Development of Block Oriented Recursive and Constrained Parameter Estimation Schemes for ARMAX Models

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Abstract: In the process industry, black box linear perturbation models are often used for the development of model predictive controllers. Maintaining a high-quality model so as to achieve good control performance in the face of changing operating conditions is a difficult task. In adaptive control schemes, the model parameters are updated online using recursive least square schemes. These recursive schemes typically update the model parameters at every sampling instant. Since in many chemical/biological processes, the model parameters change at a relatively slow rate compared to state variables, it is beneficial to update the model parameters using blocks of data points instead of updating at each sampling instant. In this work, a constrained update scheme based on blocks of data is proposed for updating ARMAX model parameters online. The inclusion of constraints ensures that the noise model is stable and inversely stable. The constrained formulation is further simplified to arrive at two unconstrained recursive parameter update schemes. The efficacy of the proposed schemes is demonstrated using a simulation study on an artificial system and experimental data obtained from a temperature control system and benchmark quadruple tank system.

Keywords: Recursive least squares, Block recursive estimation, Constrained parameter estimation

1. INTRODUCTION

Adaptive control techniques that employ data-driven linear dynamic models have been very widely used in many application domains. A wide variety of methods have been developed over the last six decades, and excellent reviews and monographs are available that summarize important developments in this area (Astrom (2001), Ydstie (1997)). At the core of any adaptive control scheme is an online recursive parameter estimation scheme. Thus, many online recursive parameter estimation schemes have been developed for the online update of model parameters (Söderström and Stoica (2001)). In fact, the success of an adaptive control scheme can be traced to a well developed online parameter estimation scheme.

Many variants of recursive parameter estimation algorithms are available in the literature. The most popular recursive update schemes update model parameters using a single data point as it arrives. Alternatively, block recursive least squares schemes have been developed that make use of data over a sliding time window. Liu and He (1995) has developed an exponentially weighted sliding window-based recursive least squares (RLS) scheme for estimation of time-varying FIR model coefficients. Jiang and Zhang (2004) have shown that a sliding window based recursive estimation scheme is better for tracking parameters of an ARX model. Recently, Ali et al. (2016) have developed a sliding window-based RLS scheme with time-varying weighting for online estimation of FIR and ARX models. This literature on sliding window recursive estimation highlights that the window-based estimation outperforms the conventional recursive estimation schemes. However, to the best of our knowledge, these schemes are developed for linear in parameter models.

Ydstie (1997) underscores the need to deal with 'parameter drift or the instability of the parameter estimator' while developing an adaptive control scheme. The parameter drift can lead to a scenario where parameters of an online recursive estimator can converge to an infeasible region or to an unstable solution when the system dynamics are stable. These difficulties can be alleviated if the online parameter update schemes are formulated as constrained optimization problems. Badwe et al. (2010) have developed a constrained recursive approach based on pseudo linear regression (PLR) for incorporating constraints arising from stability considerations.

Adaptive control formulations that employ conventional recursive estimation schemes end up changing controller parameters/prediction models at every sampling instant which increases online computations. In many chemical processes, however, the model parameter changes occur at a relatively slow rate when compared to rates at which the inputs or outputs of the system change. Thus, it is not necessary to update the model parameters at every sampling instant for implementing adaptive control of such systems. A possible approach to reduce the online computational burden is to extend the block recursive parameter estimation scheme (Crassidis and Junkins (2012)), which is based on shifting blocks of data, to the identification of the linear time series model parameters. In this work, we develop shifting window-based block recursive parameter estimation schemes for the estimation of ARMAX model parameters based on pseudo-linear regression. Taking motivation from Badwe et al. (2010), we include constraints arising from stability considerations in the optimization formulation to ensure that the noise model is stable and inversely stable. The efficacy of the proposed approach is initially studied by simulating an artificial system and later using data from two lab-scale experimental setups.

This work is organized in 4 sections. The block recursive update schemes are developed in section 2. Section 3 shows the results from applying the schemes for simulated and experimental data sets. The main conclusions are presented in section 4.

2. DEVELOPMENT OF BLOCK RECURSIVE PARAMETER ESTIMATION SCHEMES

2.1 Block Recursive Estimation for ARX model

In this work, we assume that the dynamics of an $r \times m$ MIMO system are modeled as r MISO models. Thus, consider a MISO ARX model of the form

$$A(q^{-1})y(k) = \sum_{j=1}^{m} B_j(q^{-1})u_j(k) + e(k) \tag{1}$$

which can be expressed as

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{j=1}^{m}\sum_{i=1}^{n} b_{ij} u_j(k-i) + e(k) \tag{2}$$

$$y(k) = \phi(k)^T\theta + e(k) \tag{3}$$

where

$$\phi(k)^T = [\, -y(k-1) \;..\; -y(k-n) \;\; u_1(k-1) \;..\; u_m(k-n) \,] \tag{4}$$

$$\theta = [\, a_1 \;..\; a_n \;\; b_{11} \;..\; ..b_{nm} \,]^T_{p\times 1} \tag{5}$$

where $p = n(1+m)$. If the data collected for system identification is represented as

$$S = \{(y(k), \mathbf{u}(k)) : k = 0, 1, ...N_s\} \tag{6}$$

then the least squares estimate of θ can be obtained as follows (Söderström and Stoica (2001))

$$\widehat{\theta} = (\mathbf{\Omega}^T\mathbf{\Omega})^{-1}\mathbf{\Omega}^T\mathbf{Y} \tag{7}$$

$$\mathbf{\Omega} = \begin{bmatrix} \phi(n)^T \\ .... \\ \phi(N_s)^T \end{bmatrix} \text{ and } \mathbf{Y} = \begin{bmatrix} y(n) \\ .... \\ y(N_s) \end{bmatrix} \tag{8}$$

In this work, it is desired to use sequential recursive estimation scheme for on-line estimation of model parameters θ. Thus, the data is split into subsets as as follows

$$S_0 = \{(y(k), \mathbf{u}(k)) : k = 0, 1, ...N_0\} \tag{9}$$

$$S_j = \{(y(k), \mathbf{u}(k)) : k = j_0, ..., j_f\} \tag{10}$$

$$j_0 = N_0 + (j-1) * N + 1 \text{ and } j_f = N_0 + j * N \tag{11}$$

such that for $j > 0$, the data each subset S_j consists of block of N $(> n)$ data points and $S = \bigcup S_j$. It is assumed that an initial model $(\widehat{\theta}^{(0)}, \mathbf{P}^{(0)})$ is developed using S_0 where

$$\mathbf{P}^{(0)} = \left[\left(\mathbf{\Omega}^{(0)}\right)^T \mathbf{\Omega}^{(0)}\right]^{-1} \tag{12}$$

Following the sequential block least squares scheme developed in Chapter 2 of Crassidis and Junkins (2012), the following block recursive parameter estimation scheme can be derived

$$\widehat{\theta}^{(j+1)} = \widehat{\theta}^{(j)} + \mathbf{K}^{(j+1)}\left(\mathbf{y}^{(j+1)} - \left(\mathbf{\Omega}^{(j+1)}\right)\widehat{\theta}^{(j)}\right) \tag{13}$$

where

$$\mathbf{K}^{(j+1)} = \mathbf{P}^{(j+1)}\left(\mathbf{\Omega}^{(j+1)}\right)^T \tag{14}$$

$$\left(\mathbf{P}^{(j+1)}\right)^{-1} = \left(\mathbf{P}^{(j)}\right)^{-1} + \left(\mathbf{\Omega}^{(j+1)}\right)^T\left(\mathbf{\Omega}^{(j+1)}\right) \tag{15}$$

$$\mathbf{\Omega}^{(j+1)} = \begin{bmatrix} \phi(j_0)^T \\ .... \\ \phi(j_f)^T \end{bmatrix}_{N\times p} \text{ and } \mathbf{Y}^{(j)} = \begin{bmatrix} y(j_0) \\ .... \\ y(j_f) \end{bmatrix}_{N\times 1} \tag{16}$$

The recursive update can also be cast as solution of the following optimization problem

$$\widehat{\theta}^{(j+1)} = \underset{\theta}{arg\ Min}\ J(\theta) \tag{17}$$

$$J(\theta) = \left(\theta - \widehat{\theta}^{(j)}\right)^T\left(\mathbf{P}^{(j)}\right)^{-1}\left(\theta - \widehat{\theta}^{(j)}\right) + \left(\mathbf{y}^{(j+1)} - \mathbf{\Omega}^{(j+1)}\theta\right)^T\left(\mathbf{y}^{(j+1)} - \mathbf{\Omega}^{(j+1)}\theta\right) \tag{18}$$

The optimization formulation has an advantage that constraints on parameter estimates of the form

$$\mathbf{g}(\theta) \leq \mathbf{0} \tag{19}$$

can be included in the problem formulation. The covariance update can be carried out using eq. (15) even for the constrained formulation.

2.2 Block Recursive Estimation for ARMAX Model

In this section the block recursive estimation schemes developed in the previous section are extended to a MISO ARMAX model. Consider a MISO ARMAX model of the form:

$$A(q^{-1})y(k) = \sum_{j=1}^{m} B_j(q^{-1})u_j(k) + C(q^{-1})e(k) \tag{20}$$

which can be expressed as

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{j=1}^{m}\sum_{i=1}^{n} b_{ij}u_j(k-i) + \sum_{i=1}^{n} c_i e(k-i) + e(k) \tag{21}$$

$$y(k) = \phi(k)^T\theta + e(k) \tag{22}$$

where

$$\phi(k)^T = [\, \phi_y(k)^T \;\; \phi_u(k)^T \;\; \phi_e(k)^T \,] \tag{23}$$

$$\phi_y(k)^T = [\, -y(k-1) \;....\; -y(k-n) \,] \tag{24}$$

$$\phi_u(k)^T = [\, u_1(k-1) \;..\; u_1(k-n) \;\; u_2(k-1) \;..\; u_m(k-n)\,] \tag{25}$$

$$\phi_e(k)^T = [\, e(k-1) \;..\; e(k-n)\,] \tag{26}$$

$$\theta = [\, a_1 \;..\; a_n \;\; b_{11} \;..\; ..b_{nm} \;\; c_1 \;..\; c_n\,]^T_{p\times 1} \tag{27}$$

Here, where $p = n(2+m)$.

Block Constrained Estimation Now, problem of estimating parameters using set S can be posed as an optimization problem (Söderström and Stoica (2001)):

$$\widehat{\theta} = \underset{\theta}{arg\ Min} \sum_{k=n+1}^{N_s} \left(y(k) - \phi(k)^T\theta\right)^2 \tag{28}$$

The resulting optimization problem needs to be solved using a nonlinear programming approach. Thus, to develop a block recursive estimation scheme for this case, we start by extending of optimization formulation (17) to the ARMAX model using pseudo-linear regression (Söderström and Stoica (2001)). Then, block estimates $\widehat{\theta}^{(j)}$ for $j = 1, 2, ...$ can be constructed as follows

$$\widehat{\theta}^{(j+1)} = \underset{\theta}{arg\ Min}\ J(\theta) \tag{29}$$

subject to

$$\phi_\varepsilon(k)^T = [\, \varepsilon(k-1) \;..\; \varepsilon(k-n)\,] \tag{30}$$

$$\phi(k)^T = \left[\, \phi_y(k)^T \;\; \phi_u(k)^T \;\; \phi_\varepsilon(k)^T \,\right] \tag{31}$$

$$\varepsilon(k) = y(k) - \phi(k)^T\theta \tag{32}$$

$$\text{for } k = j_0, ..., j_f \tag{33}$$

Construction of the regressor vector at the beginning of the window requires model residuals for $k < j_0$. The estimated residuals at the end of the previous window calculated after convergence of the optimization problem are used to construct these regressor vectors.

$$\mathbf{\Omega}^{(j+1)}(\theta) = \begin{bmatrix} \phi(j_0)^T \\ \phi(j_0+1)^T \\ .... \\ \phi(j_f)^T \end{bmatrix} ; \mathbf{Y}^{(j+1)} = \begin{bmatrix} y(j_0) \\ y(j_0+1) \\ .... \\ y(j_f) \end{bmatrix} \tag{34}$$

When the optimization problem converges, the covariance matrix can be updated as follows:

$$\left(\mathbf{P}^{(j+1)}\right)^{-1} = \left(\mathbf{P}^{(j)}\right)^{-1} + \left(\mathbf{\Omega}^{(j+1)}\left(\widehat{\theta}^{(j+1)}\right)\right)^T \left(\mathbf{\Omega}^{(j+1)}\left(\widehat{\theta}^{(j+1)}\right)\right) \tag{35}$$

Further, constraints of the form

$$\mathbf{g}(\theta) \leq \mathbf{0} \tag{36}$$

can be included in the optimization problem. These constraints arise from application of Jury stability criteria or a-priori knowledge about steady state gain signs. For a second order ARMAX model, the constraints arise from $C(q^{-1})$, $A(q^{-1})$ being open loop stable and possible constraints on the gains from physical insights (Badwe et al. (2010)):

$$1 + a_1 + a_2 > 0 \;;\; 1 - a_1 + a_2 > 0 \;;\; a_2 < 1 \tag{37}$$

$$1 + c_1 + c_2 > 0 \;;\; 1 - c_1 + c_2 > 0 \;;\; c_2 < 1 \tag{38}$$

Unconstrained Iterative Refinement Scheme For the unconstrained optimization problem, the necessary condition for optimality is

$$\frac{\partial J(\theta)}{\partial \theta} = 2\left(\mathbf{P}^{(j)}\right)^{-1}\left(\theta - \widehat{\theta}^{(j)}\right) + \frac{\partial}{\partial\theta}[-2\theta^T\left(\mathbf{\Omega}^{(j+1)}(\theta)\right)^T \mathbf{Y}^{(j+1)} + \theta^T\left(\mathbf{\Omega}^{(j+1)}(\theta)\right)^T\left(\mathbf{\Omega}^{(j+1)}(\theta)\right)\theta] = \mathbf{0} \tag{39}$$

This is a nonlinear equation in θ and needs to be solved iteratively. A possible way to formulate iterations to solve this problem is as follows: Let $\widetilde{\theta}^{(l)}$ represent a initial guess of (39) at iteration l. Then we approximate

$$\mathbf{\Omega}^{(j+1)}(\theta) \approx \mathbf{\Omega}^{(j+1)}\left(\widetilde{\theta}^{(l)}\right) \tag{40}$$

and substitute in eq. (39). To simplify the notations, let us define $\widetilde{\mathbf{\Omega}}^{(l)} = \mathbf{\Omega}^{(j+1)}\left(\widetilde{\theta}^{(l)}\right)$. With this approximation eq. (39) reduces to

$$\left[\left(\mathbf{P}^{(j)}\right)^{-1} + \left(\widetilde{\mathbf{\Omega}}^{(l)}\right)^T\left(\widetilde{\mathbf{\Omega}}^{(l)}\right)\right]\theta = \left(\mathbf{P}^{(j)}\right)^{-1}\widehat{\theta}^{(j)} + \left(\widetilde{\mathbf{\Omega}}^{(l)}\right)^T\mathbf{Y}^{(j+1)} \tag{41}$$

Thus, with further algebraic manipulations of eq. (41), the following block iteration scheme can be derived

$$\widetilde{\theta}^{(l+1)} = \widehat{\theta}^{(j)} + \widetilde{\mathbf{K}}^{(l+1)}\left[\mathbf{Y}^{(j+1)} - \left(\widetilde{\mathbf{\Omega}}^{(l)}\right)\widehat{\theta}^{(j)}\right] \tag{42a}$$

$$\widetilde{\mathbf{\Omega}}^{(l)} = \mathbf{\Omega}^{(j+1)}\left(\widetilde{\theta}^{(l)}\right) \tag{42b}$$

$$\widetilde{\mathbf{K}}^{(l+1)} = \left(\widetilde{\mathbf{P}}^{(l+1)}\right)\left(\widetilde{\mathbf{\Omega}}^{(l)}\right)^T \tag{42c}$$

$$\left(\widetilde{\mathbf{P}}^{(1+1)}\right)^{-1} = \left[\left(\mathbf{P}^{(j)}\right)^{-1} + \left(\widetilde{\mathbf{\Omega}}^{(l)}\right)^T\left(\widetilde{\mathbf{\Omega}}^{(l)}\right)\right] \tag{42d}$$

The iterations can be terminated after the following convergence criterion is satisfied

$$\frac{\left\|\widetilde{\theta}^{(l+1)} - \widetilde{\theta}^{(l)}\right\|}{\left\|\widetilde{\theta}^{(l+1)}\right\|} < \varepsilon \tag{43}$$

The iterations can be initialized as $\widetilde{\theta}^{(0)} = \widehat{\theta}^{(j)}$. After termination of the iterations, say for $l = l^*$, we can set

$$\widehat{\theta}^{(j+1)} = \widetilde{\theta}^{(l^*+1)} \text{ and } \left(\mathbf{P}^{(j+1)}\right)^{-1} = \left(\widetilde{\mathbf{P}}^{(l^*+1)}\right)^{-1} \tag{44}$$

Unconstrained Prediction Correction Scheme A simplified version of the iterative refinement scheme can be derived by terminating iterations at $l = 1$ i.e. set This simplification does not require iterative calculations and is more suitable for on-line parameter estimation.

3. RESULTS AND DISCUSSIONS

In this section, the performance of the proposed recursive parameter estimation schemes is demonstrated by conducting a simulation study on an artificial MISO ARMAX model. The efficacy of the proposed strategies is also evaluated on experimental data collected from temperature control lab setup (Park et al. (2020)) and benchmark quadruple tank setup (Johansson (2000)) available at Automation Lab in the department of Chemical Engineering, IIT Bombay. Performance of the proposed parameter schemes are compared using the following performance index:

$$\text{Average Error} = \frac{1}{n_b}\sum_{j=N_b-n_b}^{N_b} |(\theta_i - \hat{\theta}_i^{(j)})/\theta_i| \tag{45}$$

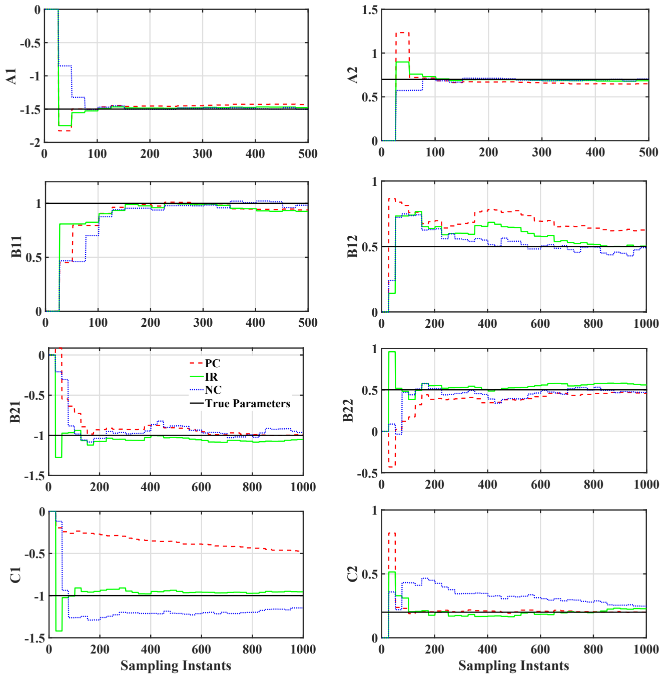

Fig. 1. Simulation study: Parameter estimates using proposed schemes with block-size 25

where, N_b represents the total number of blocks, n_b represents the number of blocks considered for averaging, $|x|$ represents the absolute value of x, $\hat{\theta}_i^{(j)}$ represents the i^{th} element of $\hat{\theta}^{(j)}$ vector and θ represents the true model parameter vector of the artificial system.

3.1 Simulation Study

The simulation study is conducted on the following 2^{nd} order ARMAX MISO model:

$$A(q^{-1}) = 1 - 1.5q^{-1} + 0.7q^{-2} \tag{46a}$$

$$B_1(q^{-1}) = q^{-1} + 0.5q^{-2} \tag{46b}$$

$$B_2(q^{-1}) = -q^{-1} + 0.5q^{-2} \tag{46c}$$

$$C(q^{-1}) = 1 - q^{-1} + 0.2q^{-2} \tag{46d}$$

The disturbance signal $e(k)$ in the ARMAX model is assumed to be a zero mean Gaussian white noise sequence with variance 0.05^2. The input-output data set is simulated by generating a pseudo-random binary signal (PRBS) of range $[0, 0.05]$ and with sampling interval of 1 second for both the inputs using *idinput*() function from MATLAB's System Identification toolbox. The performance of the proposed recursive parameter estimation schemes i.e., nonlinear constrained (NC) estimation, iterative refinement (IR) and prediction correction (PC) for block-size of 25 is plotted in Figure 1. Except for the model parameters corresponding to $C(q^{-1})$ polynomial, all three parameter estimation schemes converge to true model parameter values. In terms of average error, the performance of NC and IR schemes are comparable and better than PC. The performance of the proposed schemes for block-size of 25 and 10, in terms of Average Errors (%), is reported in Table 1 and Table 2 respectively. Changing block-size varies the converged estimates for model parameters corresponding to $C(q^{-1})$ polynomials. The performances of NC scheme are comparable for both the block-sizes. Performances of PC and IR have improved with increase in the block-size.

Table 1. Simulation study: Average Errors (%) for different schemes with block-size 25, $n_b = 2$

Model Para. →	a1	a2	b11	b12	b21	b22	c1	c2
PC	1.46	2.49	0.99	10.11	2.24	1.59	35.47	34.12
IR	0.42	0.51	1.21	1.86	3.84	9.14	2.20	5.75
NC	0.55	0.89	1.23	1.41	1.46	1.48	9.75	12.93

Table 2. Simulation study: Average Errors (%) for different schemes with block-size 10, $n_b = 5$

Model Para. →	a1	a2	b11	b12	b21	b22	c1	c2
PC	0.23	0.20	3.72	13.19	3.64	6.90	16.14	86.62
IR	0.41	0.28	2.63	11.16	4.82	9.41	0.37	29.44
NC	0.54	0.88	1.52	1.27	0.41	0.98	7.74	19.57

3.2 Experimental Study

The efficacy of the proposed schemes is also evaluated using the experimental data collected from a standard laboratory scale temperature control system and benchmark quadruple tank setup. Similar to the simulation study, the plant is perturbed by introducing PRBS signal into the plant and the input-output data set is recorded. All the open-loop input-output data set is then used to identify a batch model using the System Identification toolbox on MATLAB. The batch model is treated as true model to assess the performance of the proposed schemes.

Temperature control system (Park et al. (2020)): Experimental data was collected from the temperature control lab, a 2×2 system comprising of two heaters and two temperature sensors, and output data from temperature sensor 1 is used for testing and validation of proposed schemes. A PRBS of amplitude 10% and frequency range $[0, 0.05\pi/T]$ (where T is sampling interval) is generated for both inputs. The output data generated during the open-loop experiment by introducing the PRBS is presented in the Figure 2. Three scenarios were considered for initialization of the proposed recursive parameter estimation schemes: $N_0 = 200$, 100 and 0. The performance of the parameter estimation schemes (for $N_0 = 200$) is presented in Figure 3. The model parameters identified using the proposed schemes remain close to the batch model parameters in this case. These generated models are then validated using experimental data collected by introducing simultaneous step changes of magnitude 10% from time 100 secs to 1000 secs followed by simulataneous step changes of magnitude -10% from time 1000 secs to 1900 secs in both the inputs of the temperature control system. These models are also compared with a first order plus time delay (FOPTD) model generated using batch data in Figure 4 ($N_0 = 200$) and Figure 5 ($N_0 = 0$). While the estimated model parameters seem to vary for each proposed parameter estimation scheme, the models generated using PC and IR approaches track the

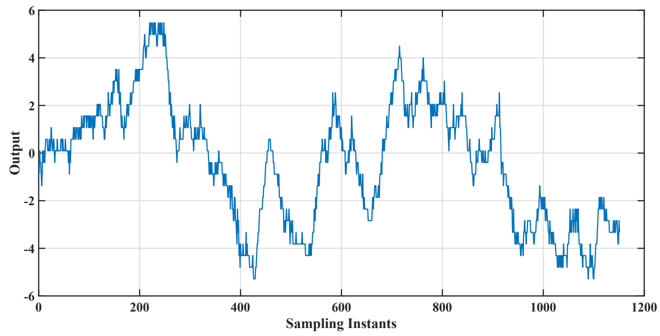

Fig. 2. Temperature control system: Temperature profile generated by PRBS signal

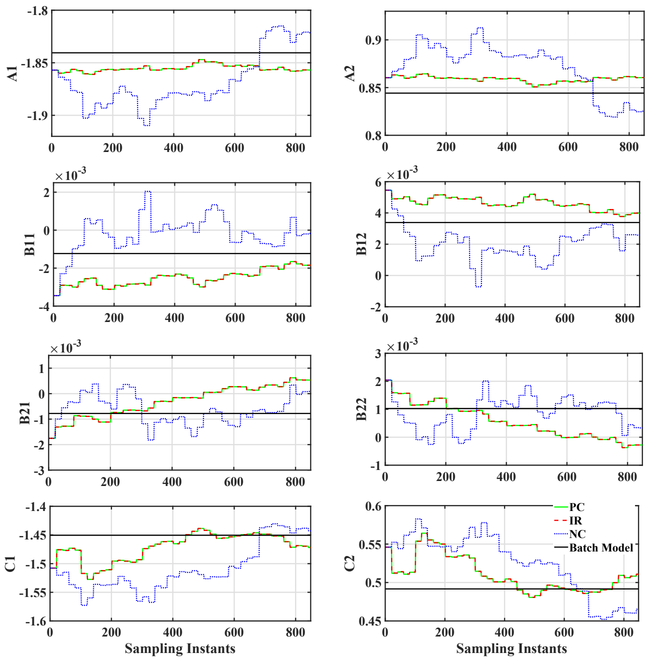

Fig. 3. Temperature control system: Parameter estimates using proposed schemes with block-size 20 using warm start (200 points)

step test data with high accuracy (ref. Table 3) for case $N_0 = 200$. This indicates that model parameter estimates for discrete-time transfer functions model may be different, but the model behavior could still give reliable predictions if these schemes are initialized using a good model. However, these approaches fail to generate good models for $N_0 = 100$ and 0. The NC approach, on the other hand, generates very good models for all N_0.

Table 3. Temperature control system: Percentage Fit (%) on validation data of proposed schemes with block-size 20 using warm start, coarse start and cold start

Proposed Scheme	Points for Start (N_0)		
	200 pts	100 pts	0 pts
NC	87.99	79.19	83.92
IR	83.57	66.45	62.42
PC	83.57	63.83	59.65
FOPTD	83.42		

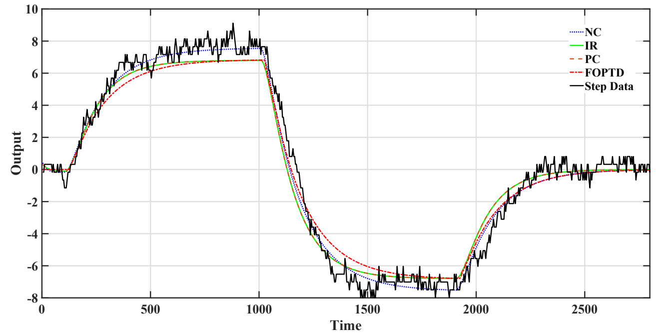

Fig. 4. Temperature control system: Model Validation for the proposed schemes with block-size 20 using warm start (200 points)

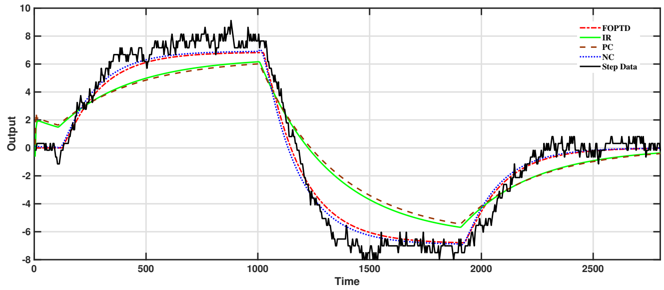

Fig. 5. Temperature control system: Model Validation for the proposed schemes with block-size 20 using cold start (no points)

Quadruple tank system (Johansson (2000)): Experimental data collected from the Quadruple Tank system (refer Karra et al. (2008) for details) is used to demonstrate the constraint handling ability of the proposed NC approach. A PRBS of amplitude 0.3 mA and frequency range $[0, 0.05\pi/T]$ was injected in both inputs. The output data generated from conducting this PRBS test on the experimental system is presented in Figure 6. Figure 7 compares the performance of NC scheme with constraints and without constraints for a block-size of 30 samples. We observe that while the final estimate is within constraint limits for both update schemes, the constraints play an important role by modifying the search so that the parameters converge to values close to that of batch model. It was also found that the stability constraints are violated at multiple sampling instants in case of unconstrained scheme (ref. Figure 8). Moreover, the NC estimator is able to converge close to the batch data estimates obtained using Matlab's System Identification toolbox while the unconstrained NC estimator generates a significantly different model.

4. CONCLUSIONS

Online model parameter estimation is the core component of an adaptive control scheme. The model parameters are typically updated online using recursive least squares at every sampling instant. In many chemical/biological processes, the model parameters change at a relatively slow rate compared to state variables. Thus, in this work, it is proposed to use a block of data points to update the model parameters instead of updating model parameters at every sampling instant. A constrained block update scheme

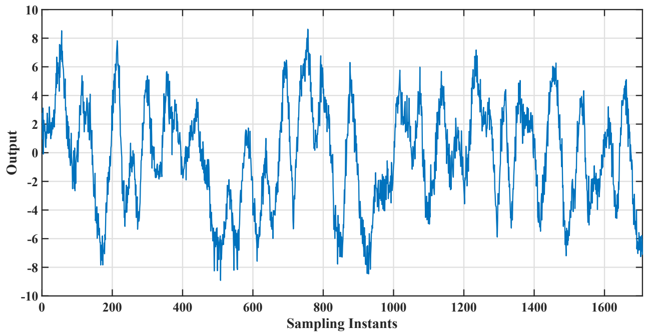

Fig. 6. Quadruple tank system: Perturbation Output from PRBS experiment

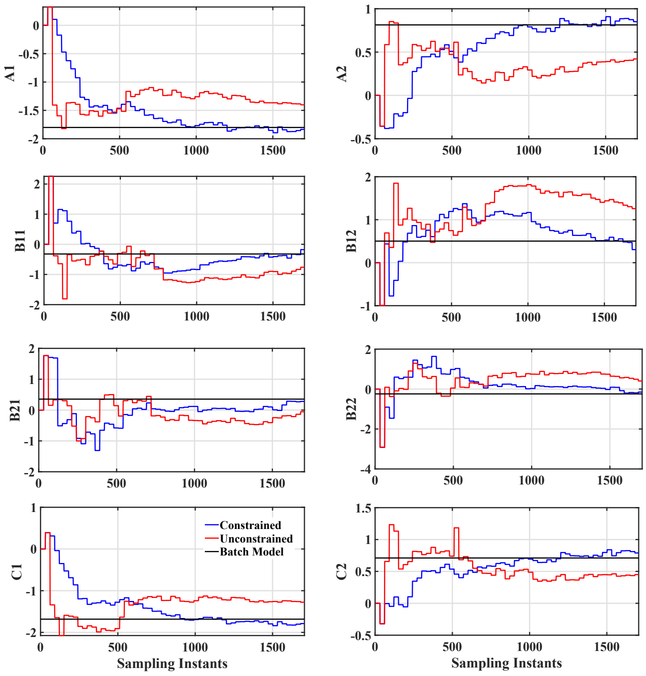

Fig. 7. Quadruple tank system: Parameter estimates using nonlinear constrained and Unconstrained schemes with block-size 30

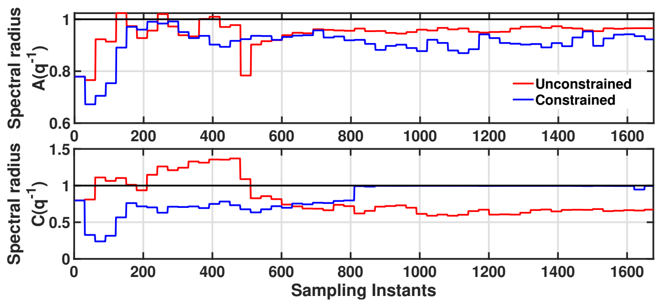

Fig. 8. Quadruple tank system: Spectral radius of $A(q{-}1)$ and $C(q{-}1)$ polynomials using NC and unconstrained NC schemes

which ensures that the noise model is stable and inversely stable is proposed to update ARMAX model parameters online. The proposed constrained formulation (NC) is further simplified to arrive at two unconstrained recursive parameter update schemes, viz iterative refinement (IR) and predictor-corrector (PC). The efficacy of the proposed schemes is evaluated by conducting a simulation study on an artificial second order ARMAX model. The analysis of the simulation study reveals that the NC and IR schemes perform better than the PC scheme. The proposed schemes are also evaluated by conducting experimental studies on a temperature control system and the benchmark quadruple tank system. It was found from the experimental study that even though the converged model parameters may be different from the batch model, but the model behavior could still give reliable predictions. Moreover, the NC approach was found to generate good models for any choice of N_0, i.e. data length used for NC initialization. It was also observed that the constraints in the proposed NC scheme modify the search so that the parameters converge to values close to that of the batch model. Extension of the proposed constrained approach to estimation of parameters of discrete and continuous time MISO models with output error structure is currently in progress.

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