

Centralized optimization of resource routing in interconnected food production units with harvesting events^{*}

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Abstract: In this work we synthesize constrained control laws monolithically to maximize production for an interconnected food production control system. The system we consider consists of a set of several plant and larvae units interconnected through valves in a circular topology. We synthesize then a centralized MPC controller after formulating the problem in the form of a nonlinear mixed integer optimization program. Then we show that the overall production increases and the energy costs decrease when the production units are connected together compared to the case when they are decoupled. These results are shown in an illustrative example on an interconnection of two production units: one containing plants (lettuce) and another containing larvae insects (*Hermetia illucens*).

Keywords: model-predictive control, interconnected system, food production units, impulsive systems, model-based control.

1. INTRODUCTION

Indoor-vertical farms with a controlled climate for biomass production are being transformed into highly sophisticated factories for food production (Kozai, 2012). This fact is driven by the need to solve food production problems such as high transportation costs, high pressure on agricultural land and water supply, high food quality requirements, fast climate changes, inconsistency of supply, unstable food prices, etc. (Despommier, 2010)

Nevertheless nowadays vertical farming still attracts several criticism that should be handled (Banerjee and Adenaer, 2014). Questions like efficiency and optimal use of energy resources are still open in these factories where sustainability is not well maintained and produced wastes are inefficiently reused or simply released as pollutants to the environment (Iddio et al., 2020). Other issues that arise, which are out of this work's scope, are lack of standardized systems that could facilitate international technology transfer, high building construction cost, non-optimal architectural design, unhealthiness of hydroponically grown food, etc. (Kozai et al., 2019)

One approach to reuse wastes produced by some of the production units in vertical farms is to interconnect them with other units that consider such wastes as input resources (Kalantari et al., 2017). For example carbon dioxide produced by units growing larvae insects could be reused by units containing plants and thus such a carbon free

production interconnected unit will produce more biomass and the energy necessary to provide such CO_2 is conserved. Also interconnecting food production units allows for a modular design where more production units could be added without changing the design and support features (Zeidler et al., 2017).

As a result, in this work we consider an interconnected system composed of a set of several plant and larvae units. The dynamic model of plant growth (lettuce) (Padmanabha et al., 2020a) is considered after some simplifications as well as the model for larvae growth proposed in (Padmanabha et al., 2020b). In this framework, the interconnection of different units is arbitrary done through valves that allow the air to be exchanged and thus influencing the temperature dynamics as well as that of the oxygen and carbon dioxide concentration.

An additional degree of freedom is also added to the models which is the instants when harvesting of plants or larvae takes place. The simplified mathematical model defines then the dynamics of biomass, CO_2 concentration, and temperature in each of the units after taking some assumptions on other variables that could affect the plant's or larva's growth. This result in an overall impulsive nonlinear system reformulation where impulses occur when harvesting takes place.

Then we formulate the problem of maximizing food production under state and input constraints in the form of a centralized mixed integer nonlinear optimization program (MINLP). The latter provides us with a model-predictive controller to be implemented online. In a biomedical application related to drug administration (Sopasakis et al., 2014; Bajcinca et al., 2020), the authors designed a MPC for a physiological pharmacokinetic impulsive model, de-

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scribing the distribution of lithium ions upon oral administration. (Sopasakis et al., 2014) considered periodic impulses and the MPC leads the linear impulsive system's state to a target set while satisfying state and input constraints in continuous time. However (Bajcinca et al., 2020) considered the impulses as control variables and after solving a mixed integer linear optimization program (MILP) the same guarantees are established but with less frequent impulses.

The main contribution in this paper is the use of MPC to control and route the resources in an interconnection of food production units. We also showed the increase of the overall production and the decrease of energy costs when the production units are coupled together compared to the case when they are decoupled. The results are demonstrated by a case study on an interconnection of several production units containing plants (lettuce) or larvae insects.

The paper is organized as follows: after the introduction we provide simplified models for plant and larvae units as well as a model for the interconnected system in Section 2. Then, in Section 3, we formulate the centralized MPC. Section 4, evaluates the proposed approach before concluding our work in Section 5.

Notations

Let \mathbb{R} , \mathbb{R}_{0+} , \mathbb{R}_+ , \mathbb{N} , \mathbb{N}_+ , denote the sets of reals, non-negative reals, positive reals, non-negative integers and positive integers, respectively. For $I \subseteq \mathbb{R}_{0+}$, let $\mathbb{N}_I = \mathbb{N} \cap I$. p_j denotes the j -th element of vector $p \in \mathbb{R}^N$. For $x \in \mathcal{S}$ and with a slight abuse of notation, $|x|$ denotes its norm on \mathcal{S} and \mathcal{B} denotes the associated unit ball.

2. PROBLEM FORMULATION

In this section, we present the models for the food production units where plants and larvae are grown. Such artificial controlled environments provide the necessary growing conditions and perform measurements of various parameters (Temperature, CO_2 concentration, etc.) with further details given in (Padmanabha et al., 2020b). Then we rewrite the models in the form of impulsive nonlinear systems, where the overall interconnected system is defined, before formulating the synthesis problem.

2.1 Case of larvae unit

We consider the case of a unit containing larvae and model the dynamics of the state $z_l(t) = [B_l(t), T_\Sigma(t), T_l(t), C_l(t)]^\top$, where B_l is the larvae dry mass, T_Σ is the development rate, T_l is the temperature, and C_l is the CO_2 concentration. v_h corresponds to a heater's input and the control inputs v_{v1} , v_{v2} , and v_{ve} correspond to 2 valves that push the air from and to the unit with neighboring units and a ventilator with the outside environment respectively. Additional input signals C_o and T_o are given and correspond to the concentration of CO_2 and the temperature in the outside environment respectively. We get the system's dynamics from (Padmanabha et al., 2020a) after posing some assumptions and simplifications. First we assume that food is always available for the larvae. Second the

food moisture is good enough to make the food ingestible without having a negative effect on the larvae's oxygen intake. Last the airflow rate is high enough and optimal for the larvae's growth rate.

Remark 1. The posed assumptions on the larvae's dynamics are not that conservative as they can be practically implemented by ensuring always that food, aeration, and moisture are within appropriate levels.

The dynamics are then given by:

$$\begin{aligned} \frac{dB_l(t)}{dt} &= \alpha_1 r_a(B_l(t), T_\Sigma(t), T(t)) B_l(t) \\ &\quad + \alpha_2 r_m(T_\Sigma(t), T_l(t)) B_l(t) \\ \frac{dT_\Sigma(t)}{dt} &= \alpha_3 r_d(T_l(t)) \\ \frac{dT_l(t)}{dt} &= \alpha_4 (T^{[N_1]}(t) - T_l(t)) v_{v1}(t) + \alpha_5 (T^{[N_2]}(t) \\ &\quad - T_l(t)) v_{v2}(t) + \alpha_6 (T_o(t) - T_l(t)) v_{ve}(t) \\ &\quad + \mu_1 v_h(t) + \beta_1(B_l(t)) \\ \frac{dC_l(t)}{dt} &= \alpha_7 (C^{[N_1]}(t) - C_l(t)) v_{v1}(t) + \alpha_8 (C^{[N_2]}(t) \\ &\quad - C_l(t)) v_{v2}(t) + \alpha_9 (C_o(t) - C_l(t)) v_{ve}(t) \\ &\quad - \beta_2(B_l(t)) \end{aligned} \quad (1)$$

Functions r_a , r_m , r_d , β_1 , and β_2 are as follows:

$$\begin{aligned} r_T(T_l(t)) &= \alpha_{10} \left(1 + \alpha_{11} \exp(-\alpha_{12}(T_l(t) - \alpha_{13})) \right)^{-1} \\ \beta_1(B_l(t)) &= \alpha_{14} N_L B_l(t) \\ \beta_2(B_l(t)) &= \alpha_{15} N_L B_l(t) \\ r_{B_m}(T_\Sigma(t)) &= \begin{cases} 1, & \text{if } T_\Sigma(t) < \alpha_{16} \\ 0, & \text{otherwise} \end{cases} \\ r_{B_a}(T_\Sigma(t), B_l(t)) &= \begin{cases} (1 - \alpha_{17}) B_l(t), \\ ((1 - \alpha_{17}) B_l(t)) \left(\frac{T_\Sigma(t) - \alpha_{19}}{\alpha_{18} - \alpha_{19}} \right), \\ 0, \end{cases} \\ r_a(B_l(t), T_\Sigma(t), T_l(t)) &= \alpha_{20} r_{B_a}(T_\Sigma(t), B_l(t)) r_T(T_l(t)) \\ r_m(T_\Sigma(t), T_l(t)) &= \alpha_{21} r_{B_m}(T_\Sigma(t)) r_T(T_l(t)) \\ r_d(B_l(t), T_\Sigma(t), T_l(t)) &= \alpha_{22} r_T(T_l(t)) \end{aligned}$$

where constants α_i , $i \in \mathbb{N}_{[1,22]}$, and μ_1 are given as in Table 1, N_L is a given initial number of larvae, first case in the function r_{B_a} is valid if $T_\Sigma(t) < \alpha_{18}$, second is valid if $\alpha_{18} \leq T_\Sigma(t) < \alpha_{19}$, and the third case holds otherwise.

We note here that the coupling bilinear terms, added to the larvae model as well as to the plant's model later on, are given by $\alpha_4 T^{[N_1]}(t) v_{v1}(t)$, $T^{[N_2]}(t) v_{v2}(t)$, $\alpha_7 C^{[N_1]}(t) v_{v1}(t)$, and $\alpha_8 C^{[N_2]}(t) v_{v2}(t)$. Practically these terms model the effect of interconnecting subsystems together by a valve that allows air to flow in between, influencing the dynamics of temperature and concentration of CO_2 .

Approximations We simplify in this work the switching functions that appear in the larvae model using a sigmoid curve. Thus the functions $r_{B_a}(\cdot)$ and $r_{B_m}(\cdot)$ are approximated by

$$r_{B_m}(T_\Sigma(t)) = \left(1 + \exp(4(T_\Sigma - 1.1\alpha_{16}))\right)^{-1}$$

$$r_{B_a}(T_\Sigma(t), B_l(t)) = (1 - \alpha_{17})B_l(t) \left(1 + \exp\left(\frac{-4}{\alpha_{18} - \alpha_{19}}\right.\right.$$

$$\left.\left.(T_\Sigma - \alpha_{18} - 0.5(\alpha_{19} - \alpha_{18}))\right)\right)^{-1}.$$

2.2 Case of plant units

We consider the case of a unit containing plants and model the dynamics of the state $z_p(t) = [B_p(t), T_p(t), C_p(t)]^\top$, where the plant's mass is B_p , the temperature is T_p , and the CO_2 concentration is C_p .

The signals v_h , v_{ve} , v_{v1} , and v_{v2} C_o and T_o are given similar to the case of units containing larvae. Based on the model in (Padmanabha et al., 2020a), and after assuming that humidity level and the water level in the growing medium are fixed, the dynamics are given by

$$\frac{dB_p(t)}{dt} = \gamma_1 r_p(B_p(t), T_p(t), C_p(t)) + \gamma_2 r_r(B_p(t), T_p(t))$$

$$\frac{dT_p(t)}{dt} = \gamma_3 T^{[N1]}(t)v_{v1}(t) + \gamma_4 T^{[N2]}(t)v_{v2}(t)$$

$$+ \gamma_5(T_o(t) - T_p(t))v_{ve}(t) + \mu_2 v_h(t)$$

$$\frac{dC_p(t)}{dt} = \gamma_6 C^{[N1]}(t)v_{v1}(t) + \gamma_7 C^{[N2]}(t)v_{v2}(t)$$

$$+ \gamma_8(C_o(t) - C_p(t))v_{ve}(t) + \gamma_9 \beta_3(B_p(t), T_p(t), C_p(t))$$
(2)

where the functions β_3 , $r_r(B_p(t), T_p(t))$, and $r_p(B_p(t), T_p(t), C_p(t))$ are as follows:

$$r_r(B_p(t), T_p(t)) = \gamma_{10} B_p(t) 2^{0.1T_p(t) - 2.5}$$

$$r_p(B_p(t), T_p(t), C_p(t)) = \gamma_{11} (1 - e^{-\gamma_{12} B_p(t)})$$

$$\left(\frac{\gamma_{13} I (-\gamma_{14} T^2 + \gamma_{15} T_p - \gamma_{16})(C_p - \gamma_{17})}{\gamma_{18} I + (-\gamma_{14} T_p^2 + \gamma_{15} T_p - \gamma_{16})(C_p - \gamma_{17})} \right)$$

$$\beta_3(B_p(t), T_p(t), C_p(t)) = r_r(B_p(t), T_p(t), C_p(t))$$

$$- r_p(B_p(t), T_p(t), C_p(t))$$

with the parameter I being the intensity of light and constants γ_i , $i \in \mathbb{N}_{[1,18]}$, μ_2 given as in Table 1.

2.3 Control system

We are interested in studying \mathcal{P} , an interconnection of $N \in \mathbb{N}_+$ control systems $\mathcal{P}^{[i]}$, $i \in \mathbb{N}_{[1,N]}$ that could be either larvae or plant's units.

As for the dynamics of $\mathcal{P}^{[i]}$, they are given, after considering harvesting events within the production units, by the impulsive nonlinear equation (Naghshtabrzi et al., 2008):

$$\dot{z}^{[i]}(t) = f^{[i]}(z(t), v^{[i]}(t))$$

$$z^{[i]}(t_k^+) = A_r^{[i]} z^{[i]}(t_k)$$
(3)

where the function $f^{[i]} : \mathbb{R}^{p_i} \times \mathbb{R}^{m_i} \rightarrow \mathbb{R}^{p_i}$ is a continuous function, $z^{[i]}(t) \in \mathbb{R}^{p_i}$ is the state of $\mathcal{P}^{[i]}$, $z(t) = [z^{[1]}(t); \dots; z^{[N]}(t)]$ is the state of \mathcal{P} , $z^{[j]}(t) \in \mathbb{R}^{p_j}$ is the state of system $\mathcal{P}^{[j]}$, $j \in \mathbb{N}_{[1,N]}$, $v^{[i]}(t) \in \mathbb{R}^{m_i}$ is the control input, $A_r^{[i]} \in \mathbb{R}^{p_i \times p_i}$ is the reset matrix, $z(t^+) = \lim_{\tau \rightarrow 0, \tau > 0} z(t + \tau)$, $T^{[i]} = (t_k^{[i]})_{k \in \mathbb{N}_+}$ is the sequence

Table 1. Values for constants in the plant and larvae models.

Constants	Value	Constants	Value
α_1	1.27×10^{-4}	α_2	-5.67×10^{-6}
α_3	2.77×10^{-4}	α_4	1.38×10^{-5}
α_5	1.38×10^{-5}	α_6	1.38×10^{-5}
α_7	0.01	α_8	0.01
α_9	0.01	α_{10}	1
α_{11}	3.65	α_{12}	0.24×10^{-4}
α_{13}	15.95	α_{14}	7.31×10^{-7}
α_{15}	3.69×10^{-6}	α_{16}	297.5
α_{17}	8.69	α_{18}	234.35
α_{19}	265.5	α_{20}	0.13
α_{21}	1.08	α_{22}	1.28
μ_1	4.6×10^{-3}	γ_1	0.544
γ_2	-2.65×10^{-7}	γ_3	1.38×10^{-5}
γ_4	1.38×10^{-5}	γ_5	1.38×10^{-5}
γ_6	0.01	γ_7	0.01
γ_8	0.01	γ_9	14.28
γ_{10}	4.87×10^{-7}	γ_{11}	1
γ_{12}	53	γ_{13}	3.55×10^{-9}
γ_{14}	5.11×10^{-6}	γ_{15}	2.3×10^{-4}
γ_{16}	6.29×10^{-4}	γ_{17}	5.2×10^{-5}
γ_{18}	3.55×10^{-9}	μ_2	4.57×10^{-4}

of reset times modeling harvesting instants in the production unit, and the initial state of $\mathcal{P}^{[i]}$ is given by $z^{[i]}(0) = z_0^{[i]}$.

We assume that $T^{[i]}$, $i \in \mathbb{N}_{[1,N]}$, is a strictly increasing sequence of impulse times without finite accumulation points, i.e. $t_{k+1}^{[i]} > t_k^{[i]}$ and $\lim_{k \rightarrow +\infty} t_k^{[i]} = +\infty$.

We note that harvesting is controlled and thus the sequence $T^{[i]}$ is designed in Section 3.

The first equation of (3) defines the continuous dynamics of $\mathcal{P}^{[i]}$, given by (1) in case $\mathcal{P}^{[i]}$ is a larvae unit and (2) otherwise, and the second describes the state jumps at impulses, where in case of a larvae unit the reset matrix is

$$A_r^{[i]} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } A_r^{[i]} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ otherwise.}$$

System formulation (3) can model any topology for the interconnection of a set of plant and larvae units. However without loss of generality and for the sake of presentation we restrict ourselves to a circular/ring topology where each subsystem (production unit) has two neighboring subsystems. Thus each subsystem $\mathcal{P}^{[i]}$ has two neighbors $\mathcal{P}^{[N_1^i]}$ and $\mathcal{P}^{[N_2^i]}$, with $N_1^i, N_2^i \in \mathbb{N}_{[1,N]}$.

Problem 1. Given \mathcal{P} , a circular interconnection of larvae and plant units, we design control inputs in each subsystem $v^{[i]} = (v_{v1}^{[i]}, v_{v2}^{[i]}, v_{ve}^{[i]}, v_h^{[i]})$ as well as the sequence of harvesting times $h_k^{[i]} = t_{k+1}^{[i]} - t_k^{[i]}$, $i \in \mathbb{N}_{[1,N]}$, to maximize production, given in terms of variables B_p and B_l , and minimize the energy consumed by the ventilators and heaters in the cubes, $(v_{ve}^{[i]}, v_h^{[i]})$, $i \in \mathbb{N}_{[1,N]}$, while satisfying constraints on the states, control inputs, and reset instants.

3. MODEL PREDICTIVE CONTROL

The goal of this section is to synthesize a centralized MPC controller for the interconnected system \mathcal{P} , whose state is $z = [z^{[1]}; \dots; z^{[N]}]$ and control input is $v = [v^{[1]}; \dots; v^{[N]}]$.

4. CASE STUDY

We propose in the following an optimization problem that designs the controller inputs $v^{[i]} = (v_{v1}^{[i]}, v_{v2}^{[i]}, v_{ve}^{[i]}, v_h^{[i]})$ and the reset times $h_k^{[i]} = t_{k+1}^{[i]} - t_k^{[i]}$, $i \in \mathbb{N}_{[1,N]}$:

Given a sampling period $T \in \mathbb{N}_+$. At every discrete time t_l , $l \in \mathbb{N}$, with $t_{l+1} = t_l + T$, we compute the solution of

$$\begin{aligned}
 \min_{\delta_n^{[i]}, v_n^{[i]}} \quad & \sum_{n=0}^{H-1} \sum_{i=0}^{N-1} -B_n^{[i]}(t_l) + \theta_1 \|v_n^{[i]}(t_l)\| \\
 \text{s.t.} \quad & z_{n+1}^{[i]}(t_l) = f_d^{[i]}(\delta_n^{[i]}, z_n(t_l), v_n^{[i]}(t_l)) \quad (4a) \\
 & \quad \quad \quad i \in \mathbb{N}_{[1,N]}, n \in \mathbb{N}_{[0,H]} \\
 & z_0^{[i]}(t_l) = z^{[i]}(t_l) \quad i \in \mathbb{N}_{[1,N]} \quad (4b) \\
 & z_n(t_l) \in \mathcal{Z} \quad n \in \mathbb{N}_{[0,H]} \quad (4c) \\
 & v_n^{[i]}(t_l) \in \Upsilon^{[i]} \quad i \in \mathbb{N}_{[1,N]} \quad (4d) \\
 & \sum_{i=0}^N \delta_n^{[i]} \leq \theta_2 \quad n \in \mathbb{N}_{[0,H]} \quad (4e) \\
 & \delta_n^{[i]} \in \{0, 1\} \quad i \in \mathbb{N}_{[1,N]}, n \in \mathbb{N}_{[0,H]} \quad (4f)
 \end{aligned}$$

with $f_d^{[i]}(\delta, z, v) = (1 - \delta)(z + Tf^{[i]}(z, v)) + \delta(A_r^{[i]}z + Tf^{[i]}(A_r z, v))$, T is the sampling period, $\delta_n^{[i]}$ are defined such that the next harvesting event is given by $t_{k+1}^i = t_k^i + \delta_0^{[i]*}$, and function $f^{[i]}(\cdot)$ is given either by (1) or (2).

The minimal reset and control sequences at each time t_l are given by $(\delta_n^{[i]*})_{n \in \mathbb{N}_{[0,H]}}$ and $(v_n^{[i]*})_{n \in \mathbb{N}_{[0,H]}}$, $i \in \mathbb{N}_{[1,N]}$, respectively. However, just $v(t_l) = v_0^*(t_l) = [v_0^{[1]*}(t_l); \dots; v_0^{[N]*}(t_l)]$ and $\delta_0^* = [\delta_0^{[1]*}; \dots; \delta_0^{[N]*}]$ are applied and held constant in between instants t_l and t_{l+1} . We note that the objective, as the cost function shows, is to maximize biomass and minimize energy costs which is weighted with respect to biomass by a design parameter θ_1 . There is a slight abuse of notation where the state $B^{[i]}$ refers to $B_p^{[i]}$ when system $\mathcal{P}^{[i]}$ corresponds to a plant unit and $B_l^{[i]}$ otherwise. Constraint (4a) refers to the dynamics of the state in each unit after discretizing the system using a sufficiently small discrete time step T . We note that $\delta_n^{[i]} = 1$ corresponds to the occurrence of harvesting events and thus a state jump occurs before the state flows using the map $f^{[i]}(\cdot)$. (4b) defines the initial state for each subsystem. (4c)-(4d)-(4f) define constraints on the states and decision/input variables. (4e) gives an upper-bound θ_2 on the number of harvesting events at each time step, translating the requirement that at most θ_2 workers can work on harvesting the production units at any given time. We note that other constraints could be added such as $\delta_n^{[i]} B_n^{[i]}(t_l) > \theta_3$ which guarantees that harvesting occur only when the biomass is larger than a parameter $\theta_3 \in \mathbb{R}_+$.

After fixing the design parameters θ_1 , θ_2 , time step T , number of larvae N_L , prediction horizon H , number of subsystems N , and sets of constraints $\mathcal{Z}, \Upsilon^{[i]}, i \in \mathbb{N}_{[1,N]}$ the MINLP (4) defines a receding horizon predictive controller which could be directly implemented on the interconnected system \mathcal{P} . We show the effectiveness of this controller in the next section.

In this section we consider two larvae units and two plant units interconnected in a ring topology through valves that allow air freely to be exchanged in between. Thus the state vector of the interconnected system is given by:

$$z(t) = [z_p^{[1]}(t), z_l^{[2]}(t), z_p^{[3]}(t), z_l^{[4]}(t)]^\top.$$

4.1 Case without disturbances

We solve five optimization problems to synthesize control inputs to maximize biomass when the plants and larvae cubes are decoupled and when they are coupled. In each, we formulate optimization problem (4) after considering no state constraints except for requiring the temperature in the units to be less than 37°C and above 10°C and the CO_2 levels to be below $0.017\text{kg}/\text{m}^3$ in the plant cubes and below $0.1\text{kg}/\text{m}^3$ in the larvae cubes, no resets ($\delta_n^{[i]} = 0$ for $i = 1, 2, 3, 4$, $n \in \mathbb{N}_{[1,70]}$ since simulations corresponds just to 24 hours), $T = 51.4\text{s}$, $\theta_1 = 10^{-6}$, $\theta_2 = 0$, and $\Upsilon^{[i]} = [0, 1]^4$, $i = 1, 2, 3, 4$. The process simulations to evaluate the optimization problem online starts with the initial state vector

$$z(t_0) = [0.25, 15, 5 \times 10^{-4}, 0.02, 124, 20, 10^{-3}, 0.05, 15, 5 \times 10^{-4}, 10^{-3}, 0, 20, 10^{-3}]^\top \quad (5)$$

with t_0 being 6am . We note that the initial number of larvae is $N_L = 2000$ and each of them has a weight given as shown in the initial state by 10^{-3} in the fourth cube. The initial weight of 0.02g in the second larvae unit corresponds to a larvae of 124 hours age, as the initial development sum shows, in optimal environment conditions. The light intensity is given by the sinusoidal signal reaching the peak amplitude at noon as shown in Figure 3. We solve the problem using the tool CASADI Andersson et al. (2012) in Matlab with an execution time of about 3 hours.

Table S2 compares the magnitude of the control inputs and the biomass in the decoupled setup to those of the coupled setup. The results show that the biomass is increased and the costs are decreased when coupling units together. The reason is two folded: First, the high CO_2 production by the larvae's unit is routed in part to the plant units through coupling, Figure 3. In the decoupled case the plants were getting the needed CO_2 through ventilation, Figure 2, and the CO_2 in the larvae's unit were just wasted to the environment. Second, there is less need in the plant's unit to activate the heater since the airflow from the larvae's unit already have a higher temperature and less airflow is coming in through ventilation from the outside environment (at 10°C). Also, the increase in the biomass production result when plants receive air from the larvae's unit having a higher concentration of CO_2 than the outside environment ($10^{-3}\text{kg}/\text{m}^3$).

4.2 Case with disturbances

We now consider another scenario with the same four production units interconnected in the same ring topology. The difference is the introduction of disturbances which are unknown apriori by the MPC controller. We introduce two types of disturbances as shown in Figure 1. The first

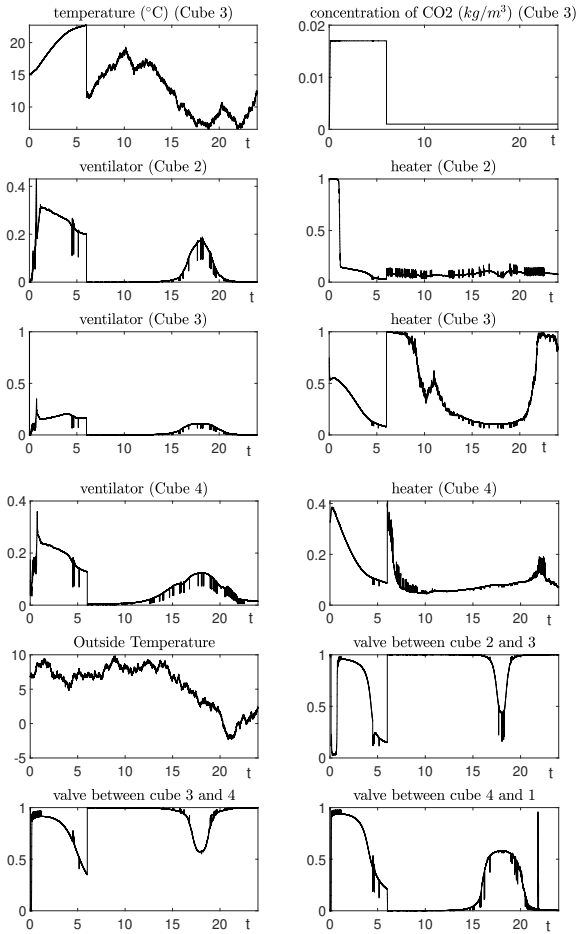


Fig. 1. Trajectories for states and control inputs in production units with an instant disturbance after time $t = 6$ hrs.

corresponds to the change in the profile of the outside temperature. The other disturbance corresponds to a sudden drop in temperature and CO_2 concentration in the third production unit which could be a result of opening a door in this plant unit. Then we study the performance of the MPC controller when implemented online to inspect the opening and closing of the valves between cubes as well as the operation of the different heaters and ventilators in reaction to the disturbances.

Figure 1 shows that, when the door is opened, the valves between different production units are opened so air inside the cubes flows to the unit undergoing a sudden temperature and CO_2 drop in order to regulate both states and return them to the optimal levels. The heaters played a significant role in this regard where the heater in the third and fourth production unit is activated significantly after the sudden drop occurred. As for the heater in the second cube it was not activated because the temperature in the corresponding cube was too high.

5. CONCLUSION

We synthesize MPC laws monolithically for an interconnected system comprised of units containing larvae and plants. The results show how the valves optimally route the air in between production units to obtain a signifi-

cant reduction in power costs and a higher biomass yield compared to the case when the units are decoupled. As a future work, we aim at solving the problem of designing the interconnection topology to maximize biomass and minimize power costs. Another future work is to synthesize the controllers in the food production units compositionally so that more units could be considered with a larger prediction horizon. This scalability problem also forbids us from considering harvesting events when solving the MPC problem centrally. The reason is that a large number of variables shall be considered as the prediction horizon should be at least 200 hours, almost the minimum time needed for a larvae to reach its maximum mass given it is growing in optimal surrounding conditions.

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Table S2. Number of steps ventilators and heaters are turned on for the coupled and decoupled setup. Total time 24 hours, discretization step = 51.4 seconds, prediction horizon = 70 .

Units	Decoupled			Units	Coupled		
	ventilator	heater	biomass		ventilator	heater	biomass
plant	2334	1135	0.3214	plant	242	195	0.332
larvae	1210	237	0.0338	larvae	377	158	0.034
sum	3544	1272	0.3552	sum	619	353	0.366

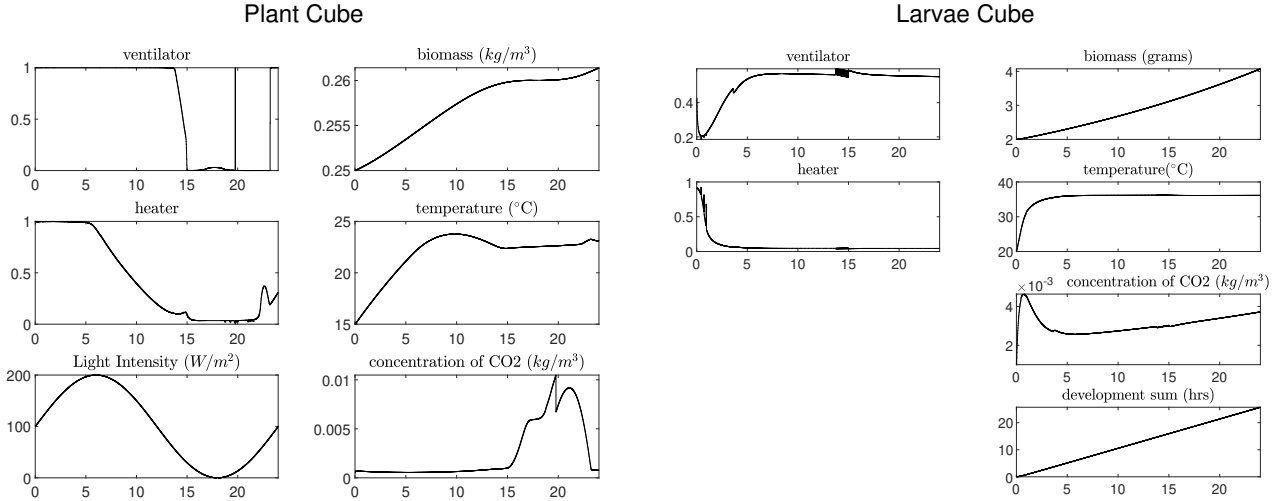


Fig. 2. Trajectories for states and control inputs in a plant's (left) and larvae' (right) unit for the decoupled setup for a duration of 24 hrs.

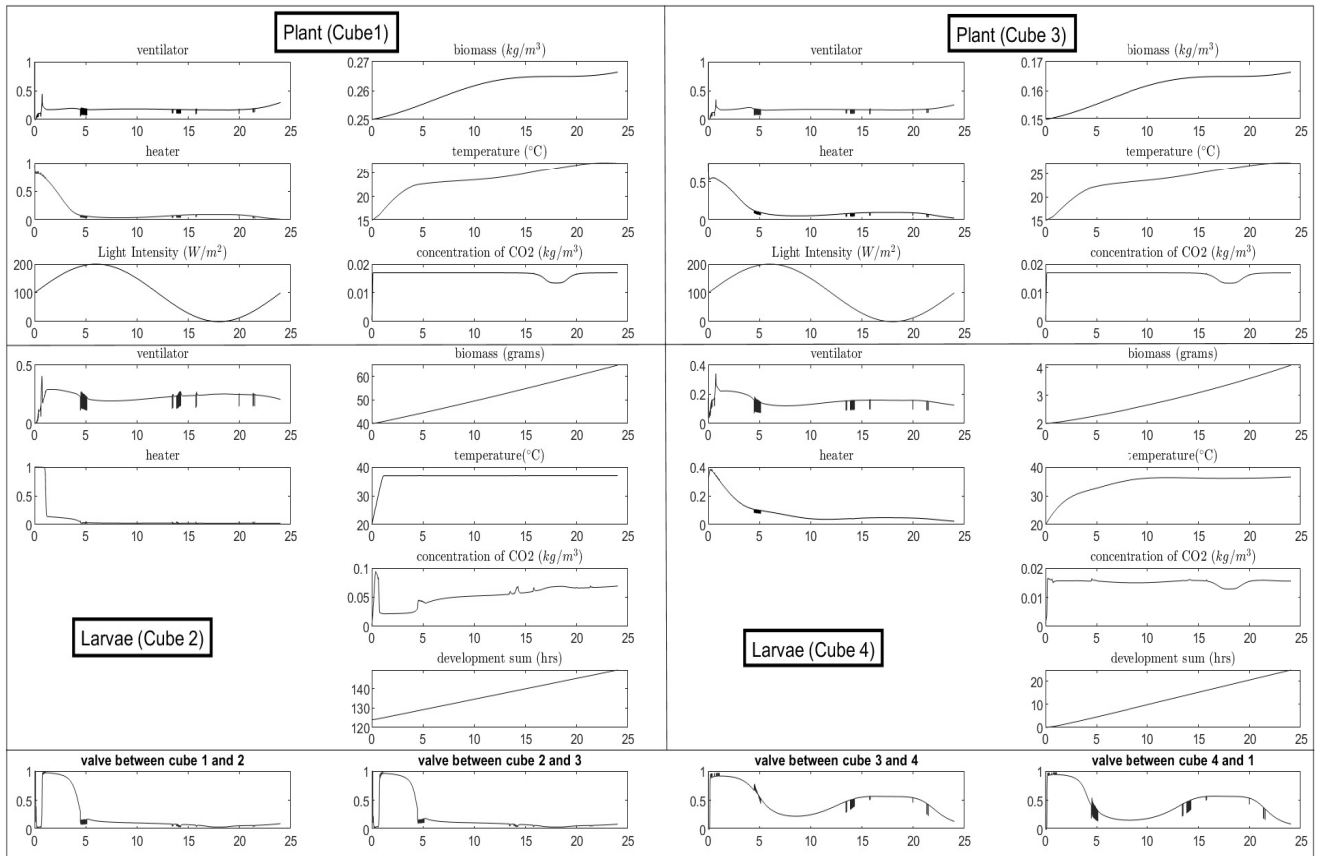


Fig. 3. Trajectories for states and control inputs of plant (Cube 1 and 3) and larvae cubes (Cube 2 and 4) connected in a ring topology for 24 hours. The bottom 4 figures show the opening and closing of valves connecting the cubes.