Plantwide Operability Analysis based on a Network Perspective: a Study on the Tennessee Eastman Process

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Abstract: Complex process plants increasingly appear in modern chemical industry due to the considerable economic efficiency that complex and interactive process designs can offer. Interactions between process units (*e.g.*, material recycle and energy integration) often cause significant difficulties in plantwide control. As such, it is important to study plantwide operability (*i.e.*, whether a plantwide process can be effectively controlled) prior to control system design and preferably during the stage of process design. This paper presents such an analysis approach based on a network perspective, where a plantwide process is modeled as a network of process units interconnected with physical mass and energy flows. This approach can be used to determine plantwide stability, stabilizability and disturbance attenuability. The proposed method is illustrated with the Tennessee Eastman Process.

Keywords: Interconnected systems, Network topologies,Operability analysis, plantwide process, Process control, Decentralized control.

1. INTRODUCTION

Process control is one of the keys to both profitable and safe operations of modern chemical plants. According to Luyben et al. (1998), the necessities to reduce waste and energy as well as to increase the safety of plant operation and tighter product specifications have resulted in complex process designs and greater demands on control system performance. However, this often leads to complex processes that are often difficult to control. For example, the use of recycle streams and heat integration has often led to a process that has operability problems, *e.g.* plant instability, difficultly in achieving operation conditions, and high sensitivity to process disturbances (Luyben (2004)).

There are several elements of process operability as elaborated by Wolff et al. (1994). In this paper, we focus on three main basic elements of operability, *i.e.* the open-loop stability and stabilizability of a plantwide process, and the analysis of disturbance attenuation. There are linear operability indicators such as process singular values by Morari (1983), relative gain array (RGA) by Bristol (1966), and the condition numbers method by Bahri et al. (1997). Some methods are also applicable to nonlinear processes, such as operability index by Vinson and Georgakis (2000) and nonlinear dynamic operability by Rojas et al. (2007a,b). The plantwide operability can also be assessed by dynamic optimization method suggested by Perkins and Walsh (1996); Kookos and Perkins (2001); Bansal et al. (2003). In modern chemical plants, the size of the plants and the interconnections between the process units are the main sources of operability problems. The above approaches treat the plantwide process as a single complex system and often become very complex when the size of the plant grows. Furthermore, these approaches cannot be used to directly analyze the effects of interactions between process units.

In this paper, an operability analysis tool based on network perspective is presented. A plantwide chemical process is viewed as an interconnected network of smaller subsystems (process units) via the flows of materials and energy. As previously stated that one major cause of operability problem is the interaction between process units, the network approach is able to represent the topology of process interconnection explicitly thus providing a key advantage for operability analysis. Since each subsystem is represented as an input-output system of mass/energy, the theory of dissipativity can be utilized to study process operability, such as stability and stabilizability. This work is based on the operability analysis framework previously developed by the authors for general nonlinear plantwide processes Rojas et al. (2009). This nonlinear framework shows the potential of the proposed approach but can be difficult to apply in practice as it requires complex nonlinear analysis and could be quite conservative. In this work, a more practical operability analysis tool based on linearized model was developed which allows the plantwide operability analysis for regulatory problems (operating around certain fixed operating points). This linear analysis approach can produce non-conservative estimates of the achievable plantwide dynamical performance of the closed-loop systems and can link plantwide operability directly to the process network topologies (the way process units are connected). Using the model developed by Downs and Vogel (1993), the proposed approach is then applied to a case study of the Tennessee Eastman (TE) process, which has complex interconnections of material and energy flows.

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This paper is organized as follows. Section 2 elaborates a brief detail of the TE process. The newly developed operability analysis tool is presented in Section 3 together with the procedures on the application to the TE process. Section 4 provides the analysis and discussions on the results obtained, followed by the conclusion.

2. PROCESS DESCRIPTION

The Tennessee Eastman Process consists of five major unit operations: a two-phase reactor, a condenser, a vapor-liquid separator, a compressor and a stripper as shown in Fig. 2. The process converts the reactants (A, C, D, and E) into the products (G, and H). A component B exists as an inert and component F is a byproduct. The process is described by a series of reactions given as follows:

$$\begin{aligned} A_{(g)} + C_{(g)} + D_{(g)} &\rightarrow G_{(l)} \\ A_{(g)} + C_{(g)} + E_{(g)} &\rightarrow H_{(l)} \\ A_{(g)} + E_{(g)} &\rightarrow F_{(l)} \\ 3D_{(g)} &\rightarrow 2F_{(l)}. \end{aligned} \quad (1)$$

There are five feed streams into the reactor (including recycle streams). Gaseous reactants are fed and react in the two-phase reactor (using a nonvolatile catalyst dissolved in liquid phase) forming liquid products. The reactor is equipped with an internal cooling-bundle due to the exothermic reactions. The products and unreacted feeds leave the reactor as vapors flowing into a condenser before entering the vapor-liquid separator. The gaseous stream is recycled back into the reactor using the compressor (a purge stream is also present to prevent inert B and byproduct F build-up). The liquid stream is then fed into the stripping column to remove remaining reactants by using one of the gaseous feed stream 4. The liquid products G, and H are then drawn from the stripper for further treatment downstream (not included in the problem). The process has six modes of operation based on different product specifications. The details are provided in the original paper of the problem by Downs and Vogel (1993).

Jockenhövel et al. (2003) proposed a dynamic model of the process that includes energy balances for the reactor, separator, stripper, and the pseudo-mixing-unit. This results in a process which is fundamentally open-loop unstable due to the exothermic nature of the reactions and the existences of recycle streams. This work uses the aforementioned dynamic model which represents the Eastman process as four smaller subsystems as follows: (1) mixing zone, (2) reactor, (3) separator with compressor-condenser, and (4) stripping column.

The focus of the operability analysis in this work is on the base mode of operation (product G/H ratio of 50/50 at base reactants feed rates). We obtained steady-state operating conditions that are slightly different from the original data in Downs and Vogel (1993) because of the use of the dynamic model that accounts for heat balances. The operating conditions for individual units are given in table 1, while the steady-state process stream data can be seen in table 2.

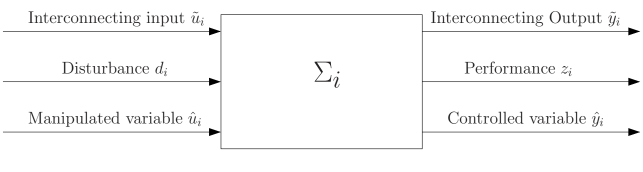

Fig. 1. Process unit as input/output system

Table 1. Steady-state operating conditions

Unit	Reactor	Separator	Stripper
Temperature (K)	393	353	337
Pressure (kPa gauge)	2769	2679	-
Heat Duty (kW)	-6618	-3074	169
Liquid volume (m^3)	14.2	5.81	4.09

This paper focuses on the operability study with *decentralized* structure of control system (each subsystem is locally controlled by a *multivariable* controller). The nonlinear dynamic model from Jockenhövel et al. (2003) with the parameters in Downs and Vogel (1993) were linearized around the operating points given in tables 1 and 2 resulting in linear time-invariant (LTI) state-space representation of each subsystem as shown in Fig. 1. It is mathematically written as follows:

$$\Sigma_i : \begin{bmatrix} \dot{x}_i \\ \widetilde{y}_i \\ z_i \\ \hat{y}_i \end{bmatrix} = \begin{bmatrix} A_i & B_{i,1} & B_{i,2} & B_{i,3} \\ C_{i,1} & D_{i,11} & D_{i,12} & D_{i,13} \\ C_{i,2} & D_{i,21} & D_{i,22} & D_{i,23} \\ C_{i,3} & D_{i,31} & D_{i,32} & D_{i,33} \end{bmatrix} \begin{bmatrix} x_i \\ \widetilde{u}_i \\ d_i \\ \hat{u}_i \end{bmatrix}. \quad (2)$$

For inputs, the physical interconnected inputs, disturbances and manipulated variables (MVs) are denoted as $\widetilde{u}$, d and $\hat{u}$ respectively. $\widetilde{y}$ denotes the physical outputs, $\hat{y}$ denotes the controlled variables (CVs) and z denotes the chosen outputs for the purpose of performance assessment. In this framework, the states x are physical states of each subsystem, such as total mass inventory or total internal energy of a subsystem. The MVs for the mixing region are the fresh feed streams, *i.e.* streams 1, 2 and 3. The reactor temperature is controlled by manipulating the cooling water flow rate. There are five MVs in the separator subsystem: recycle flow rate (stream 8), purge flow rate (stream 9), liquid flow rate to the stripping column (stream 10), compressor work and cooling water flow rate into the condenser.

3. OPERABILITY ANALYSIS

The main features of operability study include plantwide stability and stabilizability analysis as well as quantification of disturbance effects. *Dissipativity* is used as a key enabling tool for the analysis based on the *interconnections* of smaller subsystems of process units in terms of mass and energy flows.

3.1 Dissipativity

The concept of dissipative systems was introduced by Willems (1972a,b) as an extension to the concept of passivity. It is an input-output property of dynamical systems and therefore suitable for operability analysis in conjunction to the network approach. Dissipativity is mathematically expressed as follows:

Definition 1. (Dissipative Systems, Willems (1972a,b)). A dynamical system Σ is called dissipative if:

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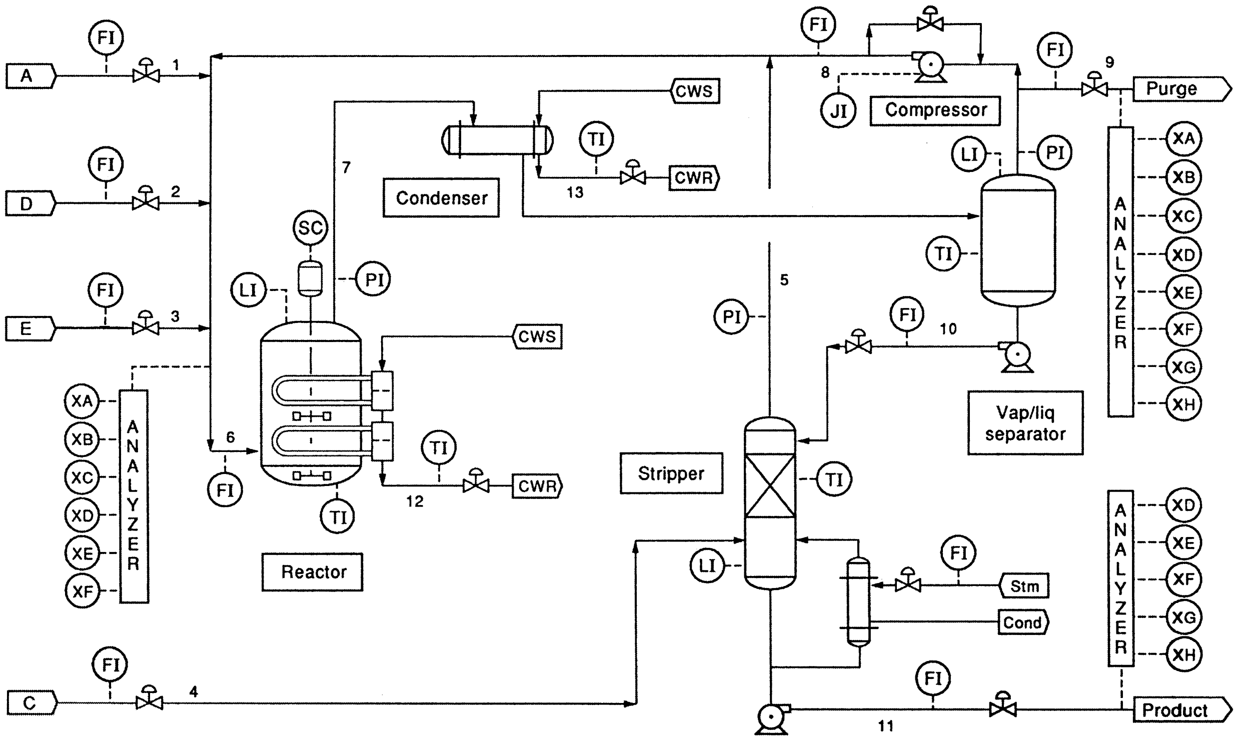

Fig. 2. Tennessee Eastman challenge problem (Jockenhövel et al. (2003))

Table 2. Steady-state process stream data

Stream	1	2	3	4	5	6	7	8	9	10	11
Flow ($kmol\ h^{-1}$)	12.3	117	98.6	421	469	1898	1476	1201	15.1	260	211
Temp. (K)	318	318	318	318	338	359	393	378	353	353	338
Mole fraction											
A	0.99990	0.00000	0.00000	0.48500	0.43515	0.33140	0.28317	0.34357	0.34357	0.00000	0.00000
B	0.00010	0.00010	0.00000	0.00500	0.00449	0.08983	0.11553	0.14017	0.14017	0.00000	0.00000
C	0.00000	0.00000	0.00000	0.51000	0.45758	0.26418	0.19683	0.23881	0.23881	0.00000	0.00000
D	0.00000	0.99990	0.00000	0.00000	0.00102	0.06922	0.00989	0.01156	0.01156	0.00202	0.00021
E	0.00000	0.00000	0.99990	0.00000	0.06575	0.17609	0.16223	0.17048	0.17048	0.12354	0.00581
F	0.00000	0.00000	0.00010	0.00000	0.00864	0.01633	0.02134	0.02243	0.02243	0.01626	0.00079
G	0.00000	0.00000	0.00000	0.00000	0.01944	0.03613	0.12527	0.04950	0.04950	0.48051	0.54697
H	0.00000	0.00000	0.00000	0.00000	0.00793	0.01681	0.08574	0.02347	0.02347	0.37766	0.44622

$$\phi(x(\tau)) - \phi(x_0) \leq \int_0^{\tau} w(u(t), y(t))\, dt, \qquad (3)$$

where $\phi(x) : X \to \mathbb{R}^+$ is a nonnegative function of the system's states called the *storage function* and $w(u, y)$ is a real valued function of the system's inputs and outputs called the *supply rate.*

In this article, the (Q, S, R)-dissipativity is adopted:

$$w(u, y) = y^T Q y + 2y^T S u + u^T R u, \qquad (4)$$

where Q, S and R constant matrices with Q and R symmetric. From Scorletti and Duc (2001), the dissipativity of an LTI system:

$$\Sigma : \begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \qquad (5)$$

can be obtained by solving the following Linear Matrix Inequality (LMI):

$$\begin{bmatrix} I & 0 \\ 0 & I \\ A & B \\ C & D \end{bmatrix}^T \begin{bmatrix} 0 & 0 & -P & 0 \\ 0 & R & 0 & S^T \\ -P & 0 & 0 & 0 \\ 0 & S & 0 & Q \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ A & B \\ C & D \end{bmatrix} > 0 \qquad (6)$$

with $P > 0$.

3.2 Stability of dissipative systems

The stability of a single (Q, S, R)-dissipative system can be determined solely on Q. Furthermore, if the dissipative system is input-output stable, then it has a finite $\mathcal{L}_2$-gain.

Theorem 2. (Hill and Moylan (1976)) Let dynamical system Σ be (Q, S, R)-dissipative and zero-state detectable (*i.e.*, if $u(t) = 0$ and $y(t) = 0$ then $\lim_{t\to\infty} x(t) = 0$). Then the free system $\dot{x} = Ax$ is stable if $Q \leq 0$ and asymptotically stable if $Q < 0$.

Theorem 3. (Moylan and Hill (1978)) Consider a (Q, S, R)-dissipative system Σ with $x_0 = 0$. If $Q < 0$, then the system has finite $\mathcal{L}_2$ gain γ:

$$\|y_\tau\|_{\mathcal{L}_2} \leq \gamma \, \|u_\tau\|_{\mathcal{L}_2} \qquad (7)$$

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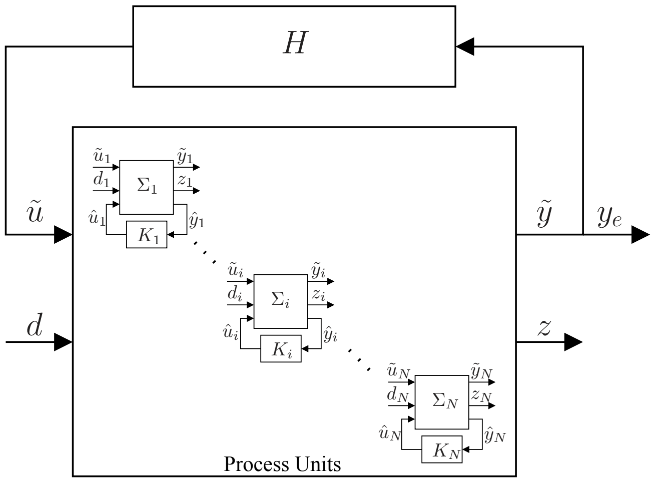

Fig. 3. Network of process units

where the subscript τ denotes truncation, *e.g.*:

$$y_\tau(t) \triangleq \begin{cases} y(t), & 0 \le t \le \tau \\ 0, & t > \tau \end{cases} \tag{8}$$

The upper bound of the system gain γ is given as:

$$\gamma = \|\hat{Q}^{-\frac{1}{2}}\| \left(\alpha + \|\hat{S}\|\right) \tag{9}$$

with $\hat{Q} = -Q$, $\hat{S} = \hat{Q}^{-\frac{1}{2}} S$ and $\hat{S}^T\hat{S} + R \le \alpha^2 I$ ($\alpha > 0$ a finite scalar).

3.3 Network perspective

A large-scale plant such as the TE process in Fig. 2 can be represented as a network of smaller subsystem as shown in Fig. 3. Each process unit is represented as an LTI system in (2). The representation also allows the use of weighting function on the performance indicators (z) which enables the analysis of frequency-based performance of the closed-loop system (from d to $\tilde{z}$). The topology of the process is then represented by the matrix H, called interaction matrix. This matrix has specific structures for different types of interconnections as follows:

- The flows of materials and energy from upstream units to downstream units (downstream connections) will only contribute to the lower triangular section of H.
- The flows of local recycle streams, *i.e.* the outlet of a particular unit is fed back into that unit will only contribute to the diagonal part of H.
- The flows of other recycle streams will contribute to the upper triangular section of H.

The TE case study will be used to illustrate how to construct the H interaction matrix. The network perspective of the TE process is given in Fig. 4. The topology of the interconnection can be mathematically written as follows:

$$H: \begin{bmatrix} \widetilde{u}_5 \\ \widetilde{u}_8 \\ \widetilde{u}_6 \\ \widetilde{u}_7 \\ \widetilde{u}_{10} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & I \\ 0 & 0 & I & 0 & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} \widetilde{y}_6 \\ \widetilde{y}_7 \\ \widetilde{y}_8 \\ \widetilde{y}_{10} \\ \widetilde{y}_5 \end{bmatrix}. \tag{10}$$

As shown in (10), the diagonal terms are zeros implying no local recycle streams. There are two recycle streams from downstream units, *i.e.* streams 5 and 8. The rest of the flows (streams 6, 7 and 10) are just simple downstream flow connections. These structures are important aspects on the plantwide operability. For example, Kiss et al. (2007) stressed that recycle can cause major operability problem such as snowball effect. This problem is reflected from the structure of H because having upper triangular entries makes it more difficult to achieve plantwide stability/stabilizability compared to process with only lower triangular entries.

3.4 Plantwide stability and stabilizability

The plantwide operability analysis based on network perspective combines the dissipativity condition of each subsystem Σ_i obtained by solving the LMI in (6) and the dissipativity of interconnection H.

In the context of open-loop stability analysis, the MVs ($\hat{u}_i$) and CVs ($\hat{y}_i$) is eliminated thus simplifying the state-space model in (2). The dissipativity of each subsystem i then becomes:

$$\dot{\phi}_i \le \begin{bmatrix} \widetilde{y}_i \\ z_i \end{bmatrix}^T \begin{bmatrix} Q_{i,1} & Q_{i,3}^T \\ Q_{i,3} & Q_{i,2} \end{bmatrix} \begin{bmatrix} \widetilde{y}_i \\ z_i \end{bmatrix} + 2 \begin{bmatrix} \widetilde{y}_i \\ z_i \end{bmatrix}^T \begin{bmatrix} S_{i,1} & S_{i,2} \\ S_{i,3} & S_{i,4} \end{bmatrix} \begin{bmatrix} \widetilde{u}_i \\ d_i \end{bmatrix} + \begin{bmatrix} \widetilde{u}_i \\ d_i \end{bmatrix}^T \begin{bmatrix} R_{i,1} & R_{i,3}^T \\ R_{i,3} & R_{i,2} \end{bmatrix} \begin{bmatrix} \widetilde{u}_i \\ d_i \end{bmatrix}. \tag{11}$$

The subscripts $1, 2, 3$ and 4 are assigned to the Q, S, R-matrices to denote the partition between different types of inputs and outputs for each subsystem i. Summing up the dissipativity of each subunit and embedding the topology information H results in the dissipativity of the overall large-scale system (with N-units) given as follows:

$$\dot{\phi} \le \begin{bmatrix} \widetilde{y} \\ z \end{bmatrix}^T \begin{bmatrix} Q_1 + S_1 H + H^T S_1^T + H^T R_1 H & Q_3^T + H^T S_3^T \\ Q_3 + S_3 H & Q_2 \end{bmatrix} \begin{bmatrix} \widetilde{y} \\ z \end{bmatrix} + 2 \begin{bmatrix} \widetilde{y} \\ z \end{bmatrix}^T \begin{bmatrix} S_2 + H^T R_3 \\ S_4 \end{bmatrix} d + d^T R_2 d \tag{12}$$

where $Q_1 = \text{diag}\ \{Q_{1,1}, \cdots, Q_{n,1}\}$, $S_1 = \text{diag}\ \{S_{1,1}, \cdots, S_{n,1}\}$, $R_1 = \text{diag}\{R_{1,1}, \cdots, R_{n,1}\}$ and the other notations follow.

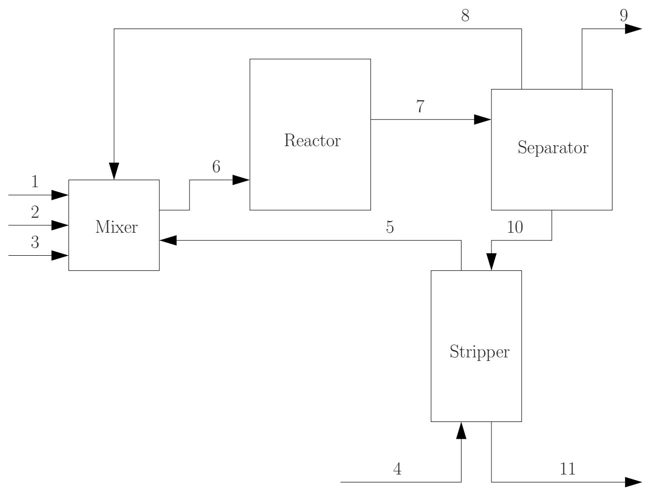

Fig. 4. TE process network

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The variable z is unnecessary in the open-loop context resulting in the plantwide (Q, S, R)-dissipativity condition as follows:

$$\begin{aligned}\dot{\phi} \leq & \widetilde{y}^T [Q_1 + S_1 H + H^T S_1^T + H^T R_1 H] \widetilde{y} \\ & + 2\widetilde{y}^T [S_2 + H^T R_3] d + d^T R_2 d.\end{aligned} \tag{13}$$

Theorem 2 is used to establish the open-loop plantwide stability based on the stability of each subsystem in (11) and the interconnection stability $Q_{overall} = Q_1 + S_1 H + H^T S_1^T + H^T R_1 H$ in (13).

Plantwide stabilizability is a more relevant subject for operability analysis. In this framework, stabilizability refers to the existence of a *decentralized* state feedback controller that can render the plantwide system stable. The same philosophy for open-loop plantwide stability analysis is applicable to the plantwide stabilizability analysis. The representation of each subsystem with a local multivariable state feedback controller $\hat{u}_i = K_i x_i$ is given as follows (the subscript i is dropped for convenience):

$$\Sigma_{i,cl} : \begin{bmatrix} \dot{x} \\ \widetilde{y} \\ z \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ \widetilde{u} \\ d \end{bmatrix} + \begin{bmatrix} B_3 \\ D_{13} \\ D_{23} \end{bmatrix} K \, [I \;\; 0 \;\; 0] \begin{bmatrix} x \\ \widetilde{u} \\ d \end{bmatrix} \tag{14}$$

(Q, S, R)-dissipativity of an LTI system is obtainable readily from the LMI in (6). However, the existence of the controller term K_i renders to problem nonlinear. Scorletti and Duc (2001); Scorletti and Ghaoui (1998) proposed the utilization of elimination lemma to convert the nonlinear matrix inequalities back into LMIs resulting in the conditions for plantwide stabilizability given as follows:

Proposition 4. There exists a decentralized state-feedback controller K that can stabilize a plantwide process if $\tilde{Q}_i < 0$, $Q_i < 0$ and $P_i > 0$ such that

$$\begin{bmatrix} I \\ D_{i,11}^T \\ D_{i,12}^T \end{bmatrix}^T \begin{bmatrix} \tilde{R}_{i,1} & \tilde{S}_{i,1}^T & 0 \\ \tilde{S}_{i,1} & \tilde{Q}_{i,1} & 0 \\ 0 & 0 & \tilde{Q}_{i,2} \end{bmatrix} \begin{bmatrix} I \\ D_{i,11}^T \\ D_{i,12}^T \end{bmatrix} > 0 \tag{15}$$

$$U_i^{\perp T} \begin{bmatrix} I & 0 \\ 0 & I \\ A_i^T & C_{i,1}^T \\ B_{i,1}^T & D_{i,11}^T \\ B_{i,2}^T & D_{i,12}^T \end{bmatrix}^T \begin{bmatrix} 0 & 0 & -P_i & 0 & 0 \\ 0 & \tilde{R}_{i,1} & 0 & \tilde{S}_{i,1}^T & 0 \\ -P_i & 0 & 0 & 0 & 0 \\ 0 & \tilde{S}_{i,1} & 0 & \tilde{Q}_{i,1} & 0 \\ 0 & 0 & 0 & 0 & \tilde{Q}_{i,2} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ A_i^T & C_{i,1}^T \\ B_{i,1}^T & D_{i,11}^T \\ B_{i,2}^T & D_{i,12}^T \end{bmatrix} U_i^{\perp} > 0 \tag{16}$$

$$\tilde{Q}_1 + \tilde{S}_1 H^T + H \tilde{S}_1^T + H \tilde{R}_1 H^T < 0 \tag{17}$$

$$\begin{bmatrix} R & S^T \\ S & Q \end{bmatrix} \begin{bmatrix} -\tilde{Q} & \tilde{S} \\ \tilde{S}^T & -\tilde{R} \end{bmatrix} = I \tag{18}$$

where $U_i = [B_{i,3}^T \, D_{i,13}^T]^T$, $U^{\perp}$ is the orthogonal complement of U defined as $U^T U^{\perp} = 0$ and $[U \, U^{\perp}]$ is full rank, $\tilde{Q}_1 = \text{diag}\ \{\tilde{Q}_{1,1}, \cdots, \tilde{Q}_{n,1}\}$, $\tilde{S}_1 = \text{diag}\ \{\tilde{S}_{1,1}, \cdots, \tilde{S}_{n,1}\}$, $\tilde{R}_1 = \text{diag}\ \{\tilde{R}_{1,1}, \cdots, \tilde{R}_{n,1}\}$ and $\tilde{Q}_2 = \text{diag}\ \{\tilde{Q}_{1,2}, \cdots, \tilde{Q}_{n,2}\}$.

Proof. First, the (Q, S, R)-dissipativity of each process unit is simplified by assuming block diagonal form, *i.e.* $Q_{i,3} = R_{i,3} = S_{i,2} = S_{i,2}^T = S_{i,3} = S_{i,3}^T = 0$. Then the elimination lemma in Scorletti and Duc (2001); Scorletti and Ghaoui (1998) states that there exists a solution for K, *i.e.* a matrix K =diag$\{K_1, \cdots, K_n\}$ such that the nonlinear matrix inequalities with state feedback control is equivalent to LMIs (15) and (16). $Q_{overall}$ in (13) is equivalent to (17) using Lemma 1 in Scorletti and Duc (2001).

The study shows that the TE process is stabilizable using a decentralized state-feedback controller. It is important to note that the framework is based on the first principle modeling. This implies the states in the model of each process unit are measurable physical states (aside from the cost factor), *e.g.* temperature, pressure, and component concentration. Each local controller on the process unit also follows the physically viable MVs as suggested in the original work by Downs and Vogel (1993).

3.5 Disturbance attenuation

In this case study, the process disturbances (d) considered in the scenario are listed as follows:

- cooling water inlet temperature to reactor and condenser,
- feed temperature of streams 1, 2 and 3,
- feed composition of stream 4.

There are two variables of interest (z) for the purpose of performance assessment: (1) product purity (% mol) and (2) the ratio of the products' mass (G/H).

Proposition 4 provides the stabilizability conditions for plantwide systems and Theorem 3 allows the quantification on the gain of the systems. Combining these two results, a method to analyze the effects of disturbance was developed (the details are omitted here due to space limit). With the full model in (14), we are able to quantify the plantwide system gain from d to z in terms of $\mathcal{L}_2$-gain defined as:

$$\|\Sigma\|_{\mathcal{L}_2} = \frac{\|z\|_2}{\|d\|_2}. \tag{19}$$

The assessment based only on direct $\mathcal{L}_2$-gain is often conservative because it only accounts for the maximum gain of the system for all frequencies. This is overcome by introducing weighting function W on z as shown in Fig. 3 to the plantwide system. The method then allows closed-loop performance specification in frequency domain. In the case of the TE process, the disturbance must be minimized at certain frequency range due to downstream requirements. Therefore a weighting function W was added to the problem formulation which specifies that the closed-loop performance from d to $\tilde{z}$ must have: (1) bandwidth of 0.295 rad/s, and (2) maximum steady-state offset of 5%. The disturbance attenuation problem was then solved giving a result that the plantwide system $\mathcal{L}_2$-gain from d to $\tilde{z}$ is 0.88.

4. DISCUSSIONS

The TE process is an example of complex chemical process mainly due to the exothermic reactions and the use of recycle streams. The proposed framework allows the decomposition of the complex TE process into a simple network representation consisting of an interaction matrix H and the individual process units as shown in Fig. 3. This decomposition allows the representation of each unit as an input/output system because the interconnections are based only on physical flows of materials and energy,

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which in turns allow the use of the theory of dissipative systems.

The main incentive to conduct the stability analysis on the TE process even though it has been well known to be open-loop unstable is because the proposed framework based on network approach is able to pin-point the problematic areas causing the operability difficulty. A detailed analysis shows that in addition to the exothermic reactions, Stream 8 (a recycle stream) has contributed to the plantwide instability. The operability problems arise due to the interactions of process units can be studied by looking at the process topology. This is called the *interaction analysis* of plantwide processes. In this framework, this is done by analyzing how different structures of interaction matrix H affects plantwide operability.

The existence of a solution to the stabilizability analysis means there exists a set of local state feedback controllers that regulate the inventory of each process unit by manipulating the local MVs of that unit and able to stabilize the entire plantwide TE process. Furthermore, the closed-loop performance was also obtained in terms of disturbance attenuation. In the case study, there are specifications on the fluctuation of the final products. This has been successfully incorporated into the stabilizability problem using the weighting function W. Since the gain of the system from d to $\tilde{z}$ was found to be less than 1, this means that with the decentralized control strategy, the required disturbance attenuation is achievable.

The approach proposed in this article is based on linearized models of process units. This allows numerical solutions to operability analysis of plantwide chemical processes by using Semi-definite programming tools. Compared to the approach for general nonlinear systems (Rojas et al. (2009)), this method does not require users to have knowledge of nonlinear control theories and can be readily used by process engineers at early process design stages. In addition, by using weighting functions, this approach can indicate more detailed achievable dynamic performance (e.g., the frequency band in which disturbances can be effectively attenuated) than Rojas et al. (2009).

5. CONCLUSION

Operability analysis is a key factor in modern process design to ensure that the plant is able to operate efficiently and safely. A framework for plantwide operability analysis is developed in this work based on a network approach. This approach allows the effects of interactions between process units to be studied based on the topology of plantwide systems and thus is able to pin-point the areas that cause operability problems. The network approach is also scalable, as shown in the case study of the TE process.

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