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Plant-Wide Control Based on Minimum Square Deviation ⋆

David Zumoffen, Gonzalo Molina, Marta Basualdo

Group of Applied Informatics to Process Engineering (GIAIP). French Argentine International Center for Information and Systems Sciences (CIFASIS-CONICET-UNR)-FRRo-UTN. 27 de Febrero 210 bis, S2000EZP Rosario, Argentina (e-mail: zumoffen/basualdo@cifasis-conicet.gov.ar).

Abstract: In this work a new systematic and generalized strategy to solve the MIMO plant-wide control problem is proposed. The methodology called Minimum Square Deviation (MSD) considers several points such as the optimal sensor location (OSL) based on the sum of square deviation (SSD) and the control structure selection (CSS) based on net load evaluation (NLE) problems simultaneously. Particularly, this work focuses on selecting the best MIMO control structure by using a new steady-state index called NLE. Thus, alternative control structures can be obtained through different interaction levels and defining a corresponding performance improvement. Two well-known chemical process are proposed here for testing this methodology. In addition, a robust stability analysis applying the classical μ-tool is performed by considering both parametric and unmodeled dynamic uncertainties.

Keywords: plant-wide control, MSD, optimal sensor location, control structure selection, NLE, RGA.

1. INTRODUCTION

The processes synthesis stage defines the connection between units and their sizing. Normally, this problem is solved by using steady-state (SS) information only without considering issues such as optimal sensor location (OSL) and control structure selection (CSS). It is crucial to identify some potential control problems at this phase for achieving a suitable plant design. However, only partial solutions exist to quantify these kind of problems with SS tools only. In this work a rigorous and generalized treatment of theses aspects are proposed without accounting any heuristic considerations.

Different approaches exist and basically can be grouped into OSL and CSS areas, addressing the problem separately. These individual and unrelated treatments usually produce sub-optimal solutions from the plant-wide control point of view. The OSL field generally uses investment costs, observability, Kalman filter theory and dynamic models to define the sensors network by means of integer optimization routines (Musulin et al., 2005; Singh and Hahn, 2005; Kadu et al., 2008; Bhushan et al., 2008). Generally, these strategies are developed on process in open loop or with an already existing control policy. None of them considers the benefit of solving the plant-wide control structure together with the OSL problems. A similar situation occurs in plant-wide CSS topics. Mainly the problem is solved by using process interaction measures without considering which would be the best way for sensing the variables. Generally, a predefined sensors network is used and the CSS problem dimension is reduced heuristically through the application of the engineering judgment. Currently the standard tools to CSS in industrial processes are still the relative gain array (RGA) proposed by Bristol (1966) and its modifications to handle non-square process (Chang and Yu, 1990), disturbances (Chang and Yu, 1994; Lin et al., 2009) and dynamic implications (McAvoy et al., 2003; He et al., 2009), among others.

In this work a new generalized and systematic strategy called Minimum Square Deviation (MSD) for optimal plant-wide control is presented for chemical process. Basically, the overall procedure can be divided in two sequential optimization problems. The OSL problem based on sum of square deviation (SSD) accounting the control effects and the net load effect (NLE) based CSS problem. In this work, only the last topic is addressed exhaustively. Detailed descriptions about the former topic can be found in Zumoffen and Basualdo (2009), Zumoffen et al. (2009) and Molina et al. (2009). Similarly, an optimal signal selection for monitoring systems design can be found in Zumoffen and Basualdo (2010) where similar mixed-integer optimization routines were proposed by using genetic algorithms (GA). In this work a new index named the NLE is proposed to decide among different control structures. This index allows to obtain a trade-off solution between servo and regulator behavior by accounting the control objectives. This can be done through a proper adjustment of the weight matrices. The overall problem results in a combinatorial one and can be efficiently solved by GA.

The optimal control structures obtained via NLE improve the overall dynamic behavior avoiding the explicit design

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of classical cascade/feedforward control approaches. In this work the μ-tools are used only to analyze the feasibility of the proposed control policies under plant-model uncertainties (realistic scenario).

2. MSD PLAN-WIDE CONTROL STRATEGY

The systematic and generalized MSD strategy for performing plant-wide control proposed here can be observed at Fig. 1. The blocks with dark grey background summarize the procedures for solving the OSL problem. Initially, during the step of SS process design can be optimized (if it is required) accounting different objective functions, closely related to the operation cost. The second step needs to define which would be the minimum control loops such that ensure the plant stability, i.e. inventory control. In this stage, the remaining degrees of freedom (if any), are used for defining the controlled variables (CV)/sensor locations. It is performed by applying GA, being the objective function the sum of square deviation (SSD), specifically developed according to the plant requirements (Zumoffen and Basualdo, 2009; Zumoffen et al., 2009; Molina et al., 2009).

Assuming that after the stabilization process the potential sensor locations are m and the available manipulated variables are n with $m > n$, then $m-n$ degrees of freedom exist. For analyzing which would be the best configuration is very useful adopting a full multivariable controller based on the internal model control (IMC) theory. It can be implemented as shown in Fig. 2. where, $\mathbf{G}_s(s)$ is a $n \times n$ transfer function matrix (TFM) containing the controlled variables (CV) of the process and $\mathbf{G}_r(s)$ is a $(m-n) \times n$ TFM representing the uncontrolled variables (UV). Similarly, $\mathbf{D}_s(s)$ and $\mathbf{D}_r(s)$ represent the disturbance models for each part of the process with dimensions $n \times p$ and $(m-n) \times p$ respectively. $\mathbf{G}_c(s)$ represents the IMC controller designed based on the process model $\tilde{\mathbf{G}}_s(s)$. The optimal selection of $\mathbf{G}_s(s)$, i.e. OSL, can be made by considering the SS (s=0) error in the UV when perfect control is supposed and both set points and disturbances changes are considered individually (Zumoffen and Basualdo, 2009; Zumoffen et al., 2009; Molina et al., 2009). Thus, the process outputs of UV can be represented as

$$\mathbf{y}_r = \mathbf{S}_{sp}\mathbf{y}_s^{sp} + \mathbf{S}_d d_* \quad (1)$$

with $\mathbf{S}_{sp} = \mathbf{G}_r\mathbf{G}_s^{-1}$ and $\mathbf{S}_d = \mathbf{D}_r - \mathbf{G}_r\mathbf{G}_s^{-1}\mathbf{D}_s$. From (1) can be observed that the SS error only depends on the selected subprocess $\mathbf{G}_s$. The SSD index can be stated as

$$\begin{aligned} SSD &= \sum_{i=1}^{n} \|\mathbf{e}_{sp}(i)\|_2^2 + \sum_{j=1}^{p} \|\mathbf{e}_d(j)\|_2^2 \\ &= \sum_{i=1}^{n} \|\Lambda_2\mathbf{S}_{sp}\Lambda_1\mathbf{y}_{sp}^n(i)\|_2^2 + \sum_{j=1}^{p} \|\Theta_2\mathbf{S}_d\Theta_1\mathbf{d}_*^p(j)\|_2^2 \\ &= \text{trace}\left(\Lambda_1^2\mathbf{S}_{sp}^T\Lambda_2^2\mathbf{S}_{yr}\right) + \text{trace}\left(\Theta_1^2\mathbf{S}_d^T\Theta_2^2\mathbf{S}_d\right) \end{aligned} \quad (2)$$

where $\mathbf{e}_{sp}(i)$ and $\mathbf{e}_d(j)$ are the vector of deviations corresponding to the y_r outputs from their nominal operating point values when an unitary change happens in the i set point and j disturbance respectively. The vectors $\mathbf{y}_{sp}^n(i)$ and $\mathbf{d}_*^p(i)$ have an unitary entry at the location i and zero elsewhere. The diagonal weighing matrices Λ_1, Λ_2, Θ_1 and

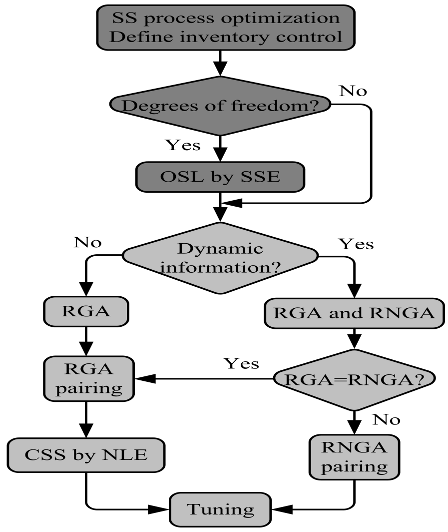


Figure 1. MSD Plant-Wide Control Strategy

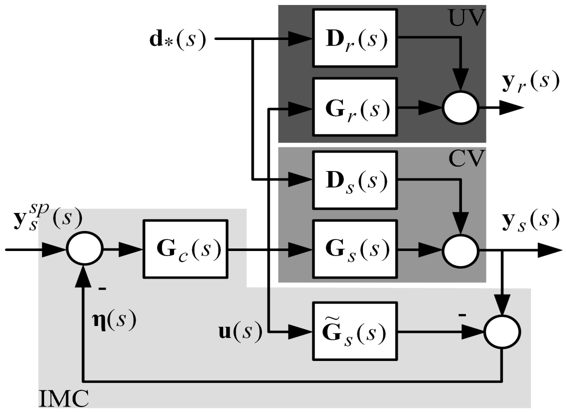


Figure 2. Generalized IMC Structure

Θ_2 allow to include the process control objectives such as set point/disturbance magnitudes and the relative degree of importance among the overall outputs. The minimization of (2) respect to the sensor location is a combinatorial problem that can be suitably solved by using GA.

2.1 Control Structure Selection

Assuming now that the problem stated previously was solved efficiently, so $\mathbf{G}_s$ was selected. In this scenario the problem to be solved is the input-output pairing, i.e. the CSS. The blocks with light grey background in the Fig. 1 allow to address this problem in a systematic and generalized way. This work is principally focussed on showing how the NLE approach allows to obtain the optimal control structure. Note that the controller structure may be diagonal (i.e decentralized/without interaction), full (i.e. full interaction) or with other some specific structure (i.e. partial interaction).

If dynamic information of the process is known (i.e. simplified linear models), so the relative normalized gain array (RNGA) is preferred as pairing tool instead of RGA. The RNGA was recently proposed by He et al. (2009) to improve the pairing selection in decentralized control structures. This approach considers the time constant and delay for each plant model subprocess as a normalization factor of the conventional RGA. If the pairing proposed by RNGA is the same as that given by the RGA or eventually there is no dynamic information available, the RGA pairing tool must be selected. In this context, the future control performance can be improved by optimal CSS via NLE.

Considering again the Fig. 2 the CV of the process can be expressed as

$$\mathbf{y}_s(s) = \tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s)\mathbf{y}_s^{sp}(s) + \left(\mathbf{I} - \tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s)\right)\mathbf{y}_s^{net}(s) \quad (3)$$

where

$$\mathbf{y}_s^{net}(s) = \mathbf{A}(s)\mathbf{y}_{sp}(s) + \mathbf{B}(s)\mathbf{d}_*(s) \quad (4)$$

$$\mathbf{A}(s) = \mathbf{C}(s)\left(\mathbf{G}_s(s) - \tilde{\mathbf{G}}_s(s)\right)\mathbf{G}_c(s) \quad (5)$$

$$\mathbf{B}(s) = \mathbf{C}(s)\mathbf{D}_s(s) \quad (6)$$

$$\mathbf{C}(s) = \left[\mathbf{I} + \left(\mathbf{G}_s(s) - \tilde{\mathbf{G}}_s(s)\right)\mathbf{G}_c(s)\right]^{-1} \quad (7)$$

being $\mathbf{y}_s^{net}(s)$ the net load effect on CV due to both set point and disturbances changes. From (3) can be observed that at SS the term $(\mathbf{I} - \tilde{\mathbf{G}}_s(s)\mathbf{G}_c(s))$ produces an integral behavior rejecting potential offset for $\mathbf{y}_s(s)$. This is true by accounting the IMC structure design where $\mathbf{G}_c(s) = \tilde{\mathbf{G}}_s^{-1}(s)\mathbf{F}(s)$, and $\mathbf{F}(s)$ the low-pass matrix filter. However, in the transient response $\mathbf{y}_s(s)$ is influenced directly by the evolution of $\mathbf{y}_s^{net}(s)$ and its SS gain. Analyzing this last case (s=0), (5) and (6) can be reduce to

$$\mathbf{A} = \mathbf{I} - \tilde{\mathbf{G}}_s\mathbf{G}_s^{-1} \quad (8)$$

$$\mathbf{B} = \tilde{\mathbf{G}}_s^{-1}\mathbf{G}_s\mathbf{D}_s. \quad (9)$$

Equation (8) shows that the full IMC structure case, $\tilde{\mathbf{G}}_s = \mathbf{G}_s$, allows to reject the set point effects completely ($\mathbf{A} = \mathbf{0}$), but the disturbance effects in (9) enter to the process without modifications ($\mathbf{B} = \mathbf{D}_s$). From (9) can be observed that a specific selection of $\tilde{\mathbf{G}}_s$ may decrease these effects. Anyway, a trade-off solution is necessary to adopt between servo and regulator problem. Then, parameterized the model selection as

$$\tilde{\mathbf{G}}_s(\Gamma) = \mathbf{G}_s \otimes \Gamma, \quad \text{with} \quad \Gamma = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & \ddots & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{bmatrix} \quad (10)$$

where $\otimes$ is the element by element product and the γ_{ij} can be $0, 1$ indicating the selection ($\gamma_{ij} = 1$) or not (i.e. $\gamma_{ij} = 0$) of the process element ij. Thus, a new index called the net load effect (NLE) in a sum square sense is proposed to decide between different control structures for multivariable systems.

$$NLE(\Gamma) = \text{trace}\left(\Delta_2^2\mathbf{A}_\Gamma^T\Delta_1^2\mathbf{A}_\Gamma\right) + \text{trace}\left(\Xi_2^2\mathbf{B}_\Gamma^T\Xi_1^2\mathbf{B}_\Gamma\right) \quad (11)$$

where $\mathbf{A}_\Gamma$ and $\mathbf{B}_\Gamma$ are the net load matrices from (8) and (9) parameterized by the model selection proposed in (10). The diagonal weighing matrices Δ_1, Δ_2, Ξ_1 and Ξ_2 allow to include the process control objectives such as set point/disturbance changes and the relative degree of importance between the outputs. An optimal solution can be obtained by the minimization of (11) by searching Γ in the parameter space.

$$\min_{\Gamma} NLE(\Gamma), \quad \text{subject to} \quad \det(\tilde{\mathbf{G}}_s(\Gamma)) \neq 0 \quad (12)$$

Table 1. Reduced Shell Process

	u_1	u_2	u_3	d_1	d_2
y_1	$\frac{4.05e^{-27s}}{(50s+1)}$	$\frac{1.77e^{-28s}}{(60s+1)}$	$\frac{5.88e^{-27s}}{(50s+1}$	$\frac{1.20e^{-27s}}{(45s+1)}$	$\frac{1.44e^{-27s}}{(40s+1)}$
y_2	$\frac{5.39e^{-18s}}{(50s+1)}$	$\frac{5.72e^{-14s}}{(60s+1)}$	$\frac{6.90e^{-15s}}{(40s+1)}$	$\frac{1.52e^{-15s}}{(25s+1)}$	$\frac{1.83e^{-15s}}{(20s+1)}$
y_3	$\frac{4.38e^{-20s}}{(33s+1)}$	$\frac{4.42e^{-22s}}{(44s+1)}$	$\frac{7.20}{(19s+1)}$	$\frac{1.14}{(27s+1)}$	$\frac{1.26}{(32s+1)}$

Table 2. Weighing Matrices for Shell Process

	Δ_1	Δ_2	Ξ_1	Ξ_2
Γ_1	diag(0,0,0.5)	diag(1,1,1)	diag(0.5,0.5)	diag(1,1,1)
Γ_2	diag(0,0,0.5)	diag(0,0,0.1)	diag(0.5,0.5)	diag(1,1,1)

Note that the exhaustive search of all possible combination of Γ, in (12), is possible in cases where only a few number of variables are accounted. The combinatorial size of the problem results $2^{(n\times n)}$, which grows quickly with the number of varaibles. For large scale process is necessary some mixed-integer optimization algorithm that allows to solve the combinatorial problem in (12). In this work GA has been applied.

3. STUDIED CASES

The proposed methodology, MSD, especially the CSS by NLE is presented in the following. The OSL part of the systematic strategy in Fig. 1 is not applied here due to space limitations. However, several works summarize the results of this proposal (Zumoffen and Basualdo, 2009; Zumoffen et al., 2009; Molina et al., 2009). In this case two examples are proposed.

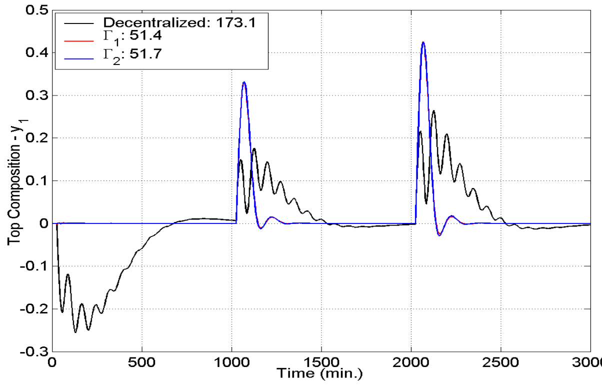
3.1 Example Nº1: The Shell Oil Fractionator

Basically, the process is a distillation column (Maciejowski, 2002). The overall plant has 7 potential variables to measure, 3 possible manipulated variables, and 2 disturbances. The control objectives propose to keep as lowest as possible the variability in 3 variables (i.e. this is economically advantageous). This requirements left the control problem without degrees of freedom. Thus, the CSS by NLE can be stated by using both $\mathbf{G}_s(s)$ and $\mathbf{D}_s(s)$ presented at Table 1 with dimensions of 3×3 and 3×2 respectively. Note that the time constant and the delays are expressed in minutes.

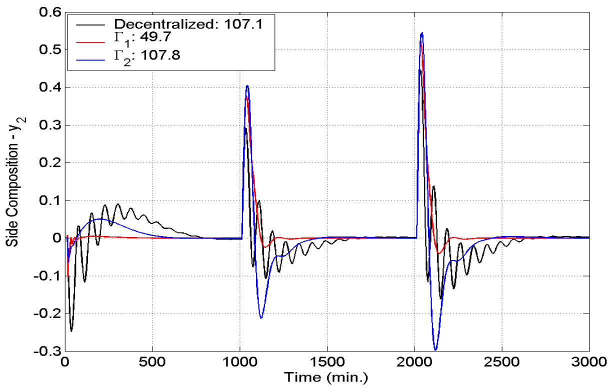
The RGA and RNGA analysis drives to the same pairing structure and suggests a decentralized control strategy, $u_1 - y_1$, $u_2 - y_2$, and $u_3 - y_3$. Therefore, a NLE analysis is necessary to define the best control structure under this conditions. Considering in Γ the decentralized loops fixed, this parametrization allows to decide on the off-diagonal elements. The size of the combinatorial problem becomes $2^{(3\times3-3)} = 64$, and this problem can be solved efficiently by exhaustive search as well as GA. Solving the problem stated in (12) with the parameters setting shown at Table 2 the following solution were found

$$\Gamma_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \Gamma_2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (13)$$

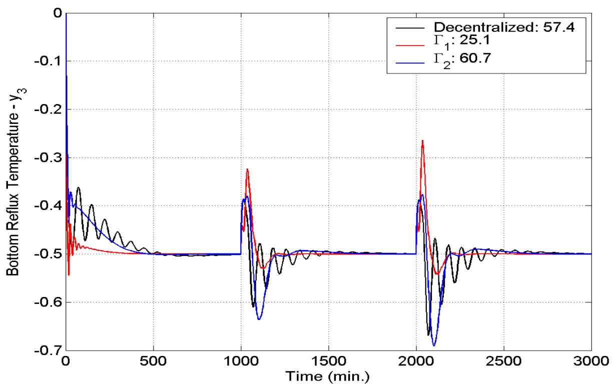
Where Γ_1 suggests a full multivariable controller taking into account the original control objectives. On the other hand, Γ_2 proposes a generalized control structure by



(a) Top Product Composition



(b) Side Product Composition



(c) Bottom Reflux Temperature

Figure 3. Shell Responses with Different Control Structures

Table 3. Total IAE for Shell Process

	Decentralized	Γ_1	Γ_2
IAE_t	337.6	126.2	220.2

coupling only some loops (i.e. two loops off-diagonal). The last structure has been obtained by relaxing the requirements on set point changes.

Figure 3 displays the outputs of the Shell process when different control structures are used, decentralized (not interacting), Γ_1 (fully interacting), and Γ_2 (partially interacting). The last two structures were obtained by NLE index optimization. In addition, the integral absolute error (IAE) is shown for each output, in the legend, as well as the corresponding control structure. In these figures a set point change of -0.5 occurs for y_3 at $t = 0$ min. and sequentially two step disturbance effects at $t = 1000$ min. and $t = 2000$ min. for d_1 and d_2 respectively with magnitud of 0.5. Clearly, the best control structure is the full IMC case, as can be observed at Table 3 considering the individual IAEs sum, $IAE_t = \sum_i IAE_i$. However, note that the partially interacting case, Γ_2, presents similar IAE values with only two additional loops (equation 13). All the controllers were designed using the IMC theory and first order models without delay information.

3.2 Example N^{o}2: The CL Column

In this case two heat-integrated distillation columns developed by Chiang and Luyben (1988) are analyzed. Here the methanol-water separation with low product purities (96/4) is proposed. The feed-split configuration was used giving a 4×4 MIMO process model with one disturbance signal that can be observed at Table 4. The control objectives are to maintain the four compositions, y_1 to y_4, (overhead and bottom for each column) at their desired values. The manipulated variables are u_1: reflux ratio in the high pressure column, u_2: heat input, u_3: reflux ratio in the low pressure column, and u_4: the feed split. The unmeasured disturbance signal is the feed composition, d_1.

In this case again the process objectives left the control problem without degrees of freedom. Thus, the CSS by NLE can be stated by using both $\mathbf{G}_s(s)$ and $\mathbf{D}_s(s)$ presented at Table 4 with dimensions of 4×4 and 4×1 respectively. Note that the time constant and the delays are expressed in minutes.

The RGA and RNGA analysis generates the same pairing information and suggests the following ones: $u_1 - y_1$, $u_2 - y_2$, $u_3 - y_3$ and $u_4 - y_4$. Therefore, a NLE analysis is necessary to define the best control structure under these conditions. Considering Γ parameterized as in the previous example, the size of the combinatorial problem becomes $2^{(4\times4-4)} = 4096$, and this problem can be solved efficiently by exhaustive search as well as GA. Solving the problem stated in (12) with the parameters setting shown at Table 5 the solutions Γ_1 and Γ_2 were found. These are shown in (14) and are compared with the solution obtained by Chang and Yu (1994) opportunely recognized as Γ_{CY}.

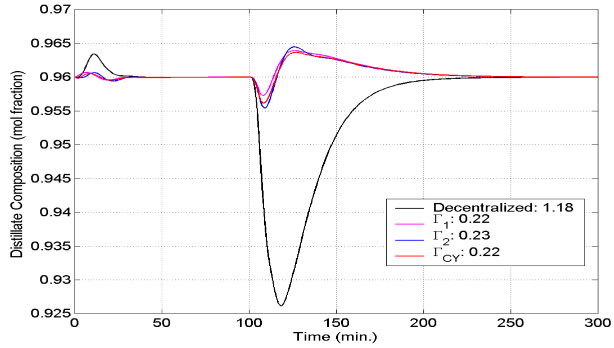
$$\Gamma_1 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \Gamma_{CY} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{14}$$

The model selection given by Γ_2 proposes a generalized control structure by coupling only some loops (i.e. four off-diagonal loops). This solution considers the original control objectives (i.e. equally weighted). On the other hand, Γ_1, suggests a full multivariable controller and its structure has been obtained by relaxing the requirements on disturbance. The model selection suggested by Chang and Yu (1994), Γ_{CY}, is an almost triangular structure obtained via generalized relative disturbance gain.

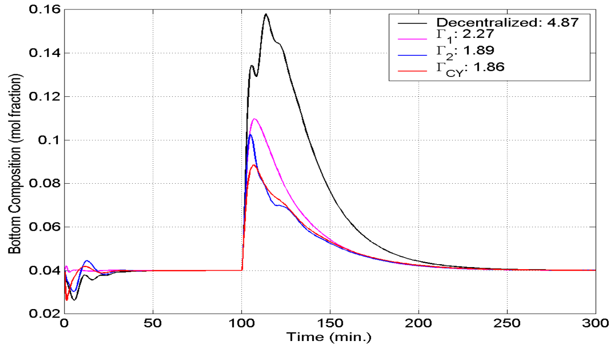
Figure 4 displays the outputs of the CL process when different control structures are used, decentralized (not interacting), Γ_1 (fully interacting), Γ_2 (partially interacting), and the proposed by Chang and Yu (1994), Γ_{CY} (almost triangular interacting). Both Γ_1 and Γ_2 were obtained by NLE index via GA optimization. The Fig. 4 also shows the

Table 4. CL Process

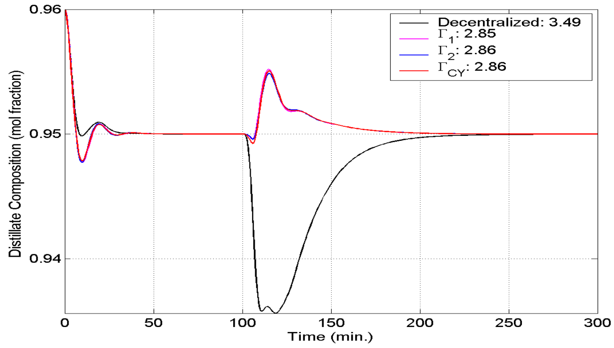
	u_1	u_2	u_3	u_4	d_1
y_1	$\frac{4.45}{(14s+1)(4s+1)}$	$\frac{-7.4}{(16s+1)(4s+1)}$	0	$\frac{0.35}{(25.7s+1)(2s+1)}$	$\frac{1.02}{(25s+1)(2s+1)^2}e^{-4.5s}$
y_2	$\frac{17.3}{(17s+1)(0.5s+1)}e^{-0.9s}$	$\frac{-41}{(21s+1)(s+1)}$	0	$\frac{9.2}{(20s+1)}$	$\frac{19.7}{(25s+1)(s+1)}e^{-0.3s}$
y_3	$\frac{0.22}{(17.5s+1)(4s+1)}e^{-1.2s}$	$\frac{-4.66}{(13s+1)(4s+1)}$	$\frac{3.6}{(13s+1)(4s+1)}$	$\frac{0.042(78.7s+1)}{(21s+1)(11.6s+1)(3s+1)}$	$\frac{0.75}{(15.6s+1)(2s+1)^2}e^{-5s}$
y_4	$\frac{1.82}{(21s+1)(s+1)}e^{-s}$	$\frac{-34.5}{(20s+1)(s+1)}$	$\frac{12.2}{(18.5s+1)(s+1)}e^{-0.9s}$	$\frac{-6.92}{(20s+1)}e^{-0.6}$	$\frac{16.1}{(25s+1)(2s+1)}e^{-0.6s}$



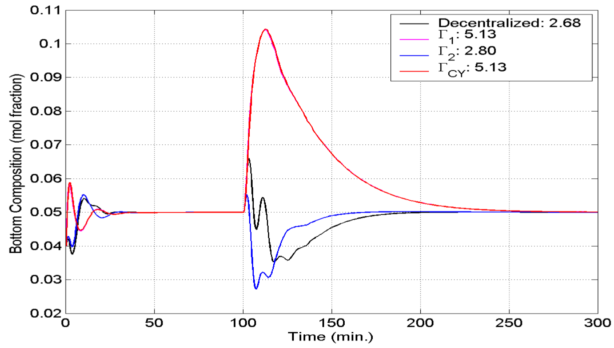
(a) Top Composition High Pressure Column



(b) Bottom Composition High Pressure Column



(c) Top Composition Low Pressure Column



(d) Bottom Composition Low Pressure Column

Figure 4. CL Responses - Different Control Structures

Table 5. Weighing Matrices for CL Column

	Δ_1	Δ_2	Ξ_1	Ξ_2
Γ_1	diag(1,1,1,1)·0.5	diag(1,1,1,1)	0.05	diag(1,1,1,1)·0.1
Γ_2	diag(1,1,1,1)·0.5	diag(1,1,1,1)	0.05	diag(1,1,1,1)

Table 6. Total IAE for CL Process

	Decentralized	Γ_1	Γ_2	Γ_{CY}
IAE_t	12.22	10.47	7.72	10.07

IAE values for each output, indicated at the legend, as well as the corresponding control structure. In this simulation case a set point change is proposed for the low pressure column at $t = 0$ min. which magnitude is about -0.01 mol fraction and 0.01 mol fraction for the overhead and bottom compositions respectively. Sequentially a disturbance appears at $t = 100$ min. which magnitude is 0.05 mol fraction in the feed composition. Clearly, the best control structure is the proposed here, Γ_2, as can be observed at Table 6 considering the individual IAEs sum. All the controllers were designed using the IMC theory and first order models without delay information.

4. ROBUST STABILITY AND PERFORMANCE

The robust stability and performance evaluation when different controller structures are used in MIMO systems is a very difficult task. The structured singular value (SSV) or μ-analysis (Skogestad and Postlethwaite, 2005) is a popular methodology to evaluate these characteristics when model uncertainties are present. The SSV generalizes the singular value decomposition (SVD) by considering the uncertainty structure.

Considering that $\mathbf{P}_s(s) = [\mathbf{G}_s(s), \mathbf{D}_s(s)]$ is the nominal process model, then the linear fractional transformation (LFT) (Skogestad and Postlethwaite, 2005) concept can be used to represent this nominal model in a generalized version, $\mathbf{P}_s^*(s)$ as is shown at Fig. 5. The robust stability of the relation $\mathbf{e}_p = \mathbf{F}(\Delta)[\mathbf{d}_*, \mathbf{y}_s^{sp}]^T$ can be analyzed by the following determinant condition

$$\det\left(\mathbf{I} - \mathbf{M}\Delta(j\omega)\right) \neq 0, \quad \forall\omega, \forall\Delta, \quad \overline{\sigma}\left(\Delta(j\omega)\right) \leq 1, \forall\omega \tag{15}$$

where $\mathbf{M}$ is the transfer function matrix from $\mathbf{w}$ to $\mathbf{z}$, resultant of close the lower loop with the controller $\mathbf{K}$ in the Fig. 5 and opening the upper one. $\mathbf{e}_p$ represents the tracking error in each controlled variable, $\Delta = \text{diag}(\Delta_1, \ldots, \Delta_i)$ is a block diagonal matrix of stable normalized perturbations, where each Δ_i may represent a specific source of uncertainty (i.e. parametric or unmodelled dynamics) and fulfilling $\overline{\sigma}(\Delta(j\omega)) \leq 1, \forall\omega$. Where, $\overline{\sigma}$ is the maximum singular value and $j\omega$ the complex frequency. A way to generalize (15) is the application of μ concepts (Skogestad

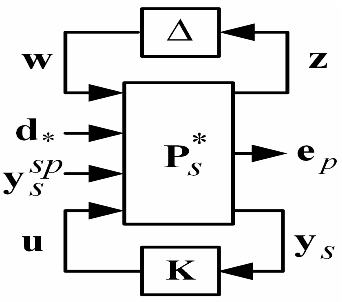
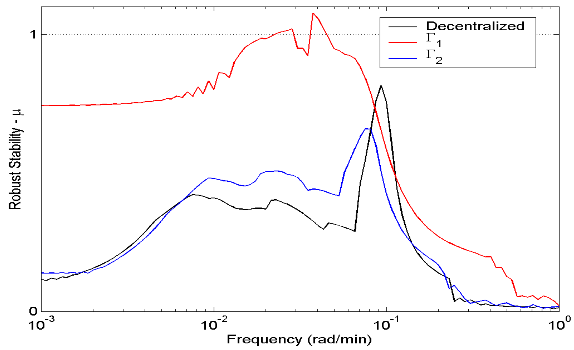
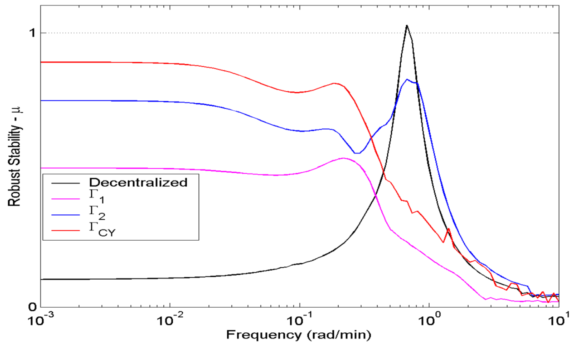


Figure 5. Generalized Process With Uncertainties



(a) Shell Process



(b) CL Process

Figure 6. μ-Analysis - Robust Stability

and Postlethwaite, 2005). The robust performance can be analyzed by applying similar ideas. In this work, the μ-analysis toolbox for Matlab® environment is used.

Here both parametric and unmodelled dynamics uncertainties were proposed for delays and gains of the process respectively. A 20% of uncertainty was selected for the dead time and a complex perturbation for the gains varying between 10% at steady-state to 200% at high frequency.

Figure 6 summarizes the uncertainty process analysis with different control structures. In the Shell case, Fig. 6(a), Γ_1 does not guarantee the robust stability ($\mu \geq 1$). On the other hand, Γ_2 presents the best behavior under these kind of uncertainties (i.e. the lower μ peak). Similarly, Fig. 6(b), displays the uncertainty CL process with different control structures. Decentralized control policy does not guarantee the robust stability under these conditions ($\mu \geq 1$). Structures Γ_1 (fully interacting) and Γ_2 (partially interacting) present the best behavior. The structure proposed by Chang and Yu (1994) is not adequate because it is located very near to the stability limit (i.e. a SS uncertainty bigger than %10 produces instability).

5. CONCLUSION

In this work several results that show how a suitable and generalized control structure selection (CSS) can improve both the overall performance and robust stability were presented. Classical control policies (decentralized and full) are not always the best solution. It is remarkable that the use of a new steady-state index, named the net load effect (NLE), proposed here represents a key element for driving the search to the most suitable MIMO control structure. In addition, it was observed how the control objectives can be included easily allowing an adequate trade-off solution between them.

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