Stochastic Optimization Model for Short-term Planning of Tanker Water Supply Systems in Urban Areas

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Abstract: Tanker-based water distribution systems are amongst one of the most prevalent methods to supply water in many developing countries facing water crisis and intermittent piped water supply. These tanker water supply systems need tighter coordination between the water sources, treatment facilities, consumers and tanker suppliers to efficiently manage timely delivery and quality water. Furthermore, accounting for the uncertain nature of customer demands while making the planning decisions would not only aid in achieving minimum operational cost but is also important in reducing the water wastage. This paper proposes a two-stage stochastic recourse programming model for an optimal planning and scheduling of tanker water supply system under daily demand uncertainty. The main objective is to supply water to maximum number of consumers with minimum total operating costs. A solution strategy combining Sample Average Approximation (SAA) and Monte-Carlo Simulation (MCS) methods, to generate an equivalent deterministic MILP (mixed integer programming problem) model with multiple scenarios of demand uncertainty realization, is adopted for problem solving. The proposed model is applied to an example tanker water supply system and the benefits of two-stage stochastic modelling in making agile decisions incorporating the effect of uncertainties are illustrated. The results also demonstrate the efficacy of adopting stochastic programming models and methods in such real-world application cases.

Keywords: Uncertainty, Urban water management, Stochastic mixed integer programming, Water tankers, Two stage recourse model.

1. INTRODUCTION

Increasing the universal access to safe drinking water and water resources is an imperative central to Sustainable Development (UNDESA, 2018). In the current situation of water crisis and lack of adequate piped distribution network infrastructure in many countries, the dependency on tanker trucks for water supply is increasing (WWAP, 2019). These tanker trucks collect water from several water sources (well, lakes, rivers, spring etc.) and after necessary treatments, supply to consumers, spanned in different locations across the city. The consumers demand water from these tankers either for their daily needs (viz. hygiene, drinking, cooking, cloth washing etc.), commercial purposes (hotels, offices, etc.) or institutions requiring high quality water (hospitals, schools etc.) (USAID, 2016). The tankers must be well equipped and maintained to avoid any (re)contamination events and the water source must be selected optimally to supply appropriate quality water at minimum cost. Such tanker-based water distribution systems are also part of municipal water supply boards to meet city water requirements, especially in urban and peri-urban areas (DJB, 2019). Furthermore, there are several uncertainties related to the water availability, consumer demands, travel time for transportation etc. that significantly affects the operations of water supply through tanker systems. However, there are very few studies in the literature that have considered research works on tanker-based water supply system (Constantine et al., 2017). Most of these studies emphasized on inadequate services and informal settlements of such tanker water supply in urban areas (Srinivasan et al., 2010; Ayalew et al., 2014; Venkatachalam, 2015; Zozmann et al., 2019) and relatively few on operational management (Maheshwari et al., 2020). Nonetheless, challenges specific to coordinated operations of water suppliers, water treatment plants, and consumers to use available water resources in a sustainable manner, along with the assurance of water quality and timely delivery through tanker water supply systems in uncertain conditions are critical to make a positive impact on enhancing universal water access and have not been dealt yet in the literature.

Incorporation of various forms of uncertainties into the tanker water supply system decision-making process will help to make better decisions in realizing the overall objective of delivering water to a maximum number of consumers while honoring all constraints related to the water demand, treatment operations, and environmental and social aspects in the system. Appropriately including these operational uncertainties at the long-term and short-term decision-making levels within the supply system can result in better planning and significant improvement of the expected operating costs (Li and Grossmann, 2021). A previous study on short-term planning and optimization of tanker water systems have focused on deterministic problem to find tanker delivery schedule using mixed integer linear programming (MILP) method (Maheshwari et al., 2020). However, uncertainties related to water demand, availability, treatment operation and traffic congestions are not in the scope of their deterministic tanker supply system planning model. Tanker movement and treatment plant operation decisions based on such deterministic assumptions will result in sub-optimal or even impractical solutions if uncertainties surface at the later stages during tanker water delivery operations. Therefore, a stochastic optimization model is extremely crucial to incorporate the effect of uncertainties to represent the real-world nuances and challenges of a tanker water supply system.

In this direction, this paper presents a stochastic optimization formulation approach for the operational planning of tanker water supply system considering demands uncertainty using two-stage stochastic programming with recourse method. While deterministic approaches have been proposed earlier for these problems (Maheshwari et al., 2020), accommodating uncertainty in the optimization formulation could be expected to meet the demands under uncertainty in an improved manner. In order to achieve tractability, the stochastic optimization problem formulation and solution approach using Monte-Carlo Simulation based Sample Average Approximation (SAA) technique (Shapiro et al., 2014) is demonstrated in this paper on an example case study, and future research directions are discussed in the conclusion section.

2. PROBLEM DESCRIPTION

A typical tanker water supply system consists of water source sites, water treatment facilities, and demand zones (Abdul et al., 2007). The raw water is pumped into the tankers from the water sources (ground water bore holes, rivers, lakes, springs) and treated depending on the intended purpose of demanding consumers. The water from fresh water sources such as rivers for daily domestic needs viz. drinking, cooking, cleaning etc. is usually provided with chlorine-based disinfection treatment on the tanker vehicles itself. However, water taken from ground water sites needs advanced treatment in treatment facilities to bring to the required quality levels (WWAP, 2022; Krishnakumar, 2019). Accordingly, three types of water states (based on water quality of intended purpose) are considered in this problem, viz. (i) raw water from the ground water sources (RW), (ii) chlorine based disinfected water for domestic purposes (DPW), and (iii) advanced treatment based ultra-pure drinking water (UPDW). However, the optimization formulation is generic and flexible enough to include any arbitrary number of such water quality states in the problem. Thus, as shown in Figure 1, tanker movements can be divided in four sections of integrated supply system – (i) supplying untreated RW from ground water sources to treatment facilities, (ii) direct supply of DPW to consumers from fresh water sources after allowing suitable time period of disinfection on tanker vehicle itself, (iii) supply of treated water products (both DPW and UPDW) from treatment facilities to consumers and (iv) empty tanker return to tanker depots in the nearest region after supplying water in the destined consumer locations. Moreover, there exists different tanker types and capacities to transport water from sources to treatment facilities and consumers depending on the constraints of geographical locations, demands, availability and transportation cost. Thus, depending on the water demands (volume and quality), different tankers having different capacities and material of construction need to be selected. In addition, uncertain conditions considerably impact all the components of supply system operations and consumer demand fulfillments. Hence, a stochastic optimization approach based integrated model, that promotes tighter coordination between water sources, treatment facilities, consumers and tanker suppliers, is necessary to facilitate efficient planning and management of water supply through tanker-based distribution systems. In this direction, the main objective of this problem is to address following challenges peculiar to stochastic modelling of tanker water supply system operations:

(i) rigorous representation of the uncertain water consumption behavior of consumers in a tractable optimization problem

(ii) ensuring adequate water treatment (based on water sources) before supplying water to the consumers

(iii) incorporation of transit time and time delays in travelling of tankers from one location to another while scheduling tanker supply to consumers

(iv) optimally matching tanker availability and suitability to various demand types and volumes

Thus, the problem characteristics considered in this paper can be stated as follows:

Known parameters:

- Location of water sources and treatment facilities
- Cost parameters for tanker transportation, tanker availability, average vehicle speed and travel distance between sources and consumers
- The related parameters of treatment facilities (i.e., throughput capacity, yield of treatment process, raw and treated water reservoir capacity etc.)

Uncertainty:

- Demand for different water quality products in different regions

Decisions:

- The raw water supply plan from water sources to treatment facilities
- Treated water supply plan (volume and tanker type) from sources to consumers
- Schedule for operations of Treatment facilities
- Tankers movement schedule across all four transport sections of supply system
- Extra tankers to be hired at the start of every planning horizon to respond demand uncertainties

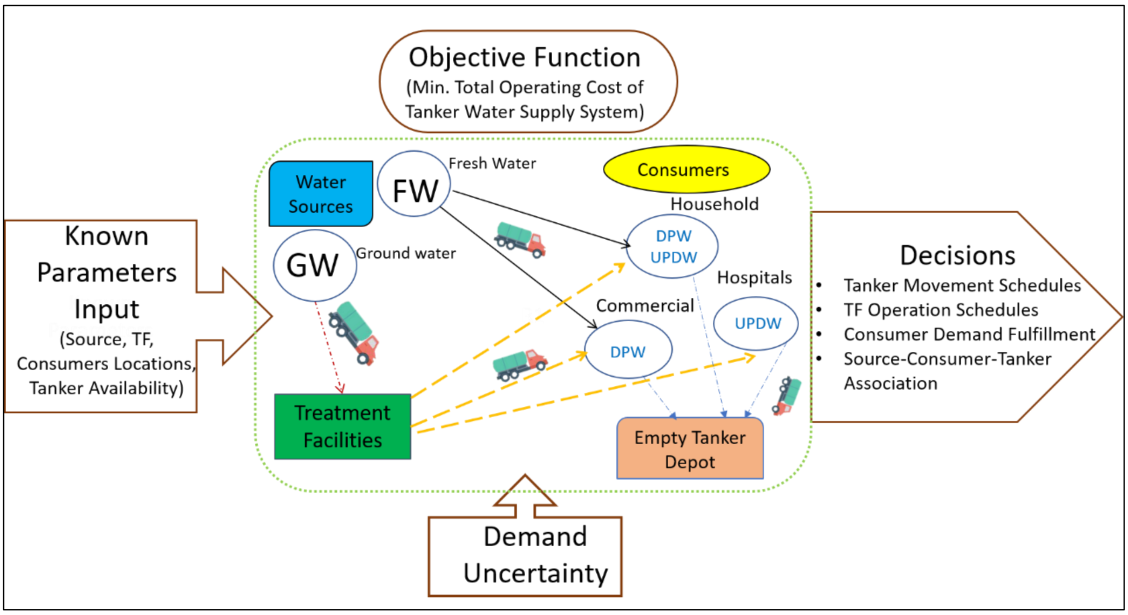

Figure 1: Schematic of Tanker Water Supply System Planning

Therefore, the problem can be considered as delivery of water product of type $p(p \in \mathbf{P})$ from a combination of sources $s(s \in \mathbf{S})$ and treatment facilities $s'(s' \in \mathbf{TF} \subset \mathbf{S})$ in a suitable tanker type $v(v \in \mathbf{V})$ to the set of consumers $c(c \in \mathbf{C})$ under a set of uncertain demand scenarios $k(k \in \mathbf{K})$ with probability π_k at minimum operating costs. The number of samples for MCS, are captured through the parameter *N*.

3. OPTIMIZATION PROBLEM FORMULATION

This paper employs the two-stage stochastic linear programming with recourse approach (Shapiro et al., 2014; Zhoua et al., 2019) to incorporate the demand uncertainties in the optimization formulation. This approach classifies decision variables in two stages: first stage is related to the "here and now" decisions, (i.e. which must be made before the realization of uncertainties), while the second-stage variables captures the "wait and see" type decisions, (i.e. those could be made only after the realization of uncertainties). In the proposed model, the decisions regarding supply quantity (viz. the quantity of raw water supplied from ground water sources to treatment facilities, and the products quantities that are sent to each consumer set), and the decisions regarding the total required distribution capacity (viz. number of tankers required to be hired/purchased at the start of the time horizon to fulfill the distribution need) are calculated at the first stage whereas, the recourse actions enforces the operational constraints and calculates the demand shortfall/surplus amount for each uncertainty realization.

The objective of this problem is to minimize the total expected operating cost subjected to uncertainties in demands and various constraints related to mass flow balance, operational, and environmental sustainability limitations. Thus, the objective function (Eq. 1) represents the total operating cost which consists of, i) tanker transportation cost from sources to treatment facilities and consumers, ii) penalties related to raw and treated water inventory violations in treatment facilities reservoirs, iii) extra tanker hiring cost, and iv) shortfall or surplus penalties in meeting demands respectively.

Total Operating Cost = *Tankers transportation cost from sources to consumers* + *Tankers transportation cost from sources to treatment facilities* + *Penalty cost on target violations in treated water reservoirs of TF*+ *Penalty cost on buffer capacity violations in raw water reservoirs of TF* + *Cost of hiring extra tanker vehicles* + *Penalties for demand shortage/surplus* (1)

The constraints that need to be satisfied include the following: (1) water treatment plant operations including raw water and treated water inventory balances, (2) consumer demand fulfillment for all water products through combination of several water sources and treatment facilities, (3) restraining ground water extraction within the maximum limit set by government regulations to prevent exploitation beyond replenishable levels, (4) tanker distribution capacity balance, and (5) overall time capacity balance. A short-term planning problem is formulated in MILP (mixed integer linear programming) framework with binary variables for scheduling treatment facility operations with mode changeover for different water quality states.

3.1 General Structure of Stochastic Optimization Model:

The general structure of the stochastic optimization model developed in this paper is presented as shown in Eq. 2, where,

x_I , x_{II} are first and second stage positive continuous variables vector respectively, y_{II} are scenario dependent binary variable vector for treatment facility operational status, Q is recourse function and ξ_k is the random vector of demand uncertainty for all consumers w.r.t water products. Further details of uncertainty modelling in the formulation are described in the next section.

$$Min \quad f(x_I) + \frac{1}{N}\sum_{k=1}^{K} Q(x_{II}, y_{II}, \xi_k)$$

s.t.

$$h_I(x_I) = 0$$

$$g_I(x_I) \leq 0$$

$$h_{II}(x_I, x_{II}, y_{II}, \xi^k) = 0 \qquad \forall\, k \in \mathrm{K}$$

$$g_{II}(x_I, x_{II}, y_{II}, \xi_k) \leq 0 \quad \forall\, k \in \mathrm{K} \qquad (2)$$

3.2 Demand Uncertainty Modelling

In the commonly used scenario-tree based method for product demand modelling in stochastic programming problems, the problem size increases exponentially with the number of scenarios (Tong et al., 2013). A very large number of scenarios make the problem computationally intractable within a considerable solution time and a very small scenario number lacks true representation of the full scenario space. Furthermore, a key difficulty in solving the above stochastic program problem is in evaluating the expectation in the objective. Therefore, in this paper, Monte Carlo Simulation method with sample average approximation (SAA) technique is adopted to generate finitely reasonable scenarios and approximating the expected cost in the objective function by Law of large numbers for identically independent samples. Thus, the projection of uncertain demands in the developed model is represented through *N* discrete scenarios generated randomly using MCS based on the underlying (known from historical data) normal probability distribution. These scenarios are incorporated in the constraints as shown in Eq. 3. The mean values of the predicted demands on daily basis ($De_{c,p,t}$) in the planning horizon are constructed using the water consumption behavior database of consumers.

$$xDe_{c,p,t,k} = xDe_{c,p,t}\left(1 + \xi_{c,p,t,k}\right) \;\; \forall\, c, p, t, k \qquad (3)$$

The uncertain demand parameter $\xi_{c,p,t,k}$ in Eq 3 are random seeds from the standard normal distribution. The range of normal distribution for generating random seeds for each scenario in MCS samples are set as -0.2 to 0.2. Scenarios and corresponding probabilities are introduced both in the objective function and all relevant second stage constraints.

4. RESULTS & DISCUSSION

This stochastic MILP optimization model developed through an example system where water supply in urban area is divided into two regions (R1, R2). As shown in Fig.1, this system consists of one water source (GW/Surface water (FW)), one treatment facility and three types of consumer clusters in each region, viz. Households (HHC), Commercial (CC) and Hospitals (HC) having different water quality demands (DPW/ UPDW). Furthermore, two types of tankers with different capacity are considered; 6 kL and 10 kL to supply water from sources to treatment facilities and consumers and the total number of available tankers of each type are provided region-wise. The tankers also differ in their materials of construction for transporting raw water and treated water. The water demands of each consumer type for both products are supplied in two delivery slots (morning and evening). The planning horizon is assumed to be of 5 days. The model statistics for the example system are provided in Table 1.

Table 1. Model Statistics for the Example Problem

Parameter	Value
Constraints	159847
Continuous variables	111575
Binary Variables	28800
Solution Time (s)	418
Relative Optimality Gap (%)	0.64%

The proposed model is formulated and solved in the FICO Xpress Optimization (Gueret et al., 2002) environment using the "mmxprs" module version 2.8.1 on a 16 GB RAM machine with an Intel i7 (3.6 GHz) processor. It is to be noted that depending on the urban settlement, there can be several treatment facilities and water sources catering to the city water supply. Therefore, the formulation presented above is flexible for applications involving much larger problems for a satisfactory solution in terms of the achieved optimality gap.

In order to accurately reflect the demand uncertainty, a convergence analysis for objective function values is performed with increasing the sample size in increments of 5 in the MCS runs (Fig.2) and it was observed that the objective function value becomes stable in the range of 1% after N=30. Therefore, 30 Monte Carlo demand scenarios were considered for solving the optimization model.

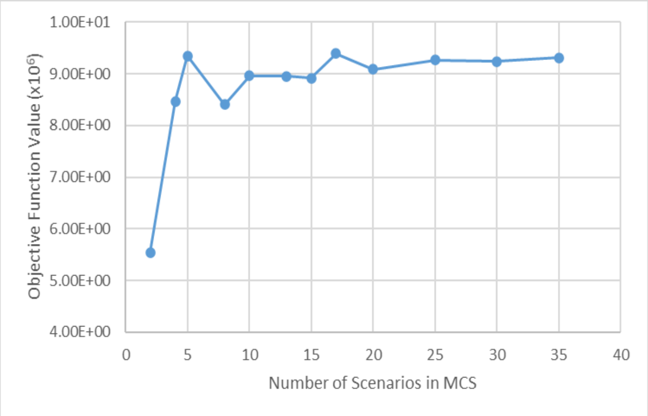

Figure 2: Objective function Convergence analysis for MCS runs

The results of total operating cost, demand shortfall penalty costs and extra tankers hired in the objective function for stochastic problem with recourse (SPR) solution are shown in

Fig. 3. Furthermore, the value of stochastic solution (VSS), i.e. advantage of performing stochastic optimization over deterministic model with mean value is shown using a quantitative metric (Birge and Louveaux, 2011) calculated as shown in Eq. 4.

$$VSS = (SPR - EVS)/EVS \quad (4)$$

Table 2: Comparison of SPR and EVS Solutions

	Stochastic Programming with Recourse Model (SPR)	**Expected Value Solution Model (EVS)**	**Value of Stochastic Solution (VSS)**
Objective Function Value (Total Operating Cost)	9.70E+06	1.11E+07	13 %

Thus, to calculate VSS, we fix the first stage solutions to the deterministic model optimal solutions and compared the model performance by solving each stage two problem separately. An expected total operating cost of 1.11 e+07 is obtained. This is also known as expected value solution (EVS) in the literature. As expected, this EVS is 13% higher than SPR solution for the example case study with the developed stochastic model in this paper. This is because the model in this first stage fixed case is oblivious of the high demand scenario and therefore hires the extra tanker capacity to be just enough to satisfy mean demand values. This led to high penalty costs on demand shortfall scenarios and consequently higher objective function value in EVS solution as also shown in results comparison in Fig. 3.

Also, the total number of extra tankers that were hired in each case, viz, (first stage fixed solution to deterministic mean demands and stochastic solution) is compared in Fig.3. Thus, the stochastic optimal solution suggests to hire approximately four times the number of tankers when compared with the deterministic solution to respond to uncertain events at minimum operating costs. The daily demand satisfaction profile for different consumers in different regions is shown using box and whisker plot (using data of all 30 scenarios) in Fig.4, where “ * ” marks the optimal solution from the SPR model accounting all scenarios and corresponding probabilities. It can be inferred herein that considering all possible realizations of daily uncertain demands, the likelihood of total demand shortfall is near to 25% for household consumers and 50% for commercial consumers in both the regions.

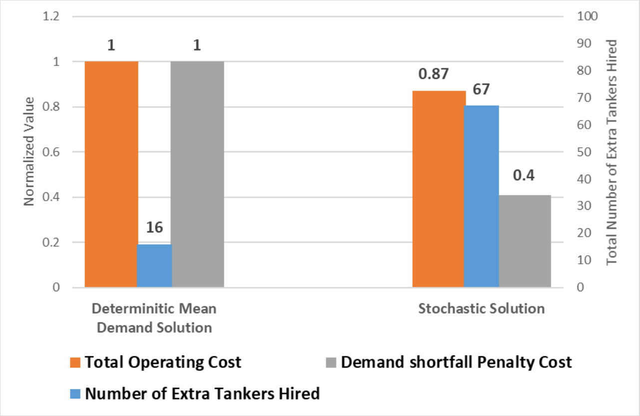

Figure 3: Comparison of stochastic solution (SPR) with expected value solution (EVS)

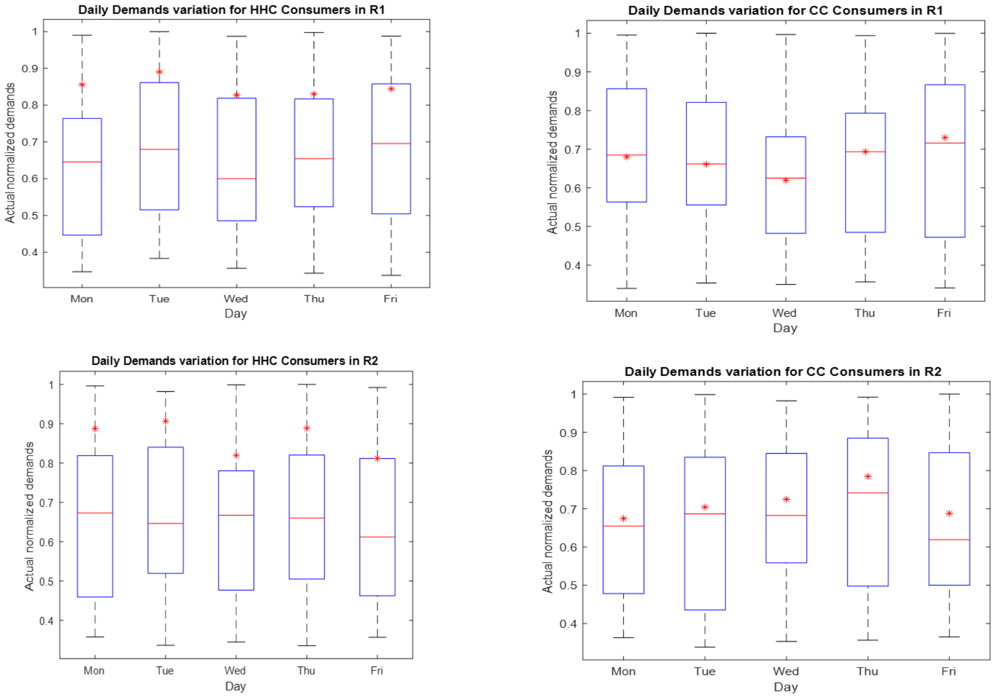

Figure 4: Daily Demand satisfaction profile of consumers in different regions

5. CONCLUSIONS & FUTURE RESEARCH

This paper presents a stochastic optimization framework for efficient planning and operations of tanker water supply systems exhibiting demand uncertainty. A two-stage recourse stochastic MILP model is developed to optimize the short-term planning and scheduling of tanker movements and water treatment facilities in the tanker-based water distribution system in urban areas. Such stochastic model-based optimal planning allows to accommodate the impacts of uncertain water demands on consumer demand fulfillment and tanker water supply (timely delivery) and treatment operations (ensured quality) without increasing the agility of decision making incorporating the effect of unforeseen circumstances. For the future work, the authors aim to extend the model for incorporating travel time uncertainty due to traffic congestions in urban areas. However, including travel time uncertainties will increase the number of scenarios required to capture the true essence of the complexities which might make the problem intractable. Therefore, future research would also direct in finding efficient modelling techniques to represent such uncertainties and solution methods to perform with higher computational efficiency. Further study should also consider uncertainties related to seasonal availability of water in long-term planning framework of tanker water supply system.

REFERENCES

Abdul, S.; Sharma, N.; Water, R. (2007). Consumption patterns in domestic households in major cities. *Econ. Political Wkly*, 42, 2190−2197.

Ayalew, M.; Chenoweth, J.; Malcolm, R.; Mulugetta, Y.; Okotto, L. G.; Pedley, S. (2014). Small Independent Water Providers: Their Position in the Regulatory Framework for the Supply of Water in Kenya and Ethiopia. *J. Environ. Law*, 26, 105−128.

Birge, J. R., and Louveaux, F. (2011). *Introduction to stochastic programming*. Heidelberg, Germany: Springer Science & Business Media.

Constantine, K.; Massoud, M.; Alameddine, I.; El-Fadel, M. (2017). The role of the water tankers market in water stressed semi-arid urban areas: Implications on water quality and economic burden. *J. Environ. Manage*., 188, 85−94.

DJB. (2019). Delhi Jal Board to Outsource Tankers for Water Supply. *The Hindu*. http://delhijalboard.nic.in/content/schedulewatertankers.

Gue′ret, C.; Prins, C.; Sevaux, M.; Heipcke, S (2002). *Applications of Optimization with Xpress-MP*, Dash Optimization Limited.

Li, C. and Grossmann, I.E. (2021). A Review of Stochastic Programming Methods for Optimization of Process Systems Under Uncertainty. *Frontiers in Chemical Engineering*, 2.

Maheshwari, A., Misra, S., Gudi, R.D. and Subbiah, S. (2020). A Short-Term Planning Framework for the Operation of Tanker-Based Water Distribution Systems in Urban Areas. *Industrial & Engineering Chemistry Research*, 59(20), pp.9575–9592.

Shapiro, A., Dentcheva, D., and Ruszczy´nski, A. (2014). *Lectures on stochastic programming: modeling and theory*. Philadelphia, Pennsylvania: SIAM.

Srinivasan, V.; Gorelick, S. M.; Goulder, L. (2010). Factors determining informal tanker water markets in Chennai, India. *Water Int*ernational, 35, 254−269.

Tong, K., Gong, J., Yue, D. and You, F. (2013). Stochastic Programming Approach to Optimal Design and Operations of Integrated Hydrocarbon Biofuel and Petroleum Supply Chains. *ACS Sustainable Chemistry & Engineering*, 2(1), pp.49–61.

UNDESA. (2018). *The Sustainable Development Goals Report*; United Nations Department of Economic and Social Affairs: New York,

USAID. (2016). *Drinking Water Supply for Urban Poor: Role of Small Water Enterprises*, United States Agency for International Development: New Delhi.

Venkatachalam, L. Informal water markets and willingness to pay for water: a case study of the urban poor in Chennai City, India. Int. J. Water Resour. Dev. 2015, 31, 134−145.

WHO. (2017). *Guidelines for drinking-water quality: fourth edition incorporating the first addendum*. World Health Organization, Geneva

WWAP (United Nations World Water Assessment Programme). (2019). *The United Nations World Water Development Report 2019: Leaving No One Behind* UNESCO, Paris.

WWAP (United Nations World Water Assessment Programme). (2022). *The United Nations World Water Development Report 2022: GROUNDWATER: Making the invisible visible* UNESCO, Paris.

Zhou, X., Zhang, H., Qiu, R., Lv, M., Xiang, C., Long, Y. and Liang, Y. (2019). A two-stage stochastic programming model for the optimal planning of a coal-to-liquids supply chain under demand uncertainty. *Journal of Cleaner Production*, 228, pp.10–28.

Zozmann, H.; Klassert, C.; Sigel, K.; Gawel, E.; Klauer, B. (2019). Commercial Tanker Water Demand in Amman, Jordan-A Spatial Simulation Model of Water Consumption Decisions under Intermittent Network Supply. *Water*, 11, No. 254.