC controller performance, a large number of discretization points is currently required to approximate the original DPS, typically 100 points. Thus, the spatial second-order derivative is approximated using the central finite difference of second-order scheme. The integral terms are computed using the trapezoidal rule. For both control cases, the control objective is to force the temperature T_m to track the desired reference T_m^d, where

$$T_m^d(t) = 25\, (1 - e^{-t}) \tag{43}$$

while rejecting the disturbance effect caused by a sudden variation in the boundary temperature T_l, i.e., the temperature at the right-hand boundary $z = 1$. The following time profile is assumed for T_1:

$$T_1(t) = \begin{cases} 25 & t \leq 10 \\ 25\, \left(2 - e^{-(t-10)}\right) & t > 10 \end{cases} \tag{44}$$

The results presented in Figs. 3 and 6 clearly show that the designed GMC controllers force the controlled output T_m to track its desired reference T_m^d, while maintaining acceptable physical variations of the heat flux q (Figs. 4 and 7). Furthermore, in both control scenarios, the GMC controller effectively rejects the disturbance effect (Figs. 5 and 8). In the case of distributed control, the impact of disturbance is not observed on the controlled output T_m (Fig. 3). This can be explained by the fact that the action of distributed GMC controller (Figs. 4 and 5) attenuates the disturbance effect, thereby preventing their propagation through the rod. In the case of boundary control, the effect of disturbance is remarkable in the controlled output (Fig. 6). This is because the control is applied at the boundary $z = 0$, resulting in a delay before the control becomes effective and allowing the disturbance effect to remain apparent for a while. Furthermore, as shown in Figs. 4 and 7, boundary control requires more heat flux than distributed control. This difference arises

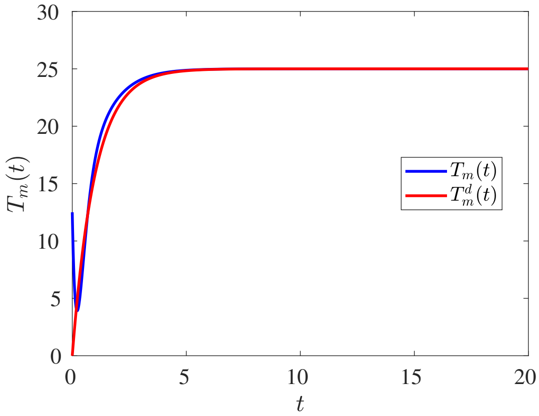

Fig. 3. Distributed control: Evolution of the controlled output T_m

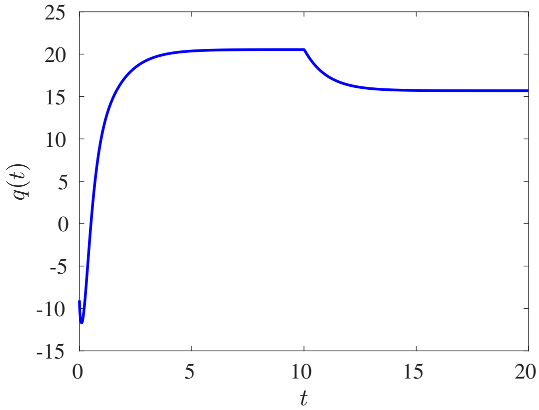

Fig. 4. Distributed control: Evolution of the heat flux q.

because distributed control applies heat across the entire spatial domain, uniformly increasing the rod temperature. In contrast, boundary control relies on diffusion from the left boundary ($z = 0$) to raise temperatures throughout the rod. Consequently, more heat flux is needed at $z = 0$ to effectively heat the points distant from this boundary. Note that quantitative performance metrics can be used to assess the impact of the tuning parameters k_1 and k_2 on the controller performance. The obtained results demonstrate the effectiveness of the GMC controllers in both output tracking and disturbance rejection, unlike the geometric controllers (Maidi and Corriou, 2011), which require to define the external variable by a robust controller in order to ensure robustness against parameter uncertainty, unmodeled dynamics, and disturbance rejection.

5. CONCLUSION

Following the late lumping approach, the GMC is extended to diffusion reaction systems. Two types of actuation, distributed and boundary, are considered to control an output defined as a spatially weighted average of the DPS state. It is shown that under the assumption of non orthogonality of the actuation and sensing shaping functions, a controller of distributed nature can be easily designed. The process design is limited to evaluating the time partial derivative of the state, which makes the use of GMC easy.

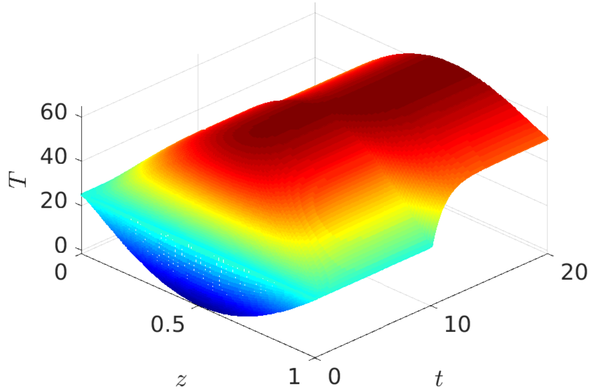

Fig. 5. Distributed control: 3D temperature profile T.

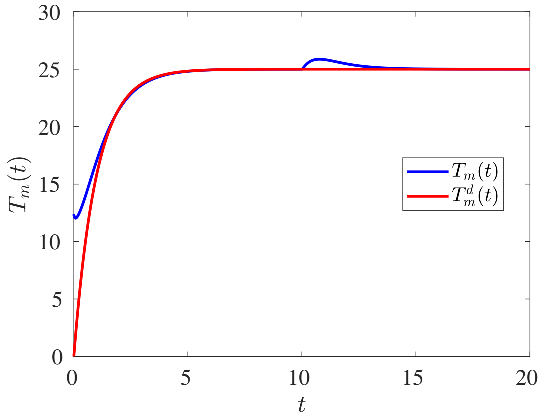

Fig. 6. Boundary control: Evolution of the controlled output T_m.

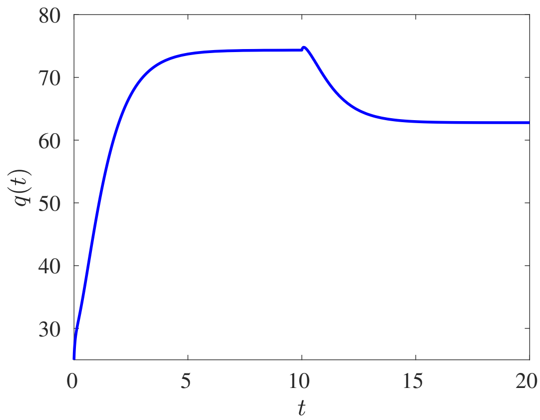

Fig. 7. Boundary control: Evolution of the heat flux q.

In the distributed control case, as the actuation acts in the spatial domain, the design of the controller is straightforward. In contrast, in the boundary control case, as the actuation acts at the boundary of the spatial domain, the design of the GMC controller is impossible. To overcome this difficulty, we proposed to convert the boundary problem to a pointwise one based on the concept of the extended operator. Using the resulting pointwise control formulation, which is a particular case of the distributed control, a GMC boundary controller is developed. Temperature control of a heated rod is taken as an application

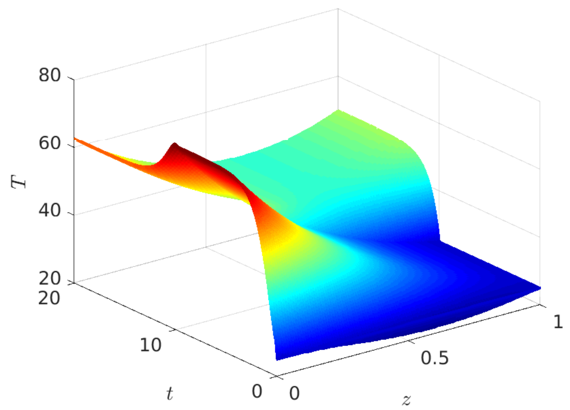

Fig. 8. Boundary control: 3D temperature profile T.

example to demonstrate the tracking and disturbance performances of the developed GMC controllers.

The present study shows that the GMC is a promising control approach for dealing with control design of DPS in the framework of late lumping approach. Future works include the application of the GMC to other classes of linear and nonlinear DPSs and the investigation of the internal stability of the closed loop using Lyapunov and semigroup theories.

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REFERENCES

Aziz, N., Hussain, M., and Mujtaba, I. (2000). Optimal control of batch reactors using generic model control (gmc) and neural network. In S. Pierucci (ed.), *European Symposium on Computer Aided Process Engineering*, volume 8, 175–180. Elsevier.

Christofides, P.D. (2001a). Control of nonlinear distributed process systems: Recent developments and challenges. *AIChE Journal*, 47(3), 514–518.

Christofides, P.D. (2001b). *Nonlinear and robust control of PDE systems: methods and applications to transport-reaction processes*. Birkhaüser, Boston.

Christofides, P.D. and Daoutidis, P. (1996). Feedback control of hyperbolic PDE systems. *AIChE Journal*, 42(11), 3063–3086.

Cong, L., Chang, L., and Liu, X. (2015). Development of extended generic model control for high-purity heat integrated distillation column using online concentration estimation. *Industrial & Engineering Chemistry Research*, 54(51), 12897–12907.

Cott, B.J. and Macchietto, S. (1989). Temperature control of exothermic batch reactors using generic model control. *Industrial & Engineering Chemistry Research*, 28(8), 1177–1184.

Farlow, S.J. (1993). *Partial differential equations for scientists and engineers*. Dover Publications, New York.

Hamdadou, D., Maidi, A., and Corriou, J.P. (2019). Boundary control of burgers' equation by input-output linearization. In M. Chadli, S. Bououden, S. Ziani, and I. Zelinka (eds.), *Advanced Control Engineering Methods in Electrical Engineering Systems*, 84–96. Springer International Publishing, Cham.

Hamid, H.B.A. and Liu, X. (2024). Adaptive generic model control scheme for an optimized external heat integrated air separation column using unscented Kalman filter. *Chemical Engineering and Processing - Process Intensification*, 204, 109956.

Lee, P.L. and Sullivan, G.R. (1988a). Generic model control (GMC). *Computers & Chemical Engineering*, 12(6), 573–580.

Lee, P.L. (1993). *Nonlinear Process Control: Applications of Generic Model Control*. Springer, London.

Lee, P. and Sullivan, G. (1988b). Generic model control – theory and applications. *IFAC Proceedings Volumes*, 21(4), 111–119. IFAC Workshop on Model Based Process Control, Atlanta, GA, USA, 13-14 June.

Li, H.X. and Qi, C. (2010). Modeling of distributed parameter systems for applications – A synthesized review from time-space separation. *Journal of Process Control*, 20(8), 891–901.

Maidi, A. and Corriou, J.P. (2011). Boundary control of nonlinear distributed parameter systems by input-output linearization. *IFAC Proceedings Volumes*, 44(1), 10910–10915. 18th IFAC World Congress.

Maidi, A., Paulen, R., and Corriou, J.P. (2024). Velocity control design of hyperbolic distributed parameter systems using zeroing dynamics and zeroing-gradient dynamics methods. *Journal of Process Control*, 138, 103210.

Meurer, T. (2013). *Control of Higher-Dimensional PDEs. Flatness and Backstepping Designs*. Springer, Berlin, Heidelberg.

Morris, K.A. (2020). *Controller Design for Distributed Parameter Systems*. Springer, Cham.

Ray, W.H. (1989). *Advanced Process Control*. Butterworth, Boston.

Singh, A. and Hahn, J. (2007). Effect of finite-dimensional approximations on observability analysis of distributed parameter models. *IFAC Proceedings Volumes*, 40(5), 197–202. 8th IFAC Symposium on Dynamics and Control of Process Systems.

Stafford, E. and Dowrick, G. (1977). Applications of extended operators to diffusive systems. *IFAC Proceedings Volumes*, 10(5), 527–537. 2nd IFAC Symposium on Control of Distributed Parameter Systems, Coventry, Great Britain, 28 June–1 July.

Vande Wouwer, A., Saucez, P., and Schiesser, W.E. (2004). Simulation of distributed parameter systems using a Matlab-based method of lines toolbox: Chemical engineering applications. *Industrial & Engineering Chemistry Research*, 43(14), 3469–3477.

Zangina, J.S., Wang, W., Qin, W., Gui, W., Sun, Y., Xu, S., Yang, C., Xie, D., Wang, Y., and Liu, X. (2021). A novel adaptive generic model control strategy for internal thermally coupled air separation columns with multivariable recursive estimation. *Computers & Chemical Engineering*, 151, 107348.