Proceedings of the 9th International Symposium on
Dynamics and Control of Process Systems (DYCOPS 2010),
Leuven, Belgium, July 5-7, 2010
Mayuresh Kothare, Moses Tade, Alain Vande Wouwer, Ilse Smets (Eds.)

A Simultaneous Approach for Correcting Differences between Units in Multi-unit Optimization ⋆

Lyne Woodward, Michel Perrier, Bala Srinivasan

Ecole Polytechnique de Montreal, Montreal, Qu. Canada (e-mail: lyne.woodward@polymtl.ca).

Abstract: Multi-unit optimization is a recently proposed extremum-seeking technique where the gradient, estimated by finite difference between two identical units, is controlled to zero. The main assumption of having two identical units is rarely verified in practice and it has been noted that differences in the static characteristics can affect the stability and the equilibrium point. So, correctors have been proposed, where optimization and correction for differences are performed alternatively. This, in turn, causes a discontinuous operation leading to a hybrid dynamics. To avoid such a scenario, an approach where optimization and correction take place simultaneously, is presented. A proof of stability of the simultaneous scheme is also provided.

Keywords: Multi-unit optimization, real-time optimization, extremum-seeking control

1. INTRODUCTION

Real-time optimization methods have the objective to bring and maintain a process to its optimal point of operation. To achieve this task, a model of the process is used to identify the descent direction which will improve the performance. The type of model used characterizes the different real-time optimization methods existing in the literature. When the objective function can be evaluated directly by the measurements, the use of an empirical model can be sufficient (Ariyur and Krstic (2003), McFarlane and Bacon (1989), Wellstead and Scotson (1990)). Otherwise, a fundamental model is required (Jang et al. (1987), Chen and Joseph (1987), Guay and Zhang (2003)).

When the performance criteria, and the constraints if any, can be evaluated directly from the available measurements, a simple empiric static linear model can be used (Krstic and Wang, 2000). Then, inputs of the process can be controlled in order to push the gradient estimated from the model to zero. Multi-units optimization method uses this type of model (Srinivasan, 2007). The gradient is estimated by finite differences between identical units of the process which are run with an offset between their inputs. When the units are identical, using a single model for all the units will bring them to their respective optima. However, when the units are similar but non identical, using a single model will not be sufficient. In order to allow the multi-unit optimization method to converge to the real optima, two models are required.

Sequential correctors were added to the original scheme in Woodward et al. (2009). Therein, a sequential approach was proposed where the multi-unit optimization was regularly interrupted to update the correctors. Even though this scheme was able to bring the two units to their respective optima, this approach presented some limitations. First of all, the interruption of the optimization needed to compute the correctors is decreasing the ability to do real-time optimization. That means that if a disturbance occurs during the corrector's adaptation period, the new optimal point is not adapted before the end of the time allowed to adapt these correctors. Also, with this sequential approach, an instantaneous change on the input is needed at each transition between optimization and correction introducing oscillations at the output. Finally, the tuning of the corrector's gains can represent a real challenge especially when the curvature of the objective function does not allow to quickly get a good approximation of the correctors. In such a case, the convergence toward the optimum is strongly dependant from the alternate between the multi-unit optimization and the correctors adaptation. In other words, the tuning of the gains depends of the moment when the corrector's adaptation is interrupted.

In this paper, a simultaneous approach is proposed where both optimization and corrector's adaptation are performed continuously. To do so, an empiric static quadratic model is added to the original multi-unit optimization structure. This model is used to identify the values of the correctors while the static linear model is still used for optimization purpose.

In Section 2, the multi-unit optimization method is presented. Section 3 reviews the effects of differences between units on the convergence of the multi-unit method. Section 4 contains the proposed simultaneous adaptive correctors approach while Section 5 shows the results obtained from the application of multi-unit optimization method with simultaneous adaptive correctors to a simulation example. Conclusions are provided in Section 6.

⋆ This work is supported by the Natural Science and Engineering Research Council of Canada.

2. MULTI-UNIT OPTIMIZATION

2.1 Problem formulation

Consider a dynamic system with state $x \in \Re^n$, input $u \in \Re^m$ that has to be operated so as to minimize a convex function $J(x, u)$ at steady state. The problem is shown below:

$$\min_u \; J(x, u) \tag{1}$$

$$s.t. \quad \dot{x} = F(x, u) \equiv 0 \tag{2}$$

where $F(x, u)$ is the function describing the dynamics of the system, which is assumed to be stable. The necessary conditions of optimality are given by :

$$\frac{dJ}{du} = \frac{\partial J}{\partial u} - \frac{\partial J}{\partial x}\left(\frac{\partial F}{\partial x}\right)^{-1}\frac{\partial F}{\partial u} = 0 \tag{3}$$

As in the steepest descent method for numerical optimization (Nocedal and Wright, 1999), extremum-seeking makes the process evolve in the opposite direction of the gradient. But instead of using the iteration index as in numerical methods of optimization, the iterations evolve in real time. The extremum-seeking control law is given by :

$$\dot{u} = -k\left(\frac{dJ}{du}\right)^T \tag{4}$$

The key problem is the estimation of the gradient, which could be addressed using several methods (Guay and Zhang (2003); Krstic and Wang (2000)). The multi-unit method provides an estimate of the gradient by finite differences as will be shown next.

2.2 The multi-unit scheme

The multi-unit optimization method uses a linear static model to estimate the gradient. The applicability of this method is guaranteed by the presence of $m + 1$ identical units in the system to optimize with m being the number of elements in the input vector u. Then, for a single input process, two identical units are required. These units are operated with inputs different one from each other with an offset of Δ, as shown in Fig. 1:

$$u_1 = u - \frac{\Delta}{2} \tag{5}$$

$$u_2 = u + \frac{\Delta}{2} \tag{6}$$

The gradient $\hat{g}$ is estimated by finite differences between the output of the units noted by J_1 and J_2:

$$\hat{g}(u) = \frac{J_2(x_2, u_2) - J_1(x_1, u_1)}{\Delta} \tag{7}$$

The extremum-seeking control law (4) is then applied:

$$\dot{u} = -k\hat{g}^T(u) \tag{8}$$

Let u^* be the equilibrium point where the multi-unit optimization algorithm converges. This means that the

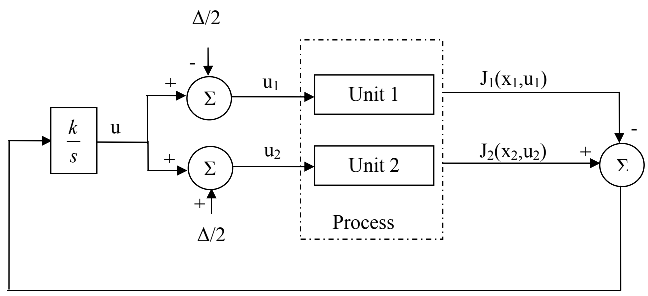

Fig. 1. Schematic for multi-unit optimization

inputs of the two units would converge to $u_1^* = u^* - \frac{\Delta}{2}$ and $u_2^* = u^* + \frac{\Delta}{2}$.

All units follow the same control law and always keep an input offset of Δ from each other. The convergence of this scheme to a ball around the optimum has been proven despite the errors caused by the dynamics (which is assumed to be stable) and the error due to finite differences.

As the units are identical, they all have the same dynamics. These dynamics are eliminated by the gradient estimation using the finite differences allowing a faster adaptation for the integral controller. As the perturbation used to estimate the gradient is not a temporal one, the use of filters is unnecessary here. The only time scale separation needed is between the adaptation and the dynamic of the system. Then, in comparison with methods using a temporal perturbation signal, a faster convergence toward the optimum can be achieved especially for slow dynamics processes. Also, as no sinusoidal perturbation is needed, no oscillations will be introduce around the optimal point. However, the assumption of having identical units is a strong assumption that does not depicts the reality. The effects of applying the multi-unit optimization method to optimize processes with non-identical units will be presented next.

3. MULTI-UNIT OPTIMIZATION WITH NON-IDENTICAL UNITS

3.1 Characterization of the difference between the units

Differences between the units of a process can take many forms. For example, the units can have different dynamics. The stability of the multi-unit optimization method applied to processes where the units have different dynamics can be guaranteed by an adequate choice of the sign of Δ (Reney et al., 2009). This choice requires a minimal knowledge of the relative response time of the units.

The static characteristics can also differ. This type of differences is the point of interest of the present paper. The following assumptions define the problem under study: i) the optimization problem has a unique manipulated variable and the process contains two similar units, ii)dynamics of the system are very fast in comparison with the time scale of the optimization, i.e. the process can be considered in quasi-static, iii)measurements are noiseless and, iv)the objective function is a convex function.

Considering $J_1(u_1)$ and $J_2(u_2)$, the static curves of unit 1 and unit 2 of the process respectively such that,

$$\left.\frac{\partial J_1}{\partial u}\right|_{u_1^{opt}} = \left.\frac{\partial J_2}{\partial u}\right|_{u_2^{opt}} = 0 \tag{9}$$

where u_1^{opt} and u_2^{opt} denote the optima of the first and second unit respectively, the relation between the static curves of the two units can be formulated as follow:

$$J_2(u) = J_1(u+\beta) + \gamma + \bar{J}(u+\beta) \tag{10}$$

where,

$$\beta = u_1^{opt} - u_2^{opt} \tag{11}$$

$$\gamma = J_2(u_2^{opt}) - J_1(u_1^{opt}) \tag{12}$$

This expression describes a shift between the two units both on the input, u (represented by β), and on the output, J (represented by γ). The function $\bar{J}$ quantifies the difference on the curvature of the static characteristics of the two units for any operational point.

The function $\bar{J}$ and its derivative, evaluated at the operational point u_1^{opt} can be obtained by evaluating the equation (10) at the point $u = u_2^{opt}$. Simplifying, these equations become:

$$\bar{J}(u_1^{opt}) = 0, \qquad \left.\frac{\partial \bar{J}}{\partial u}\right|_{u_1^{opt}} = 0 \tag{13}$$

Then, around the optimum, if the difference of curvatures between the two units at their respective optimum is negligible, i.e.$\frac{\partial^2 \bar{J}}{\partial u^2} \simeq 0$, $\bar{J}$ can be considered to be zero, $\bar{J} \simeq 0$. This assumption complete the description of the problem under study.

3.2 Equilibrium and stability of the multi-unit optimization method for processes with non identical units

Differences between the static characteristics of the units can bring the process to converge to a point of operation far from the real optimum, or worst case, can bring the process to diverge. The equilibrium point and the conditions assuring convergence of the process in such a case have been identified in Woodward et al. (2009) and are presented below.

Consider the extremum-seeking control law (8) and the multi-unit gradient estimation (7). If $\bar{J} \simeq 0$ in the neighborhood of the optimum, it was shown that:

- the equilibrium point u^*, can be approximated by:

$$u^* \simeq \frac{u_1^{opt} + u_2^{opt}}{2} - \frac{\gamma}{(\Delta+\beta)\frac{\partial^2 J_1}{\partial u^2}} \tag{14}$$

- the local stability is guaranteed iff the parameter Δ is chosen such that:

$$(\Delta)(\Delta+\beta) > 0 \tag{15}$$

4. MULTI-UNIT OPTIMIZATION WITH ADAPTIVE CORRECTORS: A SIMULTANEOUS APPROACH

In order to bring the multi-unit optimization method to converge to the real optimum, adaptive correctors are added to the original scheme as shown in Fig. 2. In the proposed approach, these correctors, $\hat{\beta}$ and $\hat{\gamma}$, are adapted simultaneously with the evolution of the process to its optimum.

Including simultaneous adaptive correctors in the multi-unit optimization scheme, requires an additional assumption. The approximation of the objective function using a quadratic model, noted J_m, should be possible, i.e.,

$$J_m = \phi^T \theta \tag{16}$$

with

$$\phi^T = [1 \quad u \quad u^2] \tag{17}$$

and θ being the adaptive parameters of the model. The assumption for which J_1 and J_2 have the same curvature around the optimum remains. The objective functions of each unit can then be modeled by:

$$J_{m1}(u_1) = \theta_{1a} u_1^2 + \theta_{1b} u_1 + \theta_{1c} \tag{18}$$

$$J_{m2}(u_2) = \theta_{2a} u_2^2 + \theta_{2b} u_2 + \theta_{c2} \tag{19}$$

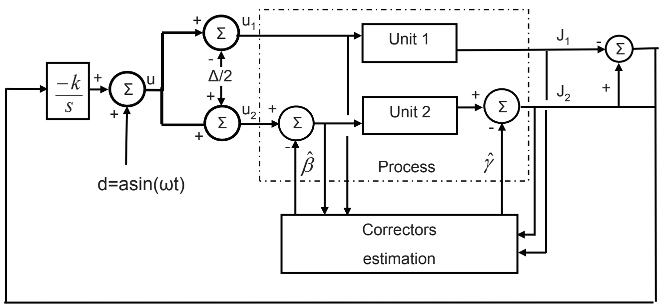

Fig. 2. Structure of the multi-unit optimization method with simultaneous adaptive correctors

As shown in Fig. 2, a temporal persistent perturbation with frequency ω is added to the input of the system, u. Then, the values of the correctors $\hat{\beta}$ and $\hat{\gamma}$ are computed using the estimated parameters $\hat{\theta}^T = [\hat{\theta}_1^T \, \hat{\theta}_2^T]$. These parameters are estimated using a recursive least square algorithm with forgetting factor Ljung (1999). The adaptive laws of this RLS algorithm are as follow:

$$\dot{\hat{R}} = \Phi^T \Phi - \lambda \hat{R} \tag{20}$$

$$\dot{\hat{\theta}} = \hat{R}^{-1} \Phi (J - \Phi^T \hat{\theta}) \tag{21}$$

where, $\hat{R}$ is the correlation matrix, λ, the forgetting factor. It is supposed here that the system is sufficiently excited to estimate the parameters $\hat{\theta}$ of the model, i.e. $\delta_1 I < R < \delta_2 I$ with $\delta_2 > \delta_1 > 0$.

The optimization is done by the multi-unit method,

$$u_1 = u - \frac{\Delta}{2} + a\sin(\omega t) \tag{22}$$

$$u_2 = u + \frac{\Delta}{2} - \hat{\beta} + a\sin(\omega t) \tag{23}$$

$$\dot{u} = -\frac{k_{mu}}{\Delta}(J_2 - J_1 - \hat{\gamma}) \tag{24}$$

simultaneously with the adaptation of the correctors $\hat{\beta}$ and $\hat{\gamma}$:

$$\dot{\hat{\beta}} = k_\beta \left(\hat{u}_1^{opt} - \hat{u}_2^{opt} - \hat{\beta}\right) \quad (25)$$

$$\dot{\hat{\gamma}} = k_\gamma \left(\hat{J}_2(\hat{u}_2^{opt}) - \hat{J}_1(\hat{u}_1^{opt}) - \hat{\gamma}\right) \quad (26)$$

where $\hat{u}_1^{opt}$ and $\hat{u}_2^{opt}$ are the estimated optimal operational points of the models of unit 1 and unit 2 respectively (from equations (18-19)). These control laws are based on the definition of β and γ themselves given by equations (11) and (12).

4.1 Convergence analysis of the multi-unit optimization method with simultaneous adaptive correctors

The stability of the equilibrium point of the multi-unit optimization method with simultaneous adaptive correctors described previously will be analyzed.

Theorem 4.1. Consider the input to be persistently exciting, i.e. the correlation matrix being bounded with $\delta_1 I < R < \delta_2 I$ with $0 < \delta_1 < \delta_2$. Then, it is possible to choose the tuning parameters k_{mu}, k_β and k_γ such that the equilibrium point describe by equations (22-21) given by $u^e = u_1^{opt}$, $\hat{\beta} = \beta$ and $\hat{\gamma} = \gamma$ is locally asymptotically stable.

Proof:

Consider the following Lyapunov function:

$$V = \frac{1}{2}w_1(u - u^e)^2 + \frac{1}{2}w_2\tilde{\beta}^2 + \frac{1}{2}w_3\tilde{\gamma}^2 + \frac{1}{2}\tilde{\theta}^T R\tilde{\theta} \quad (27)$$

Where w_1, w_2 and $w_3 > 0$, $\tilde{\beta} \equiv \hat{\beta} - \beta$, $\tilde{\gamma} \equiv \hat{\gamma} - \gamma$ and $\tilde{\theta} \equiv \hat{\theta} - \theta$. Then, $V \geq 0$ and $V = 0$ when $u = u^e$, $\tilde{\beta} = \tilde{\gamma} = \tilde{\theta} = 0$. According to the definition of $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\theta}$, the following equivalences apply:

$$\dot{\tilde{\beta}} = \dot{\hat{\beta}} \quad (28)$$

$$\dot{\tilde{\gamma}} = \dot{\hat{\gamma}} \quad (29)$$

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} \quad (30)$$

The temporal derivative of the Lyapunov function is given by the following equation:

$$\dot{V} = w_1(u - u^e)\dot{u} + w_2\tilde{\beta}\dot{\tilde{\beta}} + w_3\tilde{\gamma}\dot{\tilde{\gamma}} + \tilde{\theta}^T R\dot{\tilde{\theta}} + \tilde{\theta}^T \dot{R}\tilde{\theta} \quad (31)$$

The non-linearity of the expressions of $\dot{\tilde{\beta}}$, $\dot{\tilde{\gamma}}$ and $\dot{u}$ will introduce terms higher than the second order in the expression of $\dot{V}$. As the proof of convergence provided here is locally valid, these terms will be neglected (Khalil, 1996).

Considering again the assumption that the static characteristics of the units have the same curvature around the optimum, such that $\theta_{1a} = \theta_{2a}$, the control law (25) can be rewritten as follow:

$$\dot{\tilde{\beta}} = \dot{\hat{\beta}} = k_\beta \left(\frac{\theta_{1b} - \tilde{\theta}_{1b}}{2(\theta_{1a} + \tilde{\theta}_{1a})} - \frac{\theta_{2b} - \tilde{\theta}_{2b}}{2(\theta_{1a} + \tilde{\theta}_{2a})} - \hat{\beta}\right) \quad (32)$$

Using the equivalence $\hat{\beta} = \tilde{\beta} + \beta$ and reorganizing, we obtain:

$$\dot{\tilde{\beta}} = k_\beta \Big(-\tilde{\beta} - \frac{\tilde{\theta}_{2a}\beta - \frac{\tilde{\theta}_{2b}}{2}}{\theta_{1a} + \tilde{\theta}_{2a}} - \frac{\tilde{\theta}_{1b}}{2(\theta_{1a} + \tilde{\theta}_{1a})} + \frac{\theta_{1b}(\tilde{\theta}_{1a} - \tilde{\theta}_{2a})}{2(\theta_{1a} + \tilde{\theta}_{1a})(\theta_{1a} + \tilde{\theta}_{2a})}\Big) \quad (33)$$

A second order Taylor development of the expression $\frac{1}{\theta_{1a} + \tilde{\theta}_{2a}}$ gives:

$$\frac{1}{\theta_{1a} + \tilde{\theta}_{2a}} = \frac{1}{\theta_{1a}}\Big(\frac{1}{1 + \frac{\tilde{\theta}_{2a}}{\theta_{1a}}}\Big) \quad (34)$$

$$\approx \frac{1}{\theta_{1a}}\left(1 - \frac{\tilde{\theta}_{2a}}{\theta_{1a}} + \frac{\tilde{\theta}_{2a}^2}{\theta_{1a}^2}\right) \quad (35)$$

Using this expression and neglecting the second order terms lead to:

$$\dot{\tilde{\beta}} \approx k_\beta \left(-\tilde{\beta} - \frac{\tilde{\theta}_{2a}\beta - \frac{\tilde{\theta}_{2b}}{2}}{\theta_{1a}} - \frac{\tilde{\theta}_{1b}}{2\theta_{1a}} + \frac{\theta_{1b}(\tilde{\theta}_{1a} - \tilde{\theta}_{2a})}{2\theta_{1a}^2}\right) \quad (36)$$

Using a similar approach, we also obtain:

$$\dot{\tilde{\gamma}} \approx k_\gamma(-\tilde{\gamma} + \tilde{\theta}_{2a}\beta^2 - \frac{\beta(\tilde{\theta}_{2b}\theta_{1a} - \tilde{\theta}_{2a}\theta_{1b})}{\theta_{1a}} + \frac{\theta_{1b}(\tilde{\theta}_{1b} - \tilde{\theta}_{2b})}{2\theta_{1a}} + \frac{\theta_{1b}^2(\tilde{\theta}_{2a} - \tilde{\theta}_{1a})}{4\theta_{1a}^2} + \tilde{\theta}_{2c} - \tilde{\theta}_{1c}) \quad (37)$$

Using equation (10) in equation (24) and using the equivalences $\hat{\beta} = \tilde{\beta} + \beta$ and $\hat{\gamma} = \tilde{\gamma} + \gamma$, we get:

$$\dot{u} = \frac{-k_{mu}}{\Delta}\left(J_1(u + \frac{\Delta}{2} - \tilde{\beta}) - J_1(u - \frac{\Delta}{2}) - \tilde{\gamma}\right) \quad (38)$$

A second order Taylor expansion around the point u_e leads to:

$$\dot{u} \approx -k_{mu}\Big(\frac{\partial^2 J_1}{\partial u^2}(u - u_e) + \frac{\partial^2 J_1}{\partial u^2}(\frac{\tilde{\beta}^2}{2\Delta} - \tilde{\beta}\frac{(u - u_e)}{\Delta} + \frac{\tilde{\beta}}{4}) - \frac{\tilde{\gamma}}{2\Delta}\Big) \quad (39)$$

Keeping only the linear part, we obtain:

$$\dot{u} \approx -k_{mu}\left(\frac{\partial^2 J_1}{\partial u^2}(u - u_e) + \frac{\partial^2 J_1}{\partial u^2}(\frac{\tilde{\beta}}{4}) + k_{mu}\frac{\tilde{\gamma}}{2\Delta}\right) \quad (40)$$

Using equations (36), (37) and (40), the expression of the temporal derivative of the Lyapunov function becomes:

$$\dot{V} = -A(u - u_e)^2 - B\tilde{\beta}^2 - C\tilde{\gamma}^2 - \lambda\tilde{\theta}^T R\tilde{\theta} + D\tilde{\gamma}(u - u_e) + E\tilde{\beta}(u - u_e) + F^T\tilde{\theta}\tilde{\beta} + G^T\tilde{\theta}\tilde{\gamma} \quad (41)$$

where:

$$A = k_{mu}\omega_1\theta_{1a} > 0$$

$$B = k_\beta\omega_2 > 0$$

$$C = k_\gamma\omega_3 > 0$$

$$
\begin{aligned}
D &= \frac{k_{mu}\omega_1}{2\Delta} \\
E &= -\frac{k_{mu}\omega_1}{4}\theta_{1a} \\
F^T &= \left[\frac{k_\beta \omega_2}{2\theta_{1a}^2} \quad -\frac{k_\beta \omega_2}{2\theta_{1a}} \quad 0 \quad \frac{k_\beta \omega_2}{2\theta_{1a}^2}\beta \quad -\frac{k_\beta \omega_2}{2\theta_{1a}} \quad 0 \right] \\
G^T &= \Big[-\frac{k_\gamma \omega_3 \theta_{1b}^2}{4\theta_{1a}^2} \quad \frac{k_\gamma \omega_3 \theta_{1b}}{2\theta_{1a}} \quad -\omega_3 k_\gamma ... \\
&\quad ... \frac{k_\gamma \omega_3 \theta_{1b}^2}{4\theta_{1a}^2} + \frac{k_\gamma \omega_3 \beta \theta_{1b}}{\theta_{1a}} + k_\gamma \omega_3 \beta^2 ... \\
&\quad ... \quad k_\gamma \omega_3 \beta - \frac{k_\gamma \omega_3 \theta_{1b}}{2\theta_{1a}} \quad \omega_3 k_\gamma \Big] \\
\lambda &> 0
\end{aligned}
$$

In order to have $\dot{V} < 0$, the following expressions must be verified:

$$
\begin{aligned}
D^2 &< 4AC \\
E^2 &< 4AB \\
F^T F &< 4B\lambda R \\
G^T G &< 4C\lambda R
\end{aligned}
$$

The two last conditions may appear difficult to satisfy since the value of the elements of the matrix R are time dependant. However, since the matrix R is bounded by $\delta_1 I < R < \delta_2 I$ with $0 < \delta_1 < \delta_2$, these conditions can be met by choosing the parameters k_β, k_γ, ω_2 and ω_3 such that $FF^T < 4B\lambda\delta_1$ and $GG^T < 4C\lambda\delta_1$. The condition $\dot{V} < 0$ can then be met by an adequate choice of the values of ω_1, ω_2, ω_3, k_{mu}, k_β, k_γ, λ and Δ. This demonstrate the local asymptotic stability of the equilibrium point of the multi-unit optimization method with simultaneous adaptive correctors. □

5. ILLUSTRATIVE EXAMPLE

The problem under study is the production of green fluorescent protein (GFP) by E. coli cells. The following kinetic model, based on glucose as growth-limiting substrate, presented in Aucoin et al. (2006) was used for the simulations:

$$\dot{X} = \mu X - \left(\frac{F}{V}\right) X \tag{42}$$

$$\dot{P} = (Y_{P/S}\mu + \beta)X - \frac{F}{V}P \tag{43}$$

$$\dot{S} = \left(\frac{F}{V}\right)(S_f - S) - \frac{\mu X}{Y_{X/S}} - \frac{(Y_{P/X}\mu + \beta)X}{Y_{P/S}} - m_s\left(\frac{S}{K_{s_m} + S}\right) X \tag{44}$$

where X is the biomass concentration, F the feed rate of the substrate into the bioreactor, μ the specific growth rate of the biomass, V the volume of the bioreactor, S the substrate concentration, S_f the concentration of the substrate inlet, P the concentration of GFP, $Y_{P/S}$ is the product yield on substrate coefficient, $Y_{P/X}$ the production yield on biomass coefficient, $Y_{X/S}$ the biomass yield on substrate coefficient, β the non-growth associated product formation constant, and m_S the maintenance coefficient. The Monod model is used for the expression of μ:

$$\mu = \frac{\mu_{max} S}{K_s + S} \tag{45}$$

where μ_{max} is the maximum growth rate constant and K_s a saturation constant.

The optimization problem is to maximize the quantity of GFP in the post-induction period by adjusting the substrate flow into the bioreactor:

$$\max_F \; FP \tag{46}$$

$$s.t. \quad (42), (43), (44) \equiv 0 \tag{47}$$

The multi-unit optimization method with simultaneous adaptive correctors is applied to the system containing two similar bioreactors. Figure 3 shows the static characteristics of the two bioreactors obtained when the numerical values of Table 1 are used.

Table 1. Numerical values used for the parameters of the model

	Unit 1	Unit 2	
m_s	0.0025	0.0025	g S/(g X h)
K_S	0.4	0.4	g/L
$Y_{P/X}$	66.92	66.92	mg P/g X
S_f	60	60	g/L
$Y_{P/S}$	50	50	mg P/g S
K_{S_m}	0.04	0.04	g/L
α	0.1	0.1	mg P/(g X h)
V	20	20	L
μ_{max}	0.925	**1**	h^{-1}
$Y_{X/S}$	0.35	**0.4**	gX/gS

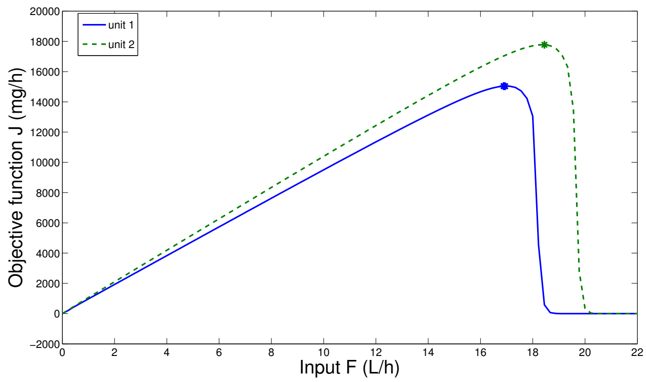

Fig. 3. Static characteristics of the two non identical bioreactors

As these static curves don't show a quadratic form, the assumption used in the proof is not verified here. Then, the control laws used for the adaptation of the correctors are different than the ones presented in the proof but they are similar to the ones used with the sequential approach:

$$\dot{\hat{\beta}} = k_\beta \left(2\hat{\theta}_{2a}(u - \hat{\beta}) + \hat{\theta}_{2b} - 2\hat{\theta}_{1a}(u) + \hat{\theta}_{1b}\right) \tag{48}$$

$$\dot{\hat{\gamma}} = k_\gamma \Big(\hat{\theta}_{2a}(u - \hat{\beta})^2 + \hat{\theta}_{2b}(u - \hat{\beta}) + \hat{\theta}_{2c} - \hat{\theta}_{1a}(u)^2 + \hat{\theta}_{1b}(u) + \hat{\theta}_{1c} - \hat{\gamma}\Big) \tag{49}$$

Figure 4 shows the optimization results obtained using the tuning parameters given in Table 2 .

Table 2. Tuning parameters of the multi-unit optimization method with simultaneous correctors

Δ	-1	L/h
k_β	-4.5 $\times 10^{-6}$	$\frac{L^2}{h^2 mg}$
k_{mu}	2.25×10^{-5}	$\frac{L^2}{h^2 mg}$
k_γ	1.2×10^{-2}	$\frac{1}{h}$
a	1	L/h
ω	0.06	rad/h
λ	0.017	
$R_{i,j\|i=j}(0)$	1	
$R_{i,j\|i\neq j}(0)$	0	

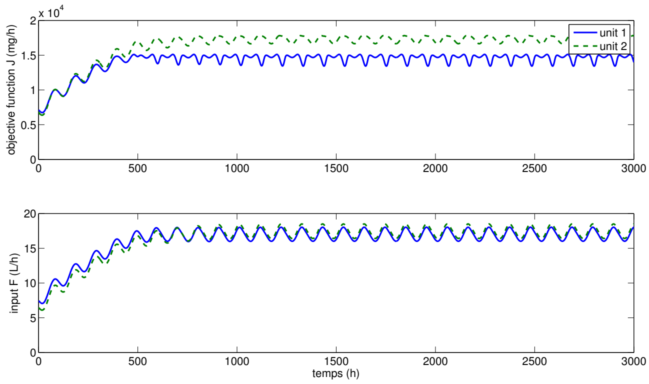

Fig. 4. Results of the multi-unit optimization method with simultaneous correctors applied to two non identical bioreactors

Even if the static curves are not quadratics, using a quadratic model makes the system converge to a point relatively close to the real optimum in about 1000 hours, which is almost the same time than when a sequential approach is chosen (results not shown). Then, the same performance is achieved without any discontinuity in the optimization. The disturbances occurring in the system will be easier to compensate by the closed loop system.

The $\hat{\gamma}$ corrector needs the values of the outputs of the units at the same point of operation. These values can be easily obtained even with a poor estimation of the model parameters. A first order model would be enough to get a good estimation of the $\hat{\gamma}$ corrector. The $\hat{\beta}$ corrector needs the estimation of the gradients of the units at the same operational point. As the real operational points of the units are different from an offset of Δ, the model must be able to take into account a change in the slope between two different points of operation. Then, a second order model is needed to get a good evaluation of the $\hat{\beta}$ corrector. Also, the amplitude of the excitation periodic signal used must be large enough to allow a good estimation of parameters $\hat{\theta}_{2a}$ and $\hat{\theta}_{1a}$ around the optimum (estimation of the curvature) in order to get an adequate value of $\hat{\beta}$. The amplitude of the persistent excitation signal is a trade-off between a good estimation of the corrector $\hat{\beta}$ and the resulting oscillations amplitude around the optimum.

6. CONCLUSION

In this paper, an improved multi-unit scheme with simultaneous adaptive correctors was presented in order to handle non- identical units. The adaptation of the correctors and multi-unit optimization are performed simultaneously using a static empiric quadratic model of the process. Local stability and convergence of the scheme to the respective optima were demonstrated.

The ideas were illustrated with two bioreactors with differences both in static and dynamic characteristics. Both bioreactors converge to a neighborhood of their real optimal point. Future work will focus on using a different type of empiric models to improve the applicability to processes described by non-quadratic objective functions.

REFERENCES

Ariyur, K. and Krstic, M. (2003). *Real-time Optimization by Extremum-Seeking Control.* John Wiley and Sons.

Aucoin, M., McMurray-Beaulieu, V., Poulin, F., Boivin, E., Chen, J., Ardelean, F., Cloutier, M., Choi, Y., Miguez, C., and Jolicoeur, M. (2006). Identifying conditions for inducible protein production in E.coli: Combining fed-batch and multiple induction. *Microbial Cell Factories*, 5(27), 1–13.

Chen, C. and Joseph, B. (1987). On-line optimization using a two-phases approach : An application study. *Industrial and Engineering Chemistry Research*, 26(9), 1924–1930.

Guay, M. and Zhang, T. (2003). Adaptive extremum seeking control of nonlinear dynamic systems with parametric uncertainties. *Automatica*, 39(7), 1283–93.

Jang, S., Joseph, B., and Mukai, H. (1987). On-line optimization of constrained multivariable chemical processes. *AIChE Journal*, 33(1), 26–35.

Khalil, H. (1996). *Nonlinear Systems.* Prentice Hall.

Krstic, M. and Wang, H.H. (2000). Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica*, 36(4), 595–601.

Ljung, L. (1999). *System Identification: Theory for the user.* Prentice-Hall Inc., Upper Saddle River, N.J.

McFarlane, R. and Bacon, D. (1989). Empirical strategies for open-loop on-line optimization. *Canadian Journal of Chemical Engineering*, 67(4), 665–677.

Nocedal, J. and Wright, S. (1999). *Numerical Optimization.* Springer, New York.

Reney, F., Perrier, M., and Srinivasan, B. (2009). Influence of differences in system dynamics in the context of multi-unit optimization. *International Symposium on Advanced Control of Chemical Processes*, Istanbul, Turkey.

Srinivasan, B. (2007). Real-time optimization of dynamic systems using multiple units. *International Journal of Robust and Nonlinear Control*, 17, 1183–1193.

Wellstead, P. and Scotson, P. (1990). Self-tuning seeking control. *IEEE Proceedings*, 137(3), 165–175.

Woodward, L., Perrier, M., and Srinivasan, B. (2009). Improved performance in the multi-unit optimization method with non-identical units. *Journal of Process Control*, 19, 205–215.