

# Data-based Sequential Design of 2×2 Multi-loop PID Controllers

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**Abstract:** This paper introduces a sequential design method for multi-loop PID controllers under the Virtual Reference Feedback Tuning (VRFT) methodology. Simulation studies are presented to illustrate the effect of different reference models on the proposed design for two-input two-output (TITO) systems. Lastly, the proposed design is compared with benchmark designs for well-known multivariable processes.

**Keywords:** MIMO control, Data-based control, Sequential design, VRFT.

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## 1. INTRODUCTION

Most industrial chemical processes involve multiple controlled and manipulated variables referred to as a multi-input and multi-output (MIMO) system. For a MIMO control system, process interaction is inherent and causes difficulties in feedback controller design. As a result, the achievable control performance of a MIMO control system is inevitably compromised depending on the magnitude of process interaction. In fact, the system can even become unstable when the magnitude of process interaction is greater than the system tolerance. Owing to these reasons, MIMO control is far more challenging and hence has been a subject of great research interest over the last few decades (Wang et al., 2008).

As far as MIMO control loops in chemical industries are concerned, decentralized control is the most common control scheme. The main reason behind its ubiquity is its ability to provide satisfactory performance despite its simple structure (Halevi et al., 1997). For a MIMO system with  $n$  manipulated and controlled variables each, a full matrix controller has  $n^2$  controllers whereas a decentralized system has only  $n$  controllers. This means that for a decentralized PID system there are  $3n$  tuning parameters as opposed to  $3n^2$  parameters for a full controller. In addition to the simplicity of decentralized control systems, they are more robust to actuator or sensor failures as the failure affects only one loop and hence can be manually stabilized with relative ease.

Decentralized control design can be roughly divided into two categories: sequential design methods (Bernstein, 1987; Chiu & Arkun, 1992; Shiu & Hwang, 1998), and independent design methods (Grosdidier & Morari, 1986; Hovd & Skogestad, 1992; Skogestad & Morari, 1989). As their names suggest, the difference between the two methods is that for independent design methods, each controller's parameters are determined independently whereas sequential design methods design controllers by using the information of controllers designed in the previous steps. This in turn makes sequential

design methods less conservative than independent design methods.

The central idea of sequential design is to decompose a MIMO design problem into a sequence of single-input single-output (SISO) design problems. Once this decomposition is achieved, tuning methods applicable to SISO problems can be readily used to design decentralized controllers for multi-loop systems. The individual channel design (ICD) method of O'Reilly and Leithead (1991) and the work of Bhalodia and Weber (1979) on the tuning of PI controllers for TITO systems are prime examples of the simplicity with which sequential design facilitates the aforementioned task.

As compared to model-based approaches, very few data-based methods have been developed for decentralized controller design. However model identification for large systems can be tedious and hence direct data-based controller design methods for MIMO systems have emerged as a promising prospect with VRFT as one of the natural choices, owing to its simple and non-iterative nature. Despite the VRFT literature on MIMO systems not being as extensive as SISO systems, there have been some notable attempts on the subject nonetheless (Campestrini et al., 2016; Da Silva et al., 2016; Nakamoto, 2004). However, these methods are either full controller design methods or independent design methods and in our knowledge, no attempts at sequential design of decentralized controllers have been made. Owing to the aforementioned reasons, an attractive alternative based on sequential design method is adopted in this paper. The method innately takes into account the process interactions and can utilize the information about the controller designed in the previous step. This gives sequential design the advantage of being less conservative than independent design methods.

The rest of this paper is organized as follows. Section 2 provides an overview of the VRFT design along with the derivation of a new reference model and the detailed steps of the proposed sequential design for a TITO system. Section 3 comprises the simulation studies for the control performance



The sequence of design steps for a TITO system is shown in Figure 4 and are as follows:

Step 1: Obtain open-loop data for all diagonal elements  $\{u_i, y_i\} \forall i \in \{1,2\}$ . Assuming a first-order plus delay time (FOPDT) between each input output pair, estimate the time constants using and time delays.

Step 2: Solve the optimization problem in Eq. (6) using  $\{u_1, y_1\}$  as the data set (Figure 4(a)). Store the initial controller  $C_1^1$  for loop 1.

Step 3: Close loop 1 using  $C_1^1$  and obtain data with pulse input change in loop 2 (Figure 4(b)).

Step 4: Solve the optimization problem in Eq. (6) using the input-output data for loop 2. Store the initial controller  $C_2^1$  for loop 2.

Step 5: Close loop 2 using  $C_2^1$  and obtain data with input change in loop 1 (Figure 4(c)).

Step 6: Solve optimization problem in Eq. (6) using the new input-output data for loop 1. Store the updated controller  $C_1^2$ .

Step 7: Repeat steps 3 to 7 to redesign controllers for the two loops until the controller parameters converge.

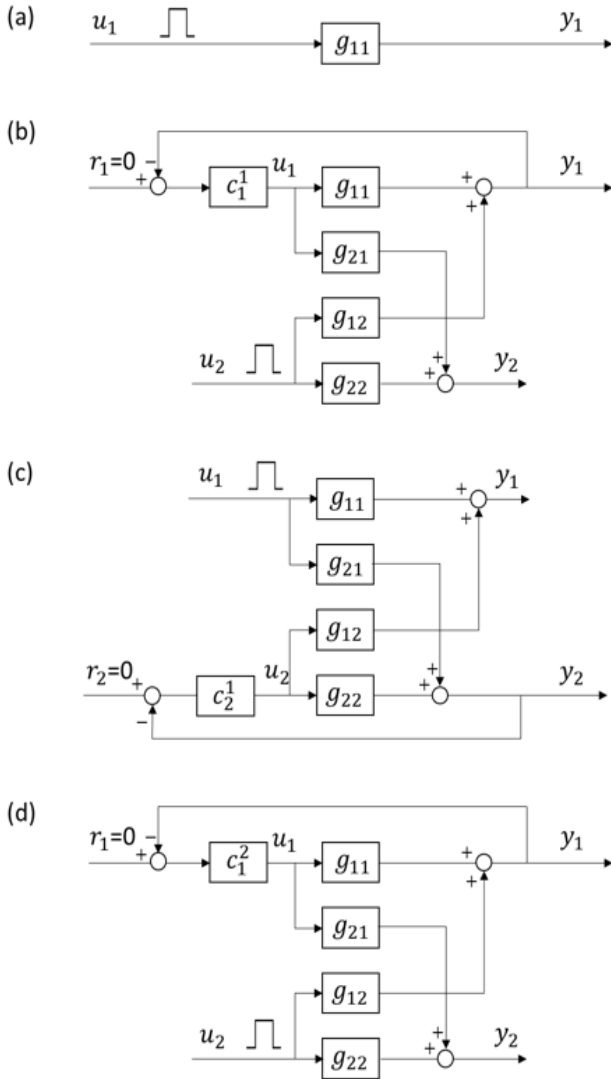


Figure 4: VRFT-based sequential design illustrated with a TITO control system

### 2.3 Reference model (RM)

The selection of an appropriate reference model is extremely important in the VRFT design. In this work, we will consider two reference models and discuss their pros and cons based on simulations on an industrial TITO process. The first reference model to be considered is a FOPDT model as given below:

$$M_i = \frac{e^{-\theta_{c_i}s}}{\tau_{c_i}s + 1} \forall i \in \{1,2\} \quad (12)$$

whereas the second reference model chosen is the loop function discussed by Huang and Jeng (2002) given as follows:

$$L_i = e^{-\theta_{c_i}s} \frac{0.76(1 + 0.47\theta_{c_i}s)}{\theta_{c_i}s} \forall i \in \{1,2\} \quad (13)$$

In order to solve the optimization problem in Eq. (6) with Eq. (12) as the RM, the delay term  $\theta_{c_i}$  is fixed at every step using the apparent time delay of the output response and the time constant is constrained as given below:

$$k_1 \min(\tau_{ii}, 0.2\tau_{ii_{ss}}) < \tau_{c_i} < k_2 \max(\tau_{ii}, 0.2\tau_{ii_{ss}}) \quad (14)$$

where  $k_1, k_2 \in (0,1)$  and  $\tau_{ii}$  and  $\tau_{ii_{ss}}$  are defined as the time required to reach 63.2% and 99.3% of the final steady-state value respectively. Similarly, for the optimization problem in Eq. (6) to be solved using Eq. (13) as the reference model, the range of the only tuning parameter  $\theta_i$  is chosen based on the apparent time delay of the output response at each step.

### 3. SIMULATION STUDIES

Consider the following industrial-scale polymerization reactor studied by Xiong and Cai (2006):

$$G_1(s) = \begin{bmatrix} \frac{22.89}{4.57s + 1} e^{-0.2s} & \frac{-11.64}{1.81s + 1} e^{-0.4s} \\ \frac{4.69}{2.17s + 1} e^{-0.2s} & \frac{5.8}{1.80s + 1} e^{-0.4s} \end{bmatrix}$$

The proposed sequential design was carried out for the process using both the reference models discussed in the previous section. PI controllers have been considered since a variety of benchmarks are available and they can be readily designed by only using the first two columns of  $\Psi$  in Eq. (9). We found that for FOPDT, the optimal  $\tau_{c_i}$  often lies on the upper bound and since it is innately associated with the speed of the loop response, different values of  $k_2$  were tried whereas  $k_1$  was fixed to be 0.2.

For the sake of brevity, the detailed steps of the proposed design have been discussed for only the case of the reference model in Eq. (13). Figure 1 contains the different steps involved in the proposed design and Table 1 lists the optimal reference model and controller parameters along with the objective function value and bandwidth. We can see from Table 1 that the controller parameters in Steps 3 and 5 are almost identical and hence the design procedure is terminated. The comparison of the closed-loop responses of the different reference models i.e. FOPDT (SD-T) with varying bounds and

the loop function  $L$  (SD-L) are shown in Figure 6. It can be seen that as the upper bound is lowered, the proposed design yields more aggressive controller parameters (see Tables 2 and 3). However, these bounds need to be chosen empirically and also need an initial estimate of the open-loop time constants. On the other hand, SD-L does not suffer from these drawbacks and at the same time yields the lowest cumulative IAE amongst all the cases considered. The only downside visible from the responses is the overshoot in the responses. However, since the aim of the design is to minimize IAE for step change, this is not of great concern, as the process requirement in this case is to reach the desired specification as fast as possible.

In addition to the reference model comparison, the proposed design is compared with three mode-based designs by Luyben (1986), Chien et al. (1999) and Xiong and Cai (2006) respectively. The closed-loop performance comparison is shown in Figure 7 and the controller parameters and IAE values are covered in Tables 2 and 3 respectively. It can be seen that the proposed design gives the least IAE amongst all the design methods.

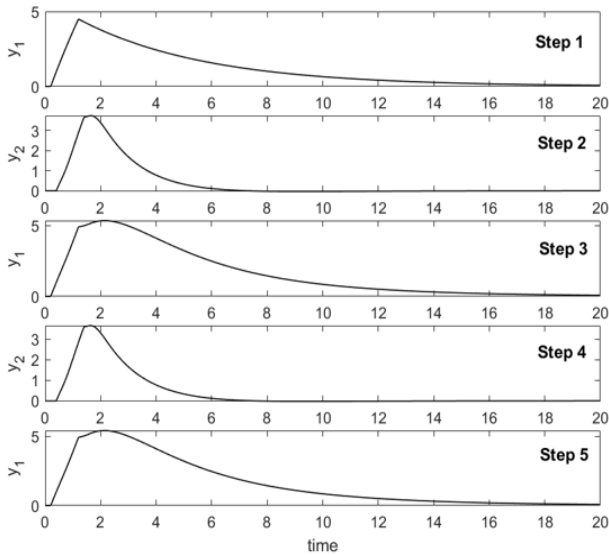


Figure 5: Process outputs for  $G_1$  at different steps of SD-L

Table 1. Design parameters at each step for  $G_1$  for SD-L

Step	$\theta$	$\omega_b$	$K_C$	$\tau_I$
1	0.37	7.89	0.43	5.01
2	0.95	3.09	0.18	1.91
3	0.32	9.17	0.43	8.83
4	0.92	3.19	0.19	1.93
5	0.32	9.20	0.43	8.87

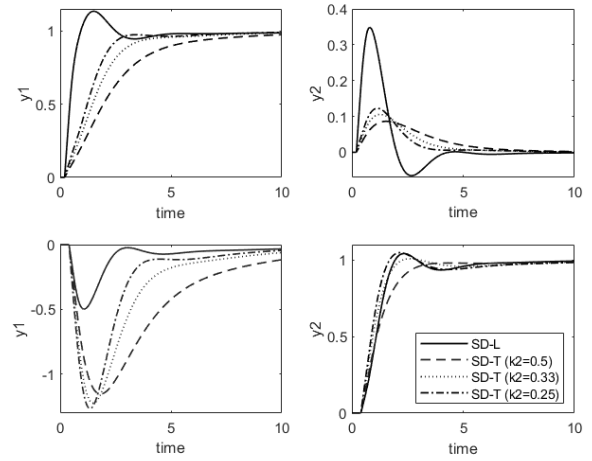


Figure 6: Closed-loop performance comparison for different reference models ( $G_1$ )

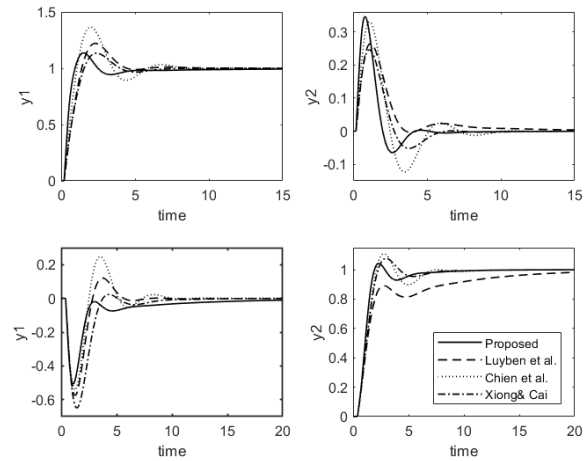


Figure 7: Closed-loop performance comparison of SD-L with benchmarks for  $G_1$

Table 2. Controller parameters for different design methods

Design method	Loop	$K_C$	$\tau_I$
SD-L	1	0.43	8.87
	2	0.19	1.93
SD-T ( $k_2=0.5$ )	1	0.05	4.71
	2	0.24	2.84
SD-T ( $k_2=0.33$ )	1	0.08	4.72
	2	0.30	3.11
SD-T ( $k_2=0.25$ )	1	0.10	4.81
	2	0.34	3.35
Luyben	1	0.21	2.26
	2	0.18	4.25
Chien et al.	1	0.26	1.42
	2	0.16	1.77
Xiong & Cai	1	0.22	4.57
	2	0.17	1.80

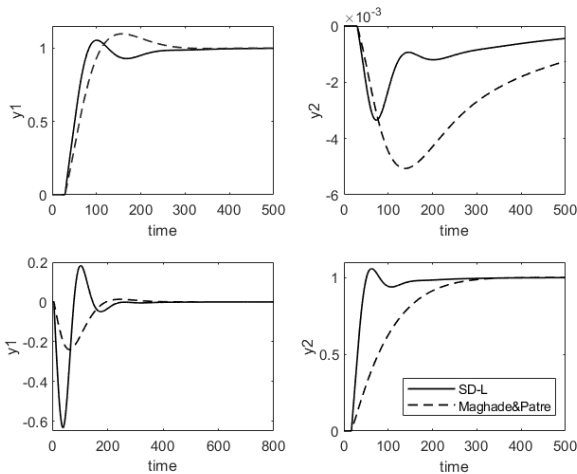
**Table 3. IAE values for different design methods for  $G_1$**

Design method	Set-point change in loop 1		Set-point change in loop 2		Total IAE
	$y_1$	$y_2$	$y_1$	$y_2$	
	<i>SD-L</i>	0.91	0.46	1.25	
<i>SD-T</i> ( $k_2=0.5$ )	2.65	0.30	5.27	1.49	9.71
<i>SD-T</i> ( $k_2=0.33$ )	1.89	0.26	3.75	1.30	7.21
<i>SD-T</i> ( $k_2=0.25$ )	1.54	0.25	3.03	1.30	6.12
<i>Luyben</i>	1.17	0.59	1.06	2.88	5.70
<i>Chien et al.</i>	1.39	0.72	1.21	1.61	4.93
<i>Xiong &amp; Cai</i>	1.06	0.48	1.33	1.50	4.37

Next, we consider a level-temperature reactor studied by Maghade and Patre (2012).

$$G_2(s) = \begin{bmatrix} \frac{1.68}{63.07s + 1} e^{-28.82s} & \frac{-0.02}{29.50s + 1} e^{-6.24s} \\ \frac{-0.03}{276.89s + 1} e^{-30s} & \frac{0.05}{62.37s + 1} e^{-16.98s} \end{bmatrix}$$

Since, from the previous example we see that SD-L outperforms SD-T, we consider only SD-L for this example and carry out the proposed design similar to  $G_1(s)$ . The performance of the proposed design is then compared to the controller designed by Maghade and Patre (2012). The closed-loop responses for the two designs are shown in Figure 8 whereas the controller parameters and the IAE values are listed in Tables 4 and 5 respectively. It can be seen that the proposed design provides approximately 32% reduction in IAE with respect to the benchmark controller.



**Figure 8: Closed-loop performance comparison of proposed design with benchmark for  $G_2$**

**Table 4. Controller parameters for two design methods for  $G_2$**

Method	Loop	$K_c$	$\tau_I$
<i>SD-L</i>	1	0.89	86.75
	2	46.38	82.78
<i>Maghade &amp; Patre</i>	1	0.54	49.54
	2	11.32	53.83

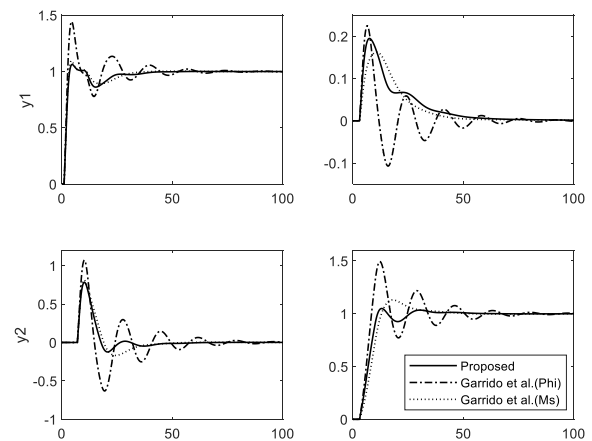
**Table 5. IAE values for different design methods for  $G_2$**

Design method	Set-point change in loop 1		Set-point change in loop 2		Total IAE
	$y_1$	$y_2$	$y_1$	$y_2$	
<i>SD-L</i>	61.50	0.60	38.32	38.34	138.76
<i>Maghade &amp; Patre</i>	73.81	1.69	21.82	106.50	203.82

Lastly, we consider the model-based frequency domain sequential design approach adopted by Garrido et al. (2021). The Wood and Berry column is considered for the comparative study.

$$G_3(s) = \begin{bmatrix} \frac{12.8}{16.7s + 1} e^{-s} & \frac{-18.9}{21s + 1} e^{-3s} \\ \frac{6.6}{10.9s + 1} e^{-3s} & \frac{-19.4}{14.2s + 1} e^{-7s} \end{bmatrix}$$

The closed-loop responses for the proposed design along with the two designs by Garrido et al. (2021) based on phase margin (Phi) and maximum sensitivity (Ms) respectively are shown in Figure 9 and the controller parameters and the IAE values are listed in Tables 6 and 7 respectively, where it is evident that the proposed design provides the least IAE.



**Figure 9: Closed-loop performance comparison of proposed design with benchmark for  $G_3$**

**Table 6. Controller parameters for different design methods for  $G_3$**

Method	Loop	$K_c$	$\tau_I$
SD-L	1	0.67	14.70
	2	-0.08	5.19
Garrido et al.(Phi)	1	0.73	3.55
	2	-0.09	3.06
Garrido et al.(Ms)	1	0.71	13.94
	2	-0.04	2.60

**Table 7. IAE values for different design methods for  $G_3$**

Design method	Set-point change in loop 1		Set-point change in loop 2		Total IAE
	$y_1$	$y_2$	$y_1$	$y_2$	
SD-L	3.83	6.03	3.33	8.07	21.26
Garrido et al.(Phi)	6.33	13.62	2.70	12.65	35.30
Garrido et al.(Ms)	3.69	7.70	2.96	10.01	24.36

#### 4. CONCLUSIONS AND FUTURE WORK

Simulation results suggest that the loop function discussed in the work by Huang and Jeng (2002) is the more appropriate reference model for the proposed sequential design. Additionally, the proposed design provides better results than the benchmark designs considered in this study and hence is an attractive alternate for the decentralized controller design of TITO systems. Future work includes further exploration of the effect of the structure and parameters of different reference models and application of the design methodology to higher order systems.

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