LSTM-based model predictive control with discrete actuators for irrigation scheduling

Bernard T. Agyeman * **Soumya R. Sahoo** * **Jinfeng Liu** * **Sirish L. Shah** *

* *Department of Chemical and Materials Engineering, University of Alberta, Edmonton AB, Canada T6G 1H9 (e-mail: jinfeng@ualberta.ca).*

Abstract: The development of well-devised irrigation scheduling methods is desirable from the perspectives of plant quality and water conservation. In this article, a model predictive control (MPC) with discrete actuators is developed for irrigation scheduling, where a long short-term memory (LSTM) model of the soil-water-atmosphere system is used to evaluate the objective of ensuring optimal water uptake in crops while minimizing total water consumption and irrigation costs. A heuristic method involving a sigmoid function is used in this framework to enhance the computational efficiency of the scheduler. The scheduling scheme is applied to a homogeneous field and the results indicate that the LSTM-based MPC with discrete actuators is able to prescribe optimal or near-optimal irrigation schedules that are typical of irrigation practice.

Keywords: Irrigation scheduling; mixed-integer model predictive control; heuristic method; long short-term memory networks; sigmoid function.

1. INTRODUCTION

According to the United Nations, agriculture accounts for about 70% of global freshwater withdrawals, the vast majority of which are used for irrigation purposes (UN Report (2018)). At the same time, the global water scarcity crisis is worsening, due to increased stress on freshwater resources resulting from population growth and climate change. Given the rising freshwater shortages, there is a pressing need for enhanced and precise irrigation management strategies that will enable efficient and sustainable water use while ensuring optimal plant development.

Precision irrigation can be realized by implementing well-devised irrigation control and scheduling operations on an hourly or daily basis for a predetermined planning horizon (Ali and Talukder (2001)). Traditionally, most control and scheduling operations in irrigation management are implemented in an open-loop fashion, where there is no direct connection between the supplied irrigation volume and the prevailing soil water status. Open-loop implementations are known to be imprecise and thus do not guarantee optimal plant yield and enhanced water use efficiency. Precision irrigation methods have been advocated as a means of alleviating the drawbacks that are associated with open-loop irrigation operations (Navarro-Hellín et al. (2015)). In the context of systems engineering, precision irrigation can be realized by closing the irrigation decision support loop to form a closed-loop system (Shah et al. (2021)).

Irrigation scheduling seeks to provide crops with the right amount of water at appropriate times. Among all the irrigation scheduling methods that have been recommended and developed, two main categories can be clearly distinguished: (1) model-free methods, and (2) model-based methods (Gu et al. (2020)). In model-free methods, soil moisture content is inferred from plant stress variables/sensor measurements/crop evapotranspiration values and the irrigation event is triggered when the inferred soil moisture exceeds a particular threshold. While these methods are computationally efficient, inaccuracies in the inferred soil moisture content often lead to a false triggering of the irrigation scheduler. To obtain a more precise and robust scheduler, agro-hydrological models have been used to determine irrigation schedules. In this regard, mechanistic agro-hydrological models such as the Richards equation (Park et al. (2009)), the AquaCrop model (Delgoda et al. (2016)), and the Root Zone Water Quality Model (Nguyen et al. (2017)) have been used to determine irrigation schedules. While these methods have been largely successful, mechanistic models are more difficult to handle from a numerical point of view and hence they render the resulting scheduling scheme computationally inefficient.

Recent studies have examined the use of statistical or data-driven models, also known as black box models, in irrigation scheduling. For example in Nahar et al. (Nahar et al. (2019)), a linear parameter varying model was used to develop a closed-loop scheduler and controller. Data-driven machine learning approaches such as adaptive neuro-fuzzy inference systems (Karandish and Šimůnek (2016)), support vector machines (Deng et al. (2016)), and feedforward neural networks (Capraro et al. (2008)) are another group of statistical models that have been used to develop model-based irrigation schedulers. Similarly, recurrent neural networks, particularly long short-term memory networks, have been used to determine irrigation schedules (Adeyemi et al. (2018)) due to their ability to learn long-term temporal dependencies in sequential data.

A number of optimal control approaches such as dynamic programming (Naadimuthu et al. (1988)), set-point tracking model predictive control (MPC) (McCarthy et al. (2014)), and MPC with zone control (Nahar et al. (2019)) have been used to schedule irrigation. When irrigation is to be scheduled on a daily basis, the determination of the irrigation time reduces to a discrete decision of whether or not the irrigation event should be performed on the days that make up the planning horizon. Thus, the daily irrigation scheduling problem can be transformed into an optimal control problem with both continuous- (irrigation volume or depth) and integer- (irrigation time) valued control variables. In the MPC framework, improvements in optimization software and computing performance permit the optimal selection of discrete-valued controls by directly including them in the MPC design. MPC with discrete actuators has seen applications in different areas such as energy systems (Risbeck (2018)) and heat pumps (Lee et al. (2019)). Following the convention in (Risbeck (2018); Rawlings and Risbeck (2017)), we refer to MPC with both continuous- and discrete-valued control variables as a mixed-integer MPC in the rest of this paper.

Motivated by the above, this work develops an irrigation scheduler in the framework of a mixed-integer MPC with zone control for agro-hydrological systems that utilize an LSTM model to predict the dynamics of soil moisture. The LSTM model is initially developed based on a dataset generated from extensive open-loop simulations of a mechanistic agro-hydrological model, specifically the Richards equation. Subsequently, a mixed-integer MPC with zone objectives is developed based on the identified LSTM model. Due to the inherently complex nature of mixed-integer programs, this work further proposes a heuristic method that can be used to simplify the mixed-integer MPC in order to reduce its computation time. The main contributions of this work include:

(1) A method to identify an LSTM model for the prediction of soil moisture content in an agro-hydrological system.
(2) A detailed closed-loop irrigation scheduler design in the framework of a mixed-integer MPC with zone control that ensures optimal root water uptake while minimizing irrigation costs and total water consumption.
(3) A heuristic method, using a sigmoid function, that simplifies the mixed-integer MPC in order to reduce its computation time.

2. PRELIMINARIES

2.1 Agro-hydrological system

In this paper, we consider an agro-hydrological system that details the movement of water between crops, the soil, and the atmosphere. Fig. 1 provides a simple illustration of an agro-hydrological system. The transport of water in soil can be modeled using the Richards equation. The Richards equation can be expressed in capillary pressure head form as:

$$c(\psi)\frac{\partial \psi}{\partial t} = \nabla.\left(K(\psi)\nabla\left(\psi + z\right)\right) - S(\psi, z) \tag{1}$$

In Eq. (1), ψ is the capillary pressure head (m), which describes the status of water in soil, t represents time, z is the spatial coordinate, $K(\psi)$ is the unsaturated

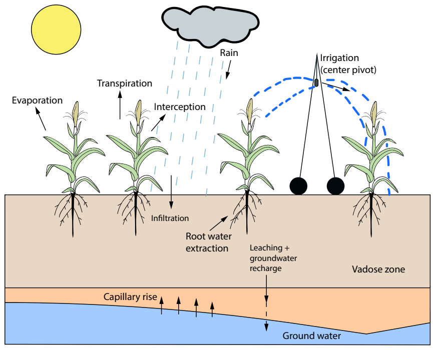


Fig. 1. An agro-hydrological system.

hydraulic water conductivity (ms^{-1}), $c(\psi)$ is the capillary capacity (m^{-1}). $K(\psi)$ and $c(\psi)$ are parameterized by models of Maulem (Maulem (1976)) and van Genuchten (Van Genuchten (1980)). $S(\psi, z)$ denotes the sink term ($\mathrm{m}^3\mathrm{m}^{-3}\mathrm{s}^{-1}$) and it is expressed as:

$$S(\psi, z) = \alpha(\psi)\mathcal{R}\left(K^c, ET^0, z_r\right) \tag{2}$$

$\alpha(\psi)[-]$ is a dimensionless stress water factor, $\mathcal{R}(\cdot)$ is the root water uptake model which is a function of the crop coefficient $K^c[-]$, the reference evapotranspiration $ET^0[\mathrm{LT}^{-1}]$, and the rooting depth $z_r[\mathrm{L}]$.

3. PROPOSED APPROACH

3.1 Development of the LSTM model

A. Data generation

In this work, we focus on infiltration processes in agro-hydrological systems. Infiltration is often assumed to be a one-dimensional (1D) process in the vertical direction (Farthing and Ogden (2017)); thus, the 1D version of Eq. 1 is used in this work. The 1D Richards equation is solved numerically using the method of lines approach. The central difference scheme is used to approximate the spatial derivative and implicit schemes, specifically the Backward Differentiation Formulas (BDFs), are used to approximate the time derivative. The discretized 1D Richards equation is solved for the following initial and boundary conditions:

$$\psi(t = 0) = \psi^{\mathrm{init}} \tag{3}$$

$$\left.\frac{\partial(\psi + z)}{\partial z}\right|_{z=H_z} = 1 \tag{4}$$

$$\left.\frac{\partial \psi}{\partial z}\right|_{z=0} = -1 - \frac{u^{\mathrm{irrig}}}{K(\psi)} \tag{5}$$

where H_z and u^{irrig} (ms^{-1}) in Eqs. (4) and (5) represent the depth of the soil column and the irrigation rate, respectively. The depth dependent root water uptake model proposed by (Feddes et al. (1993)) is used as the sink term in this work. The discretized 1D Richards equation together with the initial and boundary conditions is expressed in state space form as:

$$x_{k+1} = \mathcal{F}(x_k, u_k) + \omega_k \tag{6}$$

where $x_k \in \mathbb{R}^{N_x}$ represents the state vector containing N_x capillary pressure head values for the corresponding spatial nodes. u_k represents the input vector containing the irrigation amount, precipitation, daily reference evapotranspiration, and the crop coefficient. ω_k is the model disturbance.

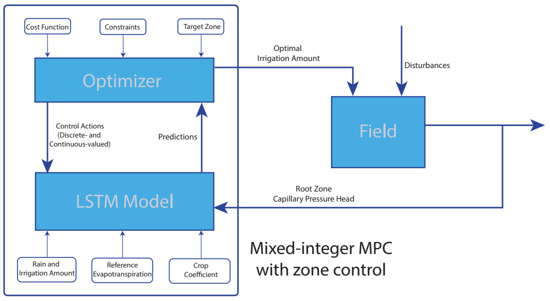


Fig. 2. A block diagram of the proposed irrigation scheduler.

Extensive open-loop simulations are conducted to generate a dataset that captures the soil water content dynamics for the state x and the inputs u. Using randomly generated initial states x_0, Eq. (6) is solved for randomly generated inputs in order to obtain a large number of state trajectories. In order to ensure a small temporal truncation error, the open-loop simulations are performed with a small time step size. Model uncertainty is included in the open-loop simulations to improve the generalization ability and robustness of the LSTM model. Finally, the time-series data obtained from the open-loop simulations are partitioned into training, validation, and test datasets.

B. Proposed LSTM model of an agro-hydrological system

For irrigation scheduling purposes, it suffices to focus on the soil moisture dynamics in the root zone. Thus, we propose a multiple input, single output LSTM model that predicts the root zone capillary pressure head in an agro-hydrological system. Specifically, the LSTM is trained to predict the the one-day-ahead root zone capillary pressure head x_{t+1} using the present and the past l root zone capillary pressure head $x(t = 0, ..., l)$, irrigation amount $u^{\text{irrig}}(t = 0, ..., l)$, rain $r(t = 0, ..., l)$, crop coefficient $K^c(t = 0, ..., l)$, and reference evapotranspiration $ET^0(t = 0, ..., l)$ inputs. In order to realize the proposed LSTM model of the soil-water-atmosphere system, the states outside the root zone are discarded from the datasets and the resulting datasets are resampled to a time frame of 1 day. The time lag l used in the model development is determined through experimentation. In addition to approximating the complex 1D Richards equation, the proposed LSTM model can also be thought of as a reduced model since it has fewer states compared to Eq. (6).

Prior to training the LSTM model, the datasets are normalized to rescale the input and output variables. The LSTM model is trained with the Keras Deep Learning Library in Python. The optimal number of layers and LSTM units are determined through experimentation. During the training process, an optimization problem which minimizes the modeling error is solved using an adaptive moment estimation algorithm (i.e. Adam in Keras).

3.2 Scheduler design - Mixed-integer MPC

The proposed scheduler, depicted in Fig. 2, is designed in the mixed-integer MPC with zone control framework. The scheduler considers a prediction horizon of up to a few weeks and its primary objective is to ensure optimal water uptake in crops while minimizing the total water consumption and the irrigation cost. In this design, the scheduler ensures optimal water uptake in crops by maintaining the root zone capillary pressure head within a target zone. The integer (binary) variable embedded in this design encodes the daily discrete (yes/no) irrigation decision. Using past weather data, daily weather forecast, the root zone capillary pressure head measurement, and the identified LSTM model, the scheduler prescribes the daily discrete irrigation decision and the daily irrigation amount that achieve its primary objective. Additionally, the soft constraint approach is used to realize the zone control in this design. In this approach, slack variables are introduced in the formulation to relax the limits (bounds) of the target zone. At the same time, the slack variables are included in the objective function that is to be minimized. For day d and a fixed prediction horizon of N, the scheduler $\mathbb{P}_{\text{MINLP}}(x)$ is formulated as:

$$\min_{\boldsymbol{x},\ \bar{\boldsymbol{\epsilon}},\ \underline{\boldsymbol{\epsilon}},\ \boldsymbol{u}^{\text{irrig}},\ \boldsymbol{c}} \quad \sum_{k=d+1}^{d+N} \left[\bar{Q}\bar{\epsilon}_k^2 + \underline{Q}\underline{\epsilon}_k^2\right] + \sum_{k=d}^{d+N-1} R_c c_k + \sum_{k=d}^{d+N-1} R_u u_k^{\text{irrig}} \tag{7a}$$

subject to

$$x_{k+1} = \mathcal{F}_{\text{LSTM}}(\{\gamma\}_{k-l}^{k}) \qquad k \in [d, d+N-1] \tag{7b}$$
$$x_d = x(d) \tag{7c}$$
$$\underline{\nu} - \underline{\epsilon}_k \le x_k \le \bar{\nu} + \bar{\epsilon}_k, \qquad k \in [d+1, d+N] \tag{7d}$$
$$c_k \underline{u}^{\text{irrig}} \le u_k^{\text{irrig}} \le c_k \bar{u}^{\text{irrig}}, \qquad k \in [d, d+N-1] \tag{7e}$$
$$c_k = \{0, 1\}, \qquad k \in [d, d+N-1] \tag{7f}$$
$$\underline{\epsilon}_k \ge 0, \quad \bar{\epsilon}_k \ge 0, \qquad k \in [d+1, d+N] \tag{7g}$$

where $k \in \mathbb{Z}^+$, $\boldsymbol{x} := [x_d, x_{d+1}, ..., x_{d+N}]$, $\bar{\boldsymbol{\epsilon}} := [\bar{\epsilon}_{d+1}, \bar{\epsilon}_{d+2}, ..., \bar{\epsilon}_{d+N}]$, $\underline{\boldsymbol{\epsilon}} := [\underline{\epsilon}_{d+1}, \underline{\epsilon}_{d+2}, ..., \underline{\epsilon}_{d+N}]$, $\boldsymbol{c} := [c_d, c_{d+1}, ..., c_{d+N-1}]$, $\boldsymbol{u}^{\text{irrig}} := [u_d^{\text{irrig}}, u_{d+1}^{\text{irrig}}, ..., u_{d+N-1}^{\text{irrig}}]$, and $\{\gamma\}_{k-l}^{k} := [\gamma_{k-l}, \gamma_{k-l-1}, \gamma_{k-l-2}, .., \gamma_k]$ where $\gamma \in [x, K^c, ET^0, u^{\text{irrig}}]$. $\underline{\epsilon}_k$ and $\bar{\epsilon}_k$ in Eqs. (7a) and (7g) are nonnegative slack variables that are introduced to relax the target zone ($\underline{\nu}_k$, $\bar{\nu}_k$) in Eq. (7d). $\underline{Q}$ and $\bar{Q}$ are the per-unit costs associated with the violation of the lower and upper bounds of the target zone, respectively. R_c is the fixed cost associated with the operation of the irrigation implementing system, and R_u is the per-unit cost of the irrigation amount u^{irrig}. The binary variable (c) encodes the daily discrete irrigation decision. The cost function, Eq. (7a), incorporates the objectives of maintaining the root zone capillary pressure head in a target zone in order to ensure optimal water uptake in crops by minimizing the violation of the target zone $\sum_{k=d+1}^{d+N}\left[\bar{Q}\bar{\epsilon}_k^2 + \underline{Q}\underline{\epsilon}_k^2\right]$, minimizing the irrigation cost $\sum_{k=d}^{d+N-1} R_c c_k$, and minimizing the irrigation amount $\sum_{k=d}^{d+N-1} R_u u_k^{\text{irrig}}$. Eq. (7b) corresponds to the LSTM model of the root zone capillary pressure head. The initial state is assumed to be measured and it is represented with Eq. (7c). Eq. (7e) is the amount of water that can be supplied during the irrigation event on day k. When the irrigation decision on day k is "a no decision" ($c_k = 0$), Eq. (7e) specifies that the irrigation amount must necessarily be 0. On the other hand, when the irrigation decision on a particular day is "a yes decision" ($c_k = 1$), this constraint states that the prescribed irrigation amount must be at least equal to the lower bound on the irrigation rate ($\underline{u}^{\text{irrig}}$) and must be no larger than the upper bound on the irrigation rate ($\bar{u}^{\text{irrig}}$). The solution to $\mathbb{P}_{\text{MINLP}}(x)$ is a sequence of predicted states $\boldsymbol{x}$, optimal slack variables ($\bar{\boldsymbol{\epsilon}}$,

$\underline{\boldsymbol{\epsilon}}$), optimal irrigation decisions ($\boldsymbol{c}$), and optimal irrigation amounts ($\boldsymbol{u}$).

3.3 Heuristic methods - Sigmoid function

It is desirable to develop modifications to mixed-integer problems so as to ensure that they can be executed in real-time. This is necessary because mixed-integer programming belongs to the class of $\mathcal{NP}-$complete problems and thus, can require extensive computation time and resources for problems with many integer variables. To this end, a heuristic method that approximates the binary variable in the mixed-integer formulation with a sigmoid function, which was originally applied to transmission expansion problems (Mazzini et al. (2018)) and active power losses minimization problems (Olivera et al. (2005)), is employed. Specifically, the binary variable in $\mathbb{P}_{\mathrm{MINLP}}(x)$ is replaced with a sigmoid function $\omega(r)$ which is defined as:

$$\omega(r) = \frac{1}{1+e^{-\beta r}} \tag{8}$$

where β is the slope of the sigmoid function and the argument r is a real number. The inclusion of Eq. (8) in $\mathbb{P}_{\mathrm{MINLP}}(x)$ results in the modified problem $\mathbb{P}_{\mathrm{SIG}}(x)$:

$$\min_{\boldsymbol{x},\, \bar{\boldsymbol{\epsilon}},\, \underline{\boldsymbol{\epsilon}},\, \boldsymbol{u}^{\mathrm{irrig}},\, \boldsymbol{r}} \sum_{k=d+1}^{d+N} \left[\bar{Q}\bar{\epsilon}_k^2 + \underline{Q}\underline{\epsilon}_k^2\right] + \sum_{k=d}^{d+N-1} R_c\omega(r_k) + \sum_{k=d}^{d+N-1} R_u u_k^{\mathrm{irrig}} \tag{9a}$$

subject to

$$\begin{aligned}
&x_{k+1} = \mathcal{F}_{\mathrm{LSTM}}(\{\gamma\}_{k-l}^{k}) && k \in [d, d+N-1] && \text{(9b)}\\
&x_d = x(d) && && \text{(9c)}\\
&\underline{\nu} - \underline{\epsilon}_k \le x_k \le \bar{\nu} + \bar{\epsilon}_k, && k \in [d+1, d+N] && \text{(9d)}\\
&\omega(r_k)\underline{u}^{\mathrm{irrig}} \le u_k^{\mathrm{irrig}} \le \omega(r_k)\bar{u}^{\mathrm{irrig}}, && k \in [d, d+N-1] && \text{(9e)}\\
&r_{\min} \le r_k \le r_{\max}, && k \in [d, d+N-1] && \text{(9f)}\\
&\underline{\epsilon}_k \ge 0, \quad \bar{\epsilon}_k \ge 0, && k \in [d+1, d+N] && \text{(9g)}
\end{aligned}$$

where $\boldsymbol{r} := [r_d, r_{d+1}, ..., r_{d+N-1}]$. The modified problem is an nonlinear program (NLP) and can thus be solved with a suitable NLP algorithm.

A. Selection of β

$\omega(r)$ converges to binary elements for higher values of its slope β. However, the use of very large β values often results in an ill-conditioned optimization. To handle this issue, an algorithm is proposed to improve the convergence of the sigmoid function to binary elements while reducing ill-conditioning issues. This process involves successively solving $\mathbb{P}_{\mathrm{SIG}}(x)$ for increasing values of β (by a factor of τ in Algorithm 1) until a predetermined convergence criterion is met. This predefined criterion, for the i^{th} evaluation of $\mathbb{P}_{\mathrm{SIG}}(x)$, can be mathematically expressed as:

$$\|\omega(\boldsymbol{r}^i) - \boldsymbol{c}\|_2 \le \zeta \tag{10}$$

where $\boldsymbol{c}$ is a vector of binary elements. Each element of $\boldsymbol{c}$ corresponds to the nearest binary value of each element of $\omega(\boldsymbol{r}^i)$. ζ represents the user-defined convergence tolerance. The detailed steps are described in Algorithm 1.

4. ILLUSTRATIVE EXAMPLES

4.1 Predictive capability of the proposed LSTM model

To evaluate the predictive capability of the proposed model framework, we apply the proposed LSTM model framework to a 0.6 m loamy-sand soil column. In this example, the pressure head value at a depth of 0.5 m is chosen to characterize the root zone pressure head in the root zone. The LSTM model is designed to have two hidden layers. Each layer consists of 200 LSTM units and a sequence length of 5 days is used for the training. Consequently, the time lag l associated with the inputs of the LSTM model is 4.

Algorithm 1 Algorithm for approximating $\boldsymbol{c}$ with $\omega(\boldsymbol{r})$

Require: $x(0)$, $\boldsymbol{x}^0$, $\boldsymbol{r}^0$, $(\boldsymbol{u}^{\mathrm{irrig}})^0$, ζ, β^0, τ
  $x_1 \leftarrow x(0)$
  $\boldsymbol{x}_{\mathrm{guess}} \leftarrow \boldsymbol{x}^0$
  $\boldsymbol{r}_{\mathrm{guess}} \leftarrow \boldsymbol{r}^0$
  $(\boldsymbol{u}^{\mathrm{irrig}})_{\mathrm{guess}} \leftarrow (\boldsymbol{u}^{\mathrm{irrig}})^0$
  $\beta \leftarrow \beta^0$
  $i \leftarrow 0$
  while $\|\omega(\boldsymbol{r}^i) - \mathbf{c}\|_2 > \zeta$ **do**
  Solve $\mathbb{P}_{\mathrm{SIG}}(x)$ for β^i
    $\boldsymbol{x}_{\mathrm{guess}} \leftarrow \boldsymbol{x}^i$
    $\boldsymbol{r}_{\mathrm{guess}} \leftarrow \boldsymbol{r}^i$
    $(\boldsymbol{u}^{\mathrm{irrig}})_{\mathrm{guess}} \leftarrow (\boldsymbol{u}^{\mathrm{irrig}})^i$
    $\beta^{i+1} \leftarrow \tau\beta^i$
    $i \leftarrow i+1$
  end while

In Fig. 3(**A**), the one-day-ahead predictions obtained from the LSTM model are compared with the actual pressure head values in the test dataset. It is evident from Fig. 3(**A**) that the identified LSTM model is able to accurately model the root zone pressure head while capturing its general trend. In model predictive algorithms, it is required that at any given time, the process outputs be predicted many time-steps into the future. To this end, the identified LSTM model is used to predict the root zone pressure head for long periods of time (at least up to the prediction horizon of the predictive controller). Particularly, these multistep-ahead predictions are produced recursively by iterating the identified one-step-ahead LSTM model in which previously predicted pressure head values are used as inputs in successive predictions. From Figure 3(**B**), it can be seen that the recursive use of the identified LSTM model produces accurate pressure head predictions and the predictive performance is comparable to that of the 1D Richards equation.

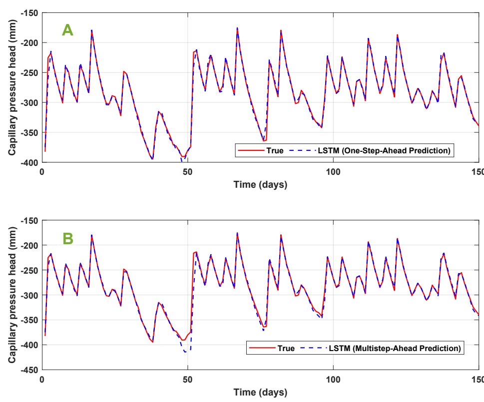


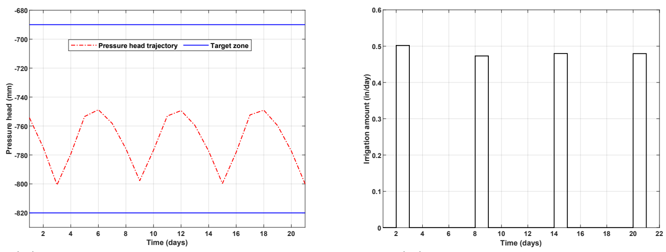
Fig. 3. Actual root zone pressure head (red solid line) and the predicted root zone pressure head (blue dash-dot) using the test dataset, (**A**) One-day-ahead prediction, and (**B**) Multistep-ahead prediction.

4.2 Utility of the proposed scheduler

In this section, the scheduler designs $\mathbb{P}_{\text{MINLP}}(x)$ and $\mathbb{P}_{\text{SIG}}(x)$ are used to prescribe irrigation schedules for a uniform field composed of loamy soil. In this simulation experiment, an LSTM model is first identified for a 0.6 m loamy soil column using the method outlined in section 3.1. The lower and upper bounds of the target zone are chosen as -820 mm and -690 mm, respectively and these values are chosen to lie within the field capacity and the permanent wilting point of loamy soil. The per unit costs associated with the violation of these zones are chosen as $\bar{Q} = \underline{Q} = 9000$. R_c and R_u are chosen as 50 and 20, respectively. The scheduler is evaluated for initial state of -795 mm and past root zone capillary pressure head values of [-748 mm, -735 mm, -740 mm, -754 mm] are used in this simulation. A prediction horizon of 14 days is used and the daily reference evapotranspiration values for the simulation are generated randomly between 2.0 mm and 2.7 mm. A constant crop coefficient value of 0.5 is used in the simulation. The mixed-integer and NLP problems arising from $\mathbb{P}_{\text{MINLP}}(x)$ and $\mathbb{P}_{\text{SIG}}(x)$ are solved using the BONMIN and the IPOPT solvers, respectively.

We consider a closed-loop implementation of the scheduler, specifically the receding horizon control (RHC) implementation. This closed-loop implementation is known to provide some degree of inherent robustness to multiplicative (imperfect knowledge of the model) and additive uncertainties. In RHC, only the first control input of the optimal control sequence is implemented, and, to incorporate feedback into this control strategy, the process is repeated at the next time instant using newly obtained information of the state. In this paper, the scheduler is evaluated each day for a period of 20 days.

A. Simulation results - $\mathbb{P}_{\text{MINLP}}(x)$



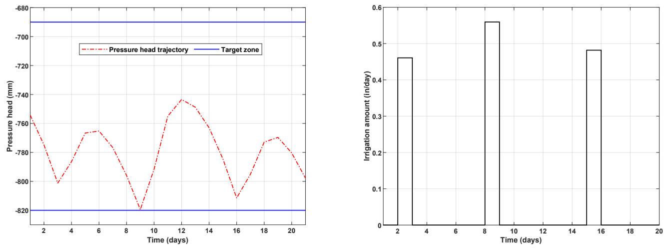
(a) Capillary pressure head. (b) Irrigation amount.

Fig. 4. Closed-loop trajectories under $\mathbb{P}_{\text{MINLP}}(x)$.

Figs. 4(a) and 4(b) show the closed-loop irrigation amount and root zone pressure head trajectories under $\mathbb{P}_{\text{MINLP}}(x)$. From these figures, it is evident that the scheduler is able to maintain the root zone pressure head in the target zone by prescribing irrigation amounts of 0.5 inches/day, 0.47 inches/day, 0.48 inches/day, and 0.48 inches/day on days 3, 8, 15, and 20 of the simulation period.

B. Simulation results - $\mathbb{P}_{\text{SIG}}(x)$

This simulation study revealed that a slope of 25 was able to provide a good convergence of the sigmoid function to the binary elements while preventing ill-conditioning issues. The closed-loop results summarized in Figs. 5(a) & 5(b) reveal that by prescribing irrigation amounts of 0.46 inches/day, 0.55 inches/day, and 0.48 inches/day on days 3, 8, and 15, the scheduler design involving the sigmoid function is able to maintain the root zone pressure head in the target zone.



(a) Capillary pressure head. (b) Irrigation amount.

Fig. 5. Closed-loop trajectories under $\mathbb{P}_{\text{SIG}}(x)$.

C. Comparison between $\mathbb{P}_{\text{MINLP}}(x)$ *and* $\mathbb{P}_{\text{SIG}}(x)$

A visual comparison of the results summarized in Figs. 4 and 5 reveals that, there exist noticeable similarities between the closed-loop irrigation and root zone pressure head trajectories obtained under $\mathbb{P}_{\text{MINLP}}(x)$ and $\mathbb{P}_{\text{SIG}}(x)$. Additionally, it is evident from Table 1 that the computation speed of the proposed scheduler can be remarkably enhanced when the binary variable in $\mathbb{P}_{\text{MINLP}}(x)$ is approximated with a sigmoid function. It should be noted in instances where the sigmoid function is unable to approximate the binary elements correctly, the value of $\sum_{k=0}^{N-1} R\omega(r_k) + \sum_{k=0}^{N-1} R_u u_k^{\text{irrig}}$ in the cost function of $\mathbb{P}_{\text{SIG}}(x)$ may be smaller than that of $\sum_{k=0}^{N-1} Rc_k + \sum_{k=0}^{N-1} R_u u_k^{\text{irrig}}$ in $\mathbb{P}_{\text{MINLP}}(x)$. This will result in a smaller overall cost for formulation $\mathbb{P}_{\text{SIG}}(x)$ compared to the mixed integer formulation $\mathbb{P}_{\text{MINLP}}(x)$. Thus, a smaller overall cost in $\mathbb{P}_{\text{SIG}}(x)$ compared to $\mathbb{P}_{\text{MINLP}}(x)$ should be construed as a failure of the sigmoid function to adequately approximate the binary elements and not the ability of the heuristic approach to provide the best possible solution (in the sense of a minimization optimization problem).

5. SUMMARY AND CONCLUSIONS

In this study, an LSTM-based mixed-integer MPC with zone control for irrigation scheduling was proposed and tested. The proposed scheduling framework seeks to ensure optimal water uptake in crops while minimizing total water consumption and irrigation costs. To this end, an LSTM model was developed to describe the dynamics of the root zone capillary pressure head. This data-driven machine learning model was trained using open-loop simulated data from the Richards equation. A mixed-integer MPC with zone control was then developed using the identified LSTM model. A heuristic method using the sigmoid function was proposed to simplify the mixed-integer MPC in order to reduce the evaluation time of the scheduler. The simulation results obtained in the illustrative examples revealed that, the LSTM model was capable of performing accurate single-step and multi-step predictions of the root zone capillary pressure head. Furthermore, the closed-loop trajectories obtained in the illustrative example highlight the efficacy of the proposed scheduler, as it was able to prescribe irrigation schedules that are typical of irrigation practice. The proposed approach can thus be successfully

Table 1. Simulation metrics for formulations $\mathbb{P}_{\text{MINLP}}(x)$ and $\mathbb{P}_{\text{SIG}}(x)$.

Formulation	$\mathbb{P}_{\text{MINLP}}(x)$	$\mathbb{P}_{\text{SIG}}(x)$
Computation Time (hours)	10.5	2.0
Cost	213.4	176.6

used to maximize crop yield while minimizing the total water consumption and irrigation costs. The heuristic method involving the sigmoid function was capable of enhancing the computational efficiency of the scheduler and this underscores the capability of the proposed approach to prescribe optimal or near-optimal irrigation schedules within workable computational budgets.

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