# Hybrid Model Predictive Control for Hybrid Electric Vehicle Energy Management Using an Efficient Mixed-Integer Formulation

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**Abstract:** This paper proposes to use hybrid model predictive control (HMPC) for energy management in hybrid electric vehicle (HEV) using an efficient formulation. HEV has two sources of energy - electric motor and internal combustion engine (ICE) - allowing it an additional degree of freedom to optimize the ratio between the use of two energy sources. HEV energy management is crucial to exploit its potential to reduce fuel consumption and emissions. However, it is challenging to achieve the optimal solution along with the fast dynamics of HEVs. In this paper, instead of using a nonlinear, computationally expensive dynamic model of HEV, a piecewise affine (PWA) model is used to depict its behavior, implemented with multiple linear regression. The validation of the developed model is taken with standard drive cycles. Based on the PWA model, an optimal control problem of HMPC was formulated as mixed integer linear programming (MILP). Conventionally, mixed logical dynamical (MLD) system has been used in the process control field, including early studies of HEV control. This paper applies Big-M formulation which is efficient in its problem size and tight inequality relation for integer variables. The performance of the HMPC controller was examined in a simulated environment based on MATLAB/Simulink HEVP2 application. As a result, HMPC shows superior control performance than the equivalent consumption minimization strategy (ECMS).

*Keywords:* Model predictive control; Dynamic modelling and simulation for control and operation; Process control

## 1. INTRODUCTION

Hybrid electric vehicles (HEVs) are widely deemed as the promising technology in future transportation field. Their two different sources of power - an electric motor and an internal combustion engine (ICE) - enable them to offer better fuel economy and fewer emissions. For example, the battery storage is able to capture the braking energy in HEV, at the same time, allowing another degree of freedom to accumulate or deliver energy. To realize such improvements, the optimal energy management in HEV system has been of a great interest in the recent automobile industry. HEV energy management problem looks for the optimal ratio of torque demand assignments to the two energy sources. The problem is a nonlinear, constrained, and dynamic optimization problem, and the performance depends on the employed optimization strategy.

In early studies, Lin et al. (2003) applied numerical solutions based on dynamic programming (DP) and Onori et al. (2010) has simplified the problem using an equivalence assumption between the electrical energy and the fuel energy. The latter approach is named equivalence consumption minimization strategy (ECMS) and has been improved with adaptive parameter correction according to the current condition by Gu and Rizzoni (2006). With the known drive cycles, DP method gives the globally optimal solution, which Sciarretta and Guzzella (2007) proposed as a benchmark for the best achievable fuel economy in HEV. However, the perfect information of the drive cycle is unrealistic. On the other hand, ECMS is a closed-loop controller, but its drawback is the absence of battery level prediction power. To compromise between these two methods, a prominent control approach is model predictive control (MPC) in which the model-based optimization is performed over a moving finite horizon (Borhan et al. (2011)).

In this study, a hybrid MPC (HMPC) is applied to the HEV energy management problem. HMPC is an MPC method for hybrid dynamic systems involving integer or binary variables. Hybrid dynamical models have been used to depict various physical systems consisting of digital controllers with discrete values or switching devices. Also, they are useful to approximate nonlinear models, often as piecewise affine (PWA) systems. This enables HMPC applicable to nonlinear models. Ripaccioli et al. (2009) showed that PWA system and HMPC is able to give a successful control over HEV energy management. In his study, HMPC consumed fuel lesser than a conventional approach, without any knowledge of the future driving cycle.

There are several modeling formulations to express PWA systems in integer programming, including Big-M formulation introduced by Nemhauser and Wolsey (1988). Big-M formulation has been the most common approach to program disjunctive conditions and is improved recently by Vielma (2015) with a key idea of using multiple Big-M parameters. Trespalacios and Grossmann (2015) applied the improved Big-M formulation to mixed integer nonlinear programming (MINLP) problems such as optimal scheduling problem of multi-product batch plant (CMU). On the other hand, mixed logical dynamical (MLD) model has been widely used in process control area, thanks to the automated formulation provided in the language HYSDEL (Hybrid Systems DEscription Language) by Bemporad and Morari (1999). MLD model has been already applied for HEV control by Ripaccioli et al. (2009) with success.

However, Marcucci and Tedrake (2019) claims Big-M formulation to be a more effective way to formulate optimal control problem for PWA systems in its relatively small size and tighter inequality bound. They also showed that Big-M formulation achieves a better and faster solution in illustrative control examples. In HEV system, higher quality of an optimal solution is desirable to increase its drive range. To accomplish this desire, the Big-M formulation of HMPC for HEV energy management problem is proposed in this paper. Its performance was examined in MATLAB/Simulink HEVP2 application and compared to the conventional controller using ECMS.

## 2. MODEL FORMULATION

In HEV energy management problem is schematically depicted in Fig. 1. MPC controller receives the driver's wheel torque demand, $\tau_{whl}$, as an input. The control objective is to satisfy the demand while minimizing fuel consumption and maintaining a safe level of SOC. The controller gives the engine torque command, $\tau_{eng}$, and motor torque command, $\tau_{mot}$, as outputs, letting the engine and motor system implement the commands. For this purpose, the system can be divided into 3 components - 1) a battery model to predict the state of charge, 2) an engine torque computation model to satisfy the drive cycle, and 3) a fuel consumption rate map.

In this section, it is described how these models were reduced to linear models and how accurate they are.

### *2.1 Piecewise Affine Model Identification*

When a model is highly nonlinear, linear models cannot represent its behavior. In such cases, PWA model can be an option. PWA identification is implemented with input-output data samples from the empirical model.

Consider a PWA dynamic model in the form:

$$x_{k+1} = A_i x_k + B_i u_k + c_i \tag{1a}$$

$$\text{where} \quad F_i x_k + G_i u_k \leq h_i, i \in \mathcal{I}, k = 0, ..N-1, \tag{1b}$$

where $k$ is the time step starting from 1 to terminal time $N$, and $x_k$ and $u_k$ stand for the state and control input at time step $k$. The procedure to find the best model is a series of linear regression with different polyhedra covering the overall input space. This results in matrices $A_i, B_i, F_i$, and $G_i$ and vectors $c_i$ and $h_i$ with respect to each mode $i$ in a set of integers $\mathcal{I}$.

In this paper, a PWA identification was implemented based on a max absolute error with regard to the data with more or less than 1% error on average.

The data for PWA identification was generated using empirical models from Onori et al. (2016). As the purpose of PWA identification is to embed it into the MPC optimization problem, every state and control input data were scaled in $[0, 1]$ range.

### *2.2 Battery State of Charge Model*

Battery state of charge (SOC) predicts the future SOC level, which is used in the control objective to maintain a safe SOC level. According to Onori et al. (2016), the SOC dynamics are:

$$\frac{dSOC}{dt} = \begin{cases} -\dfrac{1}{\eta_{coul}} \dfrac{I}{Q_{nom}}, \text{if } I > 0 \\ -\eta_{coul} \dfrac{I}{Q_{nom}}, \text{if } I < 0 \end{cases} \tag{2}$$

$$I = \frac{V_{oc}}{2R_0} - \sqrt{\left(\frac{V_{oc}}{2R_0}\right)^2 - \frac{P}{R_0}}, \tag{3}$$

where $Q_{nom}$ is the nominal charge capacity, $I$ the battery current which is positive during discharge, and $\eta_{coul}$ the Coulombic efficiency known as charge efficiency. In Eq. 3, the electric power, $P$, is a function of the motor speed, $w_{mot}$, and motor torque command, $\tau_{mot}$, while the open circuit voltage, $V_{oc}$, and the resistance $R_0$ depends on SOC.

In this study, we assume that $w_{mot}$ is constant along the prediction horizon, so PWA identification was taken for each case with different $w_{mot}$ and saved offline. This approach was proven in the setting of MPC by a work of Borhan et al. (2011). To apply the saved PWA model, whenever the new measurement was injected to HMPC, the two closest PWA models for the current $w_{mot}$ were used to interpolate.

For PWA identification, the data was produced by the numerical integration of the above model in first-order with a sampling time of 0.01s. The constant parameter

Table 1. Comparison of battery SOC model PWA identification

| Number of polytopes | Absolute error Average | Maximum |
|---|---|---|
| **1** | $\mathbf{1.67 \times 10^{-6}}$ | $\mathbf{8.90 \times 10^{-3}}$ |
| 4 | $1.67 \times 10^{-5}$ | $6.44 \times 10^{-5}$ |
| 9 | $8.56 \times 10^{-6}$ | $3.21 \times 10^{-5}$ |
| 16 | $4.76 \times 10^{-6}$ | $1.71 \times 10^{-5}$ |

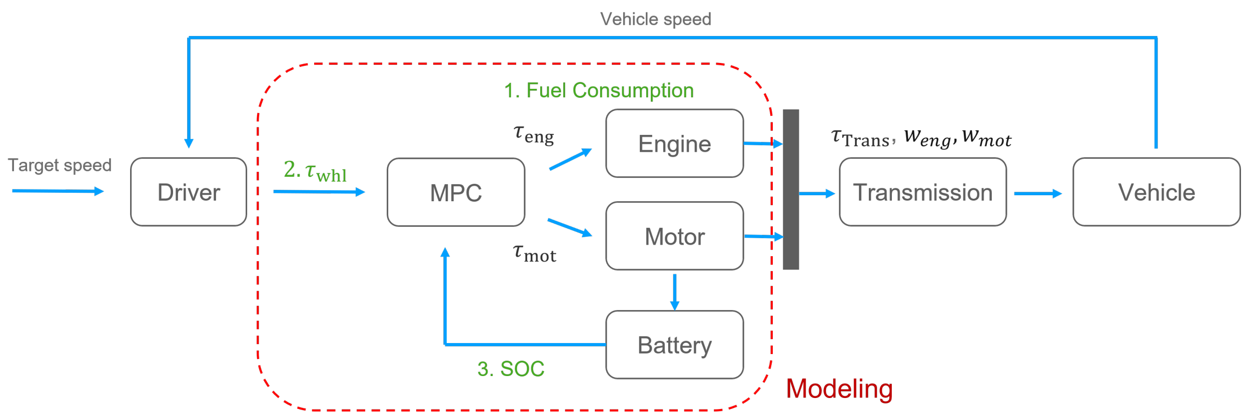


Fig. 1. General scheme of HEV energy management problem

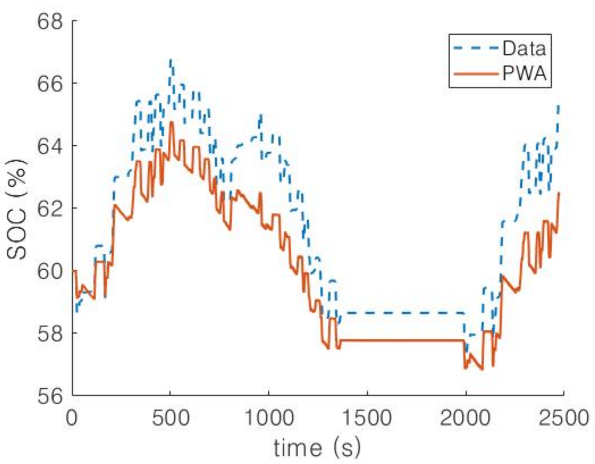


Fig. 2. Battery SOC system linear model identification on the basis of FTP-75 drive cycle

values are given from HEVP2 application. After trying the different number of polyhedra (Table 1), a simple linear regression was used, because it shows a generally linear relation with the previous state of SOC and power, given a constant $w_{mot}$. Its validation was done over the reference city drive cycle of FTP-75, given the initial SOC level and inputs along the track. Fig. 2 shows the PWA model's accuracy in a prediction horizon of 2474 seconds.

### *2.3 Fuel Consumption Rate Map*

In HEVP2, the empirically developed map is used to calculate the actual fuel consumption rate, $c_{fuel}$, of a vehicle. It is a function of engine torque command, $\tau_{eng}$, and vehicle speed, $v$. The engine torque command, $\tau_{eng}$, is computed from the relation between wheel torque demand, $\tau_{whl}$, and motor torque command, $\tau_{mot}$, which is given as:

$$\tau_{whl} = ratio(Gear)\tau_{mot}\eta_{tm}(\tau_{mot}) + \tau_{eng}\eta_{tm}(\tau_{eng}). \quad (4)$$

The function *ratio* represents the mechanical role of gear, and $\eta_{tm}$ is a transmission efficiency. According to the drive cycle, or the driver's demand velocity, a wheel torque demand, $\tau_{whl}$, is decided, and also gear. Hence, only one of $\tau_{eng}$ and $\tau_{mot}$ can be a free variable. As in the work of Ripaccioli et al. (2009), $\tau_{mot}$ is used as a control input in this study, as it has both positive and negative values. This makes $\tau_{eng}$ a dependent variable following the function of:

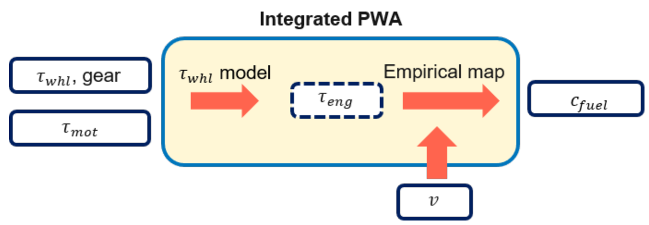


Fig. 3. Illustration for integrated PWA model for fuel consumption rate prediction: $\tau_{eng}$ model and empirical map are integrated into one big fuel consumption rate model using $\tau_{whl}$, gear, $\tau_{mot}$, and $v$

$$\tau_{eng} = f_{engine}(\tau_{whl}, \tau_{mot}). \quad (5)$$

In the HMPC scheme, we don't need an exact value for $\tau_{eng}$. Therefore, an integrated PWA model was developed to predict fuel consumption rate directly out of $\tau_{whl}$, gear, $\tau_{mot}$, and $v$ (Fig. 3). The PWA model is validated on the empirical fuel map combined with the engine model with 0.67% error on average as shown in Fig. 4. The overall prediction error is about 15% mainly in low-speed range. This is due to the engine start-up mechanism. According to Yan et al. (2012), the engine start-up process results in different fuel consumption rates for different engine speed accelerating trajectories. To cope with it, ECMS was used as a backup controller in low speed region $v \leq 5m/s$.

## 3. HYBRID MODEL PREDICTIVE CONTROL DESIGN

In this section, an MPC controller is designed for HEV energy management, based on the PWA model developed in the previous chapter. To formulate the model into mixed integer linear programming (MILP) problem, we applied the Big-M formulation for optimal control problems derived by Marcucci and Tedrake (2019).

### *3.1 Overall Hybrid Dynamical Model*

The objective function for HEV energy management is formulated as:

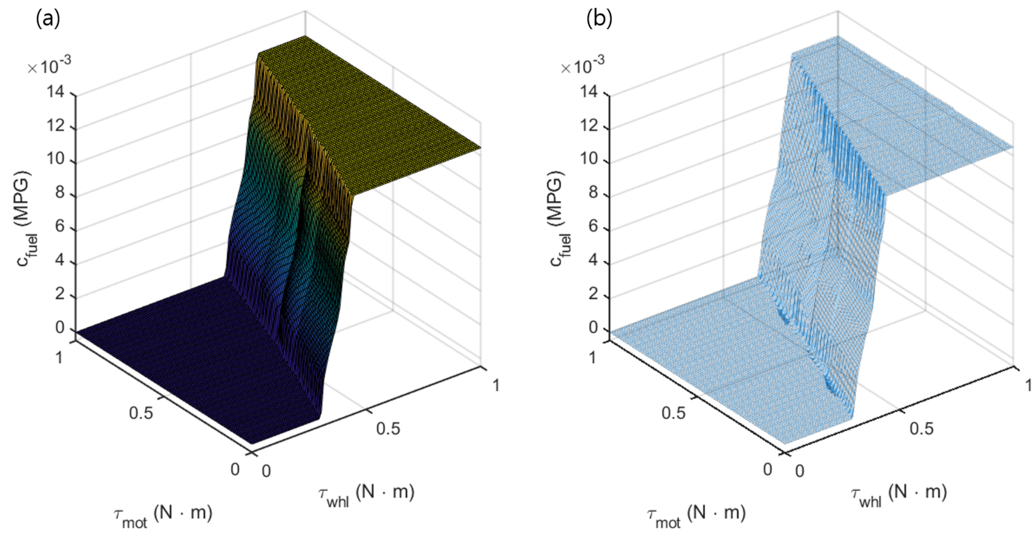


Fig. 4. Fuel consumption rate map comparison between (a) the empirical model and (b) PWA model PWA model at gear 2, vehicle speed 30 m/s

$$J = \sum_{k=0}^{N-1} \Big[ q_f c_{fuel}(k) + q_\tau |\tau_{mot}(k)| \Big] + q_f c_{fuel}(N) + q_s |SOC(N) - SOC_{ref}| + q_\rho \rho, \quad (6)$$

where $\rho$ is a deviation penalty of SOC level from its lower and upper bounds, i.e. $SOC_{lb} - \rho \leq SOC(N) \leq SOC_{ub} + \rho$ and $\rho \geq 0$. The physical meaning of the control objective is to minimize the fuel consumption rate while maintaining a safe SOC level. The absolute function $|\cdot|$ genuinely has a feature of PWA system, so it can be expressed in a similar way as in the Big-M formulation.

Using the PWA model developed from the section 2, the optimal control problem of HEV energy management can be formulated as:

$$\begin{aligned} \min_{\xi} \quad & J(\xi, x(t), z(t)) = q^T y && \text{(7a)} \\ \text{s.t.} \quad & \hat{H}_1(z(t))\xi + \hat{H}_2(z(t))x_0 \leq \hat{h}(z(t)), && \text{(7b)} \\ & \hat{G}_1 \xi + \hat{G}_2 y \leq \hat{g}, && \text{(7c)} \\ & \xi = [u_0, \mu_0, x_1, ..., u_{N-1}, \mu_{N-1}, x_N]^T && \text{(7d)} \\ & x_0 = x(t). && \text{(7e)} \end{aligned}$$

$$y = \begin{bmatrix} c_{fuel}(0) \\ |\tau_{mot}(0)| \\ c_{fuel}(1) \\ |\tau_{mot}(1)| \\ \vdots \\ |\tau_{mot}(N-1)| \\ c_fuel(N) \\ |SOC(N) - SOC_{ref}| \\ \rho \end{bmatrix}, \quad (8)$$

Here, $N$ is the prediction horizon, $x(t)$ is the state of the HEV system at sampling time $t$, and $z(t)$ is the vehicle feedback information that will be used to interpolate the pre-determined PWA matrices, namely gear, $w_{mot}$ and $v$. Following the definition of $y$ in Eq. 8, a weight vector, $q$, and the matrices $\hat{G}_1, \hat{G}_2$ and $\hat{g}$ are determined.

### *3.2 Big-M Formulation*

To start with, set a polytope:

$$\mathcal{D}_i = \{(x, u) | F_i x + F_i u \leq h_i\}, \ \forall i \in \mathcal{I}. \quad (9)$$

If a current $(x, u)$ pair is in the polytope $\mathcal{D}_i$, next state, $x_{k+1}$ follows the dynamics defined with the matrices $A_i, B_i$, and $c_i$. To distinguish which polytope should work at the current $(x, u)$ pair, define a vector $\mu$ with its $i$'th element as a binary variable, $\mu_{(i)}$, such that

$$\mu_{(i)} = \begin{cases} 1, & if \ (x, u) \in \mathcal{D}_i \\ 0, & otherwise. \end{cases} \quad (10)$$

To avoid confusion of notation, subscript index in a parenthesis $v_{(i)}$ refers to the $i$'th element of a vector $v$, and so is $M_{(i,j)}$ of a matrix $M$. The disjunctive condition can be integrated in inequalities using big-M matrices, $M_1, M_2$, and $M_3$, which is defined for two different mode indices $(i, j) \in \mathcal{I}^2$:

$$M_{1,(i,j)} = \max_{(x,u) \in \mathcal{D}_j} (A_i - A_j)x + (B_i - B_j)u + c_i - c_j \quad (11a)$$

$$M_{2,(i,j)} = \max_{(x,u) \in \mathcal{D}_j} (A_j - A_i)x + (B_j - B_i)u + c_j - c_i \quad (11b)$$

$$M_{3,(i,j)} = \max_{(x,u) \in \mathcal{D}_j} F_i x + G_i u - h_i. \quad (11c)$$

Diagonal entries in big-M matrices are 0.

Now the PWA model of Eq. 1 can be expressed as MILP constraints:

$$A_i x_k + B_i u_k + c_i - x_{k+1} \le M_{1,(i,:)}\mu_k, \quad \forall i \in \mathcal{I} \tag{12a}$$
$$x_{k+1} - A_i x_k - B_i u_k - c_i \le M_{2,(i,:)}\mu_k, \quad \forall i \in \mathcal{I} \tag{12b}$$
$$F_i x_k + G_i u_k \le h_i + M_{3,(i,:)}\mu_k, \quad \forall i \in \mathcal{I} \tag{12c}$$
$$\mathbf{1}^T \mu_k = 1, \tag{12d}$$

where $\mathbf{1}$ refers to all-ones vector with appropriate size. When $(x_k, u_k) \in \mathcal{D}_i$ and $\mu_{k,(i)} = 1$, the first two equations act as an equality constraint for the dynamics in the polytope $\mathcal{D}_i$ at time step $k$.

Note that the big-M matrices should be computed whenever the HMPC receives new feedback information at every sampling time since the system dynamics change according to $z(t)$. Computation of big-M matrices is a series of simple linear programming (LP) problems as defined in Eq. 11, which can be done in a second. However, this routine LP solving step can be computed offline, exploiting its linear property. For an arbitrary feedback input, $\hat{z} = \lambda z_1 + (1-\lambda)z_2$, one of its big-M matrices $M^{\hat{z}}_{1,(i,j)}$ is smaller than the interpolation of pre-computed big-M matrices at $z_1$ and $z_2$.

$$\begin{aligned}
M^{\hat{z}}_{1,(i,j)} &= \max_{(x,u)\in\mathcal{D}_j} (A^{\hat{z}}_i - A^{\hat{z}}_j)x + (B^{\hat{z}}_i - B^{\hat{z}}_j)u + c^{\hat{z}}_i - c^{\hat{z}}_j \\
&= \max_{(x,u)\in\mathcal{D}_j} \Big[(A^{z_1}_i - A^{z_1}_j)x + (B^{z_1}_i - B^{z_1}_j)u + c^{z_1}_i - c^{z_1}_j \\
&\quad + (A^{z_2}_i - A^{z_2}_j)x + (B^{z_2}_i - B^{z_2}_j)u + c^{z_2}_i - c^{z_2}_j\Big] \\
&\le \lambda M^{z_1}_{1,(i,j)} + (1-\lambda) M^{z_2}_{1,(i,j)}
\end{aligned}$$

Compensating strength of the ineqaulity bound, the interpolation of pre-computed big-M matrices may substitute $M^{\hat{z}}_{1,(i,j)}$. In this paper, HMPC uses the pre-computed big-M matrices at the $z$ values of the PWA model.

## 4. SIMULATION RESULTS

The closed-loop performance of the HMPC controller using Big-M formulation (HMPC-BM) was simulated on HEV P2 application environment in MATLAB/Simulink. FTP-75 drive cycle was used in the simulation because it is widely accepted as a standard. The LP problem for big-M value and MILP optimization for overall HMPC was done through MATLAB built-in function - *linprog* and *intlinprog*. Also, the HMPC-BM controller includes a heuristic rule in negative wheel torque demand. To fully utilize the vehicle's braking energy, the battery is charged whenever the vehicle decelerates.

Under a prediction horizon of $N = 5$, the design parameters for HMPC-BM are the cost weight parameters, $q_f = 5, q_\tau = 20, q_s = 0$, and $q_{rho} = 10^4$. The SOC reference tracking cost, $|SOC(N) - SOC_{ref}|$, is employed by Ripaccioli et al. (2009). However, after a number of

Table 2. Design parameter tuning: SOC reference tracking cost over the first 500 seconds of FTP-75 drive cycle

| Case # | $q_s$ | $q_\tau$ | Average $c_{fuel}$ |
|---|---|---|---|
| 1 | 0.2 | 0.2 | 33.2508 MPG |
| 2 | 0.02 | 0.2 | 33.2484 MPG |
| 3 | 0.002 | 0.2 | 33.2487 MPG |

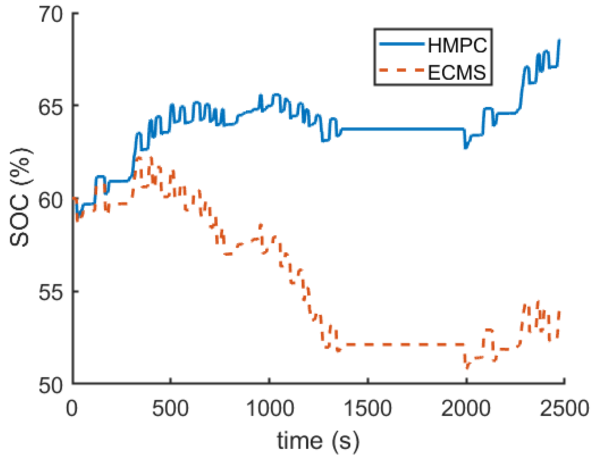


Fig. 5. SOC control performance of HMPC and ECMS for FTP-75 drive cycle simulation

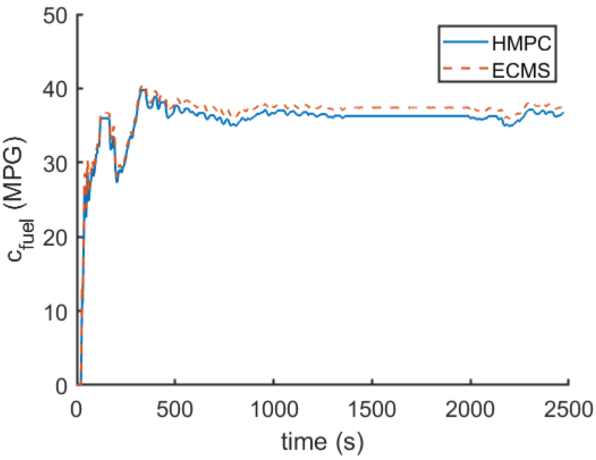


Fig. 6. Fuel consumption rate of HMPC and ECMS for FTP-75 drive cycle simulation

experiments in Table 2, the SOC reference tracking cost is meaningless, with the safety constraint cost of SOC range violation using $\rho$. The SOC prediction power in HMPC-BM's longer prediction horizon makes the safety constraint cost work.

The simulation result of a HMPC-BM with the setting is shown in Fig. 5 - 7. Its control performance is compared to the ECMS controller since it is the industrial standard at this time. HMPC-BM acquired terminal SOC 13% higher (here, % is a unit of SOC), while reducing the average fuel consumption rate by 3% with regard to the one of ECMS. In other words, HMPC-BM used the fuel energy to satisfy the driving demand and to charge up the battery more efficiently. This stems from the removal of SOC tracking cost, which made the vehicle charging strategy more flexible.

In Fig. 7, the motor torque command is compared within a period of 400 seconds. The five largest peaks of motor torque command from ECMS are not seen in the HMPC-BM control trajectory. Even without SOC tracking cost considered, the motor torque command cost provided smooth control action.

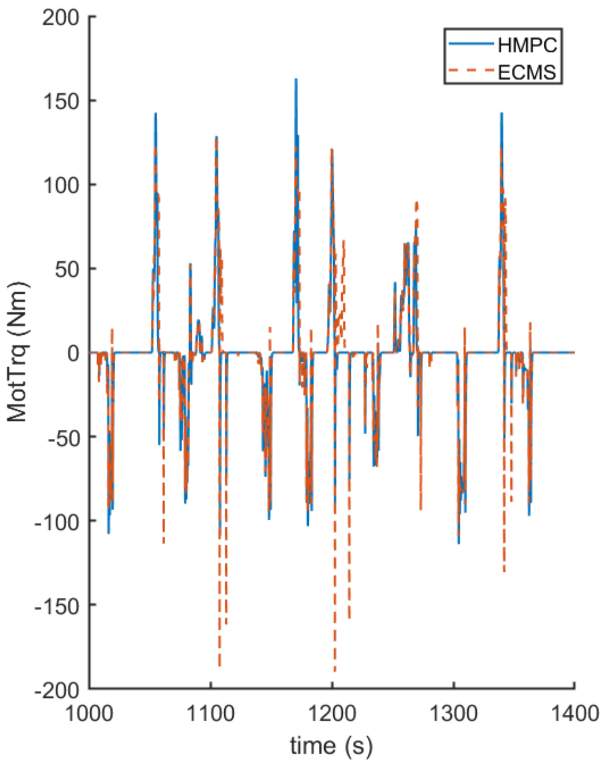


Fig. 7. Comparison of optimal motor torque command between HMPC and ECMS for FTP-75 drive cycle simulation

## 5. CONCLUSION

This paper presents the use of efficient MILP formulation for PWA systems in HEV energy management problem. PWA system for HEV dynamics was acquired using the empirical model and data from HEV P2 application and showed less than 1% fitting error. The system is converted into an MILP problem using Big-M formulation which gives a smaller and stronger model to solve. The efficacy of the developed HMPC-BM controller is shown in the MATLAB/Simulink simulation environment. The controller showed efficient use of the fuel and electric energy in the given drive cycle with its safety constraint and motor torque action cost.

## ACKNOWLEDGEMENTS

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science ICT(MSIT) (NRF-2016R1A5A1009592) and Hyundai Motor Company.

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