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Analysis and performance comparison of PID and fractional PI controllers

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Abstract: This paper compares the frequency domain performance criterion (J_v) of PI, fractional PI, and PID controllers when a step load disturbance is applied at the plant input. Process information is available in form of first-order plus dead-time (FOPDT) model. In addition, the controllers were compared using the H_∞–norm of the sensitivity function as a measure of robustness, and some comments on industrial practice are offered in this context.

Keywords: Fractional control, PID control, fractional PI, performance comparison.

1. INTRODUCTION

Despite the dramatic advancement of process control in recent decades, the proportional-integral-derivative (PID) controller continues to be the most frequently used feedback controller today, Åström and Hägglund (2001). PID control mechanism, the ubiquitous availability of reliable and cost-effective commercial PID modules, and pervasive operator acceptance are among the reasons for the success of PID controllers, Gude and Kahoraho (2007).

Fractional calculus, which is the expansion to fractional orders, has existed since the development of regular calculus. Unfortunately, fractional-order control was not originally incorporated into control engineering due to a general lack of mathematical knowledge and the limited computational power available at that time.

Several recent research activities are presently developing new tuning rules and techniques for fractional controllers, Monje et al. (2008). Gude and Kahoraho (2009a, 2009b) presents new tuning rules for PI and PI^λ control of processes that are typically found in process control. Some of these techniques are based on an extension of classical PID control theory. For example, Valério and da Costa (2006) have proposed fractional PID tuning rules that are similar to those proposed by Ziegler and Nichols. Gude and Kahoraho (2009c, 2009d) also have proposed new tuning rules for fractional PI controllers in the spirit of the Ziegler-Nichols rules.

The PI controller is the most commonly used control structure in the process industry, Åström and Hägglund (2001), wherein it is a common practice to turn off the derivative gain. Therein, two additional options should be considered. One option is to add a derivative component to the PI controller, making it a PID controller, whereas the other option is to implement a fractional PI.

This paper compares the performance of PI, PI^λ, and PID controllers. In this pursuit, the following questions will be answered: Is fractional PI control superior to PI or PID control? When and under what conditions is a PI^λ controller expected to perform better than a PI or a PID controller? The effects of robustness and control effort on controller performance are also considered.

This paper is organised as follows. Section 2 presents the different controllers and their structures. Section 3 presents the controller comparison criteria used in this study. Section 4 presents the results. Section 5 discusses the additional considerations related to robustness, control effort, and tuning rules. Section 6 concludes.

2. CONTROLLERS

This paper investigates three different types of controllers. The first controller we investigated is the fractional PI controller, which is a generalisation of the PI controller, and is a non-integer order controller of the form:

$$C(s) = k + \frac{k_i}{s^\lambda} \tag{1}$$

This type of controller is attractive because it exhibits a better flexibility that is derived from the order of λ of the fractional integral part of the control. Therefore, three parameters can be tuned in this structure (k, k_i, and λ); that is, an additional parameter relative to those available for conventional PI control (λ = 1). We can take advantage of the fractional order of λ in order to improve controller performance. The PID controller can be parameterised in parallel form as:

$$C(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N \cdot s} + 1}\right) \tag{2}$$

A PID controller typically consists of four parameters, wherein N is often considered to be a tuning parameter, Isaksson and Graebe (2002). In order to limit the complexity of the PID controller, N was fixed to a value of 10, which is a typical value that has been recommended in textbooks on process control and in specialised literature, Åström and Hägglund (1995). For the PI controller investigated in this study, T_d was set to 0.

3. COMPARISON CRITERIA

The primary objective of this paper is to provide insight into choosing between a fractional PI^{λ} and a PI(D) controller within the context of process control.

When designing a process controller, detailed process knowledge is usually considered not to be available, which is often the case in the process industry. In this paper it is assumed that process information is available in terms of a first-order plus dead-time (FOPDT) model. It is denoted by:

$$G(s) = \frac{K_P}{1+Ts} e^{-Ls} \qquad (3)$$

The comparison was performed for the model (3) modifying the value of the time constant in the interval $T \in [0.01, 10]$ while L and K_P were fixed to 1. This is the simplest way to divide process dynamics into delay-dominated and lag-dominated dynamics.

Comparison was only performed for this process. By ensuring that the investigated controllers satisfy a robustness constraint, all controller results should be valid for plants that are sufficiently close to this model.

The block diagram of the loop is shown in Figure 1. The symbol C(s) represents the controller, G(s) is the process to be controlled, l is the load disturbance that affects the system and n is measurement noise.

3.1 Performance criterion

Regulation performance is often of primary importance in the process industry because most controllers operate as regulators, Shinskey (1996).

Regulation performance is often expressed in terms of the control error obtained for certain disturbances. A load disturbance is typically applied at the process input. Specific criteria are typically used to minimise a loss function of the form:

$$I = \int_0^{\infty} t^n |e(t)|^m \cdot dt \qquad (4)$$

where the error is defined as e(t) = r(t) – y(t). Common cases include integral absolute error (IAE) (n = 0, m = 1), integral square error (ISE) (n = 0, m = 2), and integral of time weighted squared error (ITSE) (n = 1, m = 2). A related error is often referred to as integrated error (IE) defined as:

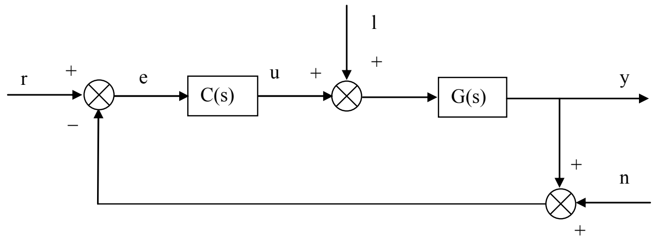

Fig. 1. Block diagram of the loop considered in the controller comparison investigated in this study.

$$IE = \int_0^{\infty} e(t) \cdot dt \qquad (5)$$

IE and IAE are the same if the error does not change sign; however, it is important to note that IE can be very small even if the error is not. One reason for using IE is that its value is directly related to the integral gain k_i of the PID controller, Åström and Hägglund (2001). For a load disturbance in the form of a unit step, IE = $1/k_i$. Thus, the performance criterion that minimises the IE is equivalent to maximizing the integral gain k_i.

Kristiansson and Lennartson (2002) have defined another performance criterion in the frequency domain based on a function of the error signal as an alternative to the aforementioned criteria. This performance criterion is formulated as:

$$J_v = \left\| \frac{1}{s} G(s) S(s) \right\|_{\infty} = \max_{\omega} \left| \frac{1}{j\omega} \cdot \frac{G(j\omega)}{1+L(j\omega)} \right| \qquad (6)$$

This performance criterion primarily measures the system's ability to handle low-frequency load disturbances. J_v also works for poorly dampened closed loop systems and can be used in MIMO systems.

3.2 Robustness constraint

Robustness is an important consideration in control design. The trade-off between robustness and performance must be taken into account when comparing the performance of different control structures. Control structure comparison should be made in such a way that guarantees that the compared control structures have the same robustness.

There are many different criteria for robustness. Many of these criteria can be expressed as restrictions on the Nyquist curve of the loop transfer function L(s) = G(s)C(s). In recent years, the maximum sensitivity function has been increasingly accepted as the premier measure of robustness, Åström and Hägglund (1995).

$$\|S(s)\|_{\infty} = \max_{\omega} |S(j\omega)| = \max_{\omega} \left| \frac{1}{1+L(j\omega)} \right| \le M_S \qquad (7)$$

The constraint (7) that the sensitivity function S(jω) is less than a given value M_S implies that the loop transfer function should be outside of a circle with radius $1/M_S$ and centred at –1.

3.3 Control activity

A reasonable ambition in all control design is to keep the control signal as small as possible. Control system design typically deals with a trade-off between performance and control effort, provided that a reasonable mid-frequency robustness can be guaranteed. Therefore, we introduce the control activity criterion:

$$J_u = \|C(s)S(s)\|_\infty = \max_\omega \left| \frac{C(j\omega)}{1+L(j\omega)} \right| \qquad (8)$$

3.4 Controller design

The design problem discussed in the controller comparison made in this paper can be formulated as an optimisation problem: *Find parameters of the different controllers that minimise the performance criterion (6) subject to the robustness constraint (7).* Additional considerations associated with control effort (8) can also be taken into account. In this paper, the Matlab Optimisation Toolbox is used for the computational comparisons. The expression *optimal controller* will from hereon refer to a controller that has been optimised as described above.

Because the PI^λ and PI(D) controller parameters are only subject to robustness constraint (7), their results do not depend on a specific controller tuning rule.

4. COMPARISON RESULTS

Figure 2 depicts values of J_v for PI, PI^λ, and PID controllers for different values of L/T in a FOPDT process and a typical value of M_S = 1.4. It shows that the benefit in using a PI^λ controller is increasing as L/T grows. In the range $[10^{-1}, 10^1]$ the PID controller gets the best optimal J_v-value if the control effort is ignored. If we have limitations on the control effort an additional analysis has to be done and the previous comparison changes for small values of L/T. For a more detailed analysis, see Section 5.

In order to simplify our discussion, the range of L/T is split into three different regions, as shown in Figure 2.

4.1 Region 1: L/T ∈ $[10^{-1}, 10^0]$

Over this region, the PI controller has a slightly higher value of J_v than the PI^λ controller, although their performance is quite similar. It is important to note that the performance curves for all the investigated controllers decrease as the FOPDT process approaches a pure lag-dominant process. The performance of the PI^λ controller approaches that of the PI controller as the L/T ratio is decreased. This phenomenon is not unexpected because when L = 0 the poles of the closed loop system can be arbitrarily placed with a PI controller. Investigations of the fractional order of λ indicate that it approaches unity for smaller values of L/T. The benefit obtained from using a PI^λ instead of a PI controller increases as a function of increasing L/T, as can be seen in Figure 2.

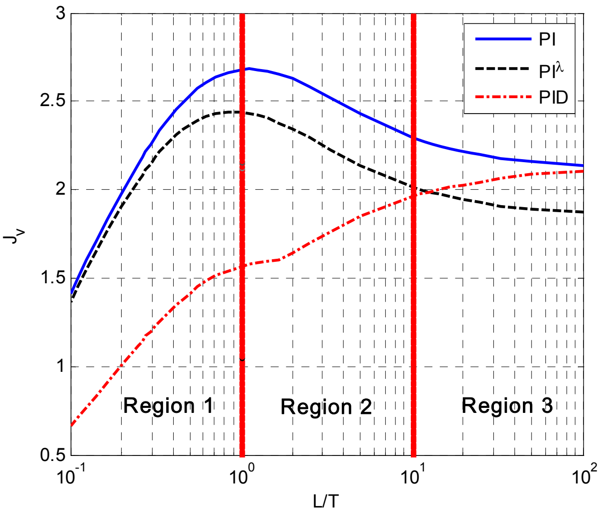

Fig. 2. Values of J_v for the different control structures and for different values of L/T.

Furthermore, in this figure, we can also observe that the PID controller performs better than the other two investigated controller structures. The largest difference between the PI^λ and PID controllers is when L/T = 10^{-1}. In addition, according to Figure 2, J_v decreases 100% when control is switched from a PI or a PI^λ to a PID controller.

It is important to note that in this comparison control effort has not been considered. The performance of the PID controller for small values of L/T often implies large values of J_u. Some control effort considerations are analysed in Section 5.

4.2 Region 2: L/T ∈ $[10^0, 10^1]$

In this region the performance of the PID controller continues to be superior to that of the PI and PI^λ controllers. The control performances of the PI^λ and PID controllers approach one another as the ratio L/T is increased, as can be observed in Figure 2 and become nearly equal when L/T ≈ 10^1. It is important to note that in this region, the control effort generated by the PID controller is more than two and seven times the control generated by the PI^λ or PI controllers for L/T = 10^1 and 10^0, respectively. Table 1 compares the control efforts of the different controllers at different values of L/T.

4.3 Region 3: L/T ∈ $[10^1, 10^2]$

In region 3, the performance of the PI^λ controller is superior to that of the other controllers. J_v decreases by 12% over the entire range after a switch from a PI controller to a fractional PI controller and decreases by 10% following a switch from a PID controller. The performance of the PID controller approaches that of the PI controller as a function of increasing L/T, which is not surprising because past investigations of the derivative gain have indicated that this is because it approaches zero for larger values of L/T.

The PI controller exhibits a higher value of J_v over this range, although its performance is similar to that of the PID controller for large values of L/T. It is important to note that the performance curves for all of the investigated controllers is level out and approach a constant value as the FOPDT process becomes a pure dead-time process.

5. ADDITIONAL CONSIDERATIONS

A value of $M_S = 1.4$, which was recommended in Åström and Hägglund (1995), was used in this study. It would be interesting to investigate the results of controller performance in terms of J_v for other values of M_S. Figures 3 and 4 indicate that the robustness constraint significantly influences the performance of all controllers investigated in this study. Figures 3 and 4 illustrate the trade-off between performance and robustness for PI and PI^λ controllers and different values of L/T.

Reasonable robustness constraint values are required to achieve good control, although it is common practice in process control to further increase constraints by detuning the controller in order to ensure good control when dynamics change due to non-linearities in the plant. Figures 3 and 4 indicate that the resulting loss of performance for PI and PI^λ controllers due to detuning may be high.

The benefit of using a PI^λ controller instead of a PI controller increases for large values of L/T. For example, for small values of M_S and an L/T of 10^{-1}, 10^0, 10^1, and 10^2, there is 7%, 20%, 22% and 22%, respectively, more benefit in using a PI^λ controller in comparison to a PI controller.

Another interesting result is that the ratio between the optimal values of J_v obtained for the PI^λ and PI controllers tend to approach unity when $M_S = 2$ in the full range of L/T–values. Therefore, it is important not to overly relax the robustness constraint in order to obtain greater performance improvement when using a PI^λ controller.

Another valuable consideration is the control effort generated by each controller. PID control effort increases as a function of decreasing L/T. It is important to note that the PID controller requires two, seven, and ten times the amount of control effort than that required by the PI^λ and PI controllers for $L/T = 10^1$, 10^0, and 10^{-1}, respectively. Figure 5 illustrates that, in order to achieve a comparable performance, the PID controller requires five and seven times more control effort than optimal PI and PI^λ controllers, respectively; however, as can be observed in Figure 6, in order to achieve the same performance as the PI^λ, the value of J_u of the PID controller needs to be doubled. The J_v–J_u curve of the PID controller approaches the optimal value of that obtained by the PI controller for small values of J_u. For large values of L/T, the control effort is similar for all of the investigated controllers and an improvement in performance can be obtained by using the PI^λ controller instead of the other controllers (see Table 1 for numerical values of J_u and J_v).

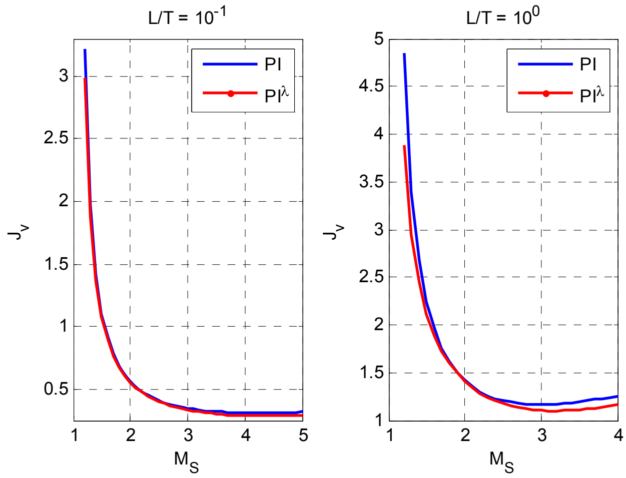

Fig. 3. Values of optimal J_v for PI and PI^λ controllers using different values of L/T.

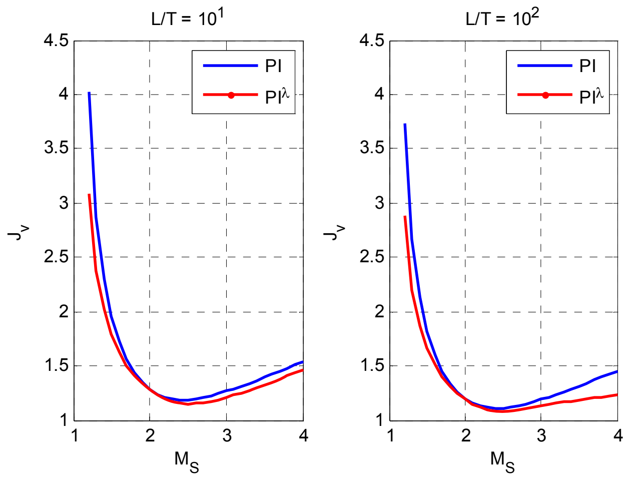

Fig. 4. Values of optimal J_v for PI and PI^λ controllers using different values of L/T.

Table 1. Performance and control effort in terms of J_v and J_u obtained for different values of L/T and different controllers

L/T	Controller	J_v	J_u
10^{-1}	PI	1.41	4.42
	PI^λ	1.37	4.58
	PID	See Figure 5	
10^0	PI	2.68	1.00
	PI^λ	2.44	1.08
	PID	See Figure 6	
10^1	PI	2.30	1.00
	PI^λ	2.02	1.06
	PID	1.96	2.34
10^2	PI	2.14	1.00
	PI^λ	1.87	1.06
	PID	2.09	1.89

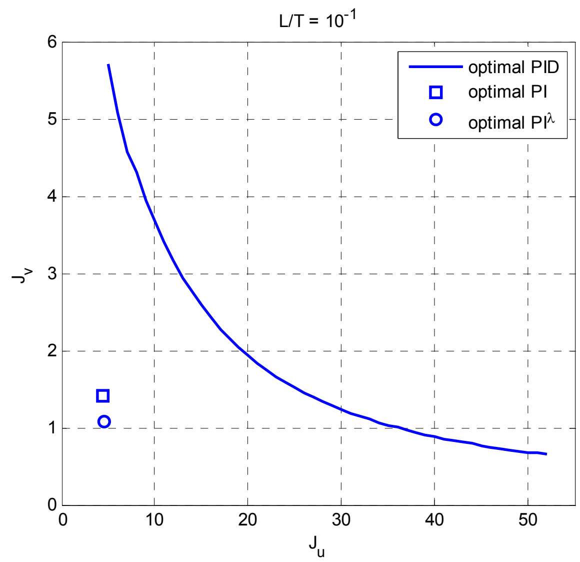

Fig. 5. Trade-off curve between performance and control effort for L/T = 10^{-1}. J_v–J_u values for PI and PI^λ controllers are also shown.

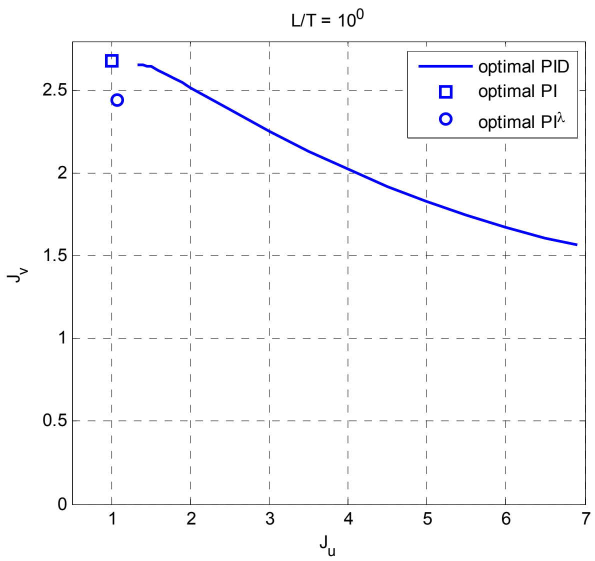

Fig. 6. Trade-off curve between performance and control effort for L/T = 10^0. J_v–J_u values for PI and PI^λ controllers are also shown.

Audits conducted in the process industry have shown that problems due to controller tuning are frequent. Typical problems are oscillations due to tight tuning or sluggishness due to loose tuning, Bialkowski (1993). Systematic tuning of PID controllers in the process industry is usually done using a tuning rule. The tuning methods for PI^λ controllers proposed in Gude and Kahoraho (2009c) (denoted by f-GK and af-GK, respectively) are compared in terms of J_v and M_S with the Ziegler-Nichols step and frequency response methods (ZN step and ZN frequency), the Cohen-Coon method (CC), the Chien-Hrones-Reswick 0% and 20% methods (CHR 0% and CHR 20%), and Åström and Hägglund's kappa-tau method (κ-τ), respectively. See Åström and Hägglund (1995) for reference of all these methods. Figures 7 and 8 depict the J_v and M_S values for tuning rules for PI and PI^λ controllers.

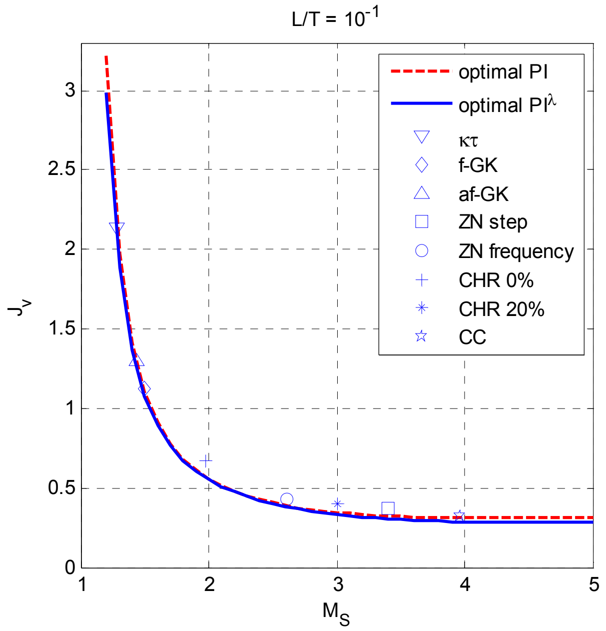

Fig. 7. Optimal J_v for PI and PI^λ controllers as a function of M_S with some common tuning rules. The process is $G(s) = e^{-s}/(10s+1)$.

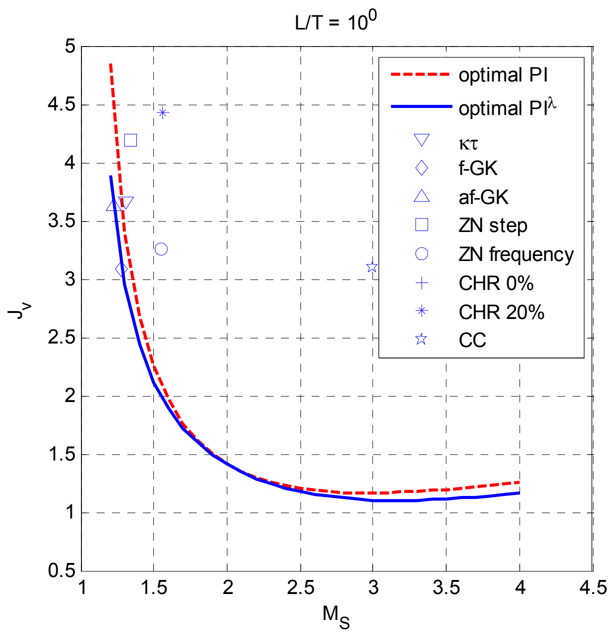

Fig. 8. Optimal J_v for PI and PI^λ controllers as a function of M_S with some common tuning rules. The process is $G(s) = e^{-s}/(s+1)$.

For L/T = 10^{-1}, all the rules yield controllers with performance close to the optimal one. However, the robustness varies considerably for the rules considered. Many of them have poor robustness.

For L/T = 10^0, the robustness is slightly better but the performance differs from the optimal one considerably. Use of these rules without careful consideration to robustness is likely to result in problems later on when process dynamics change.

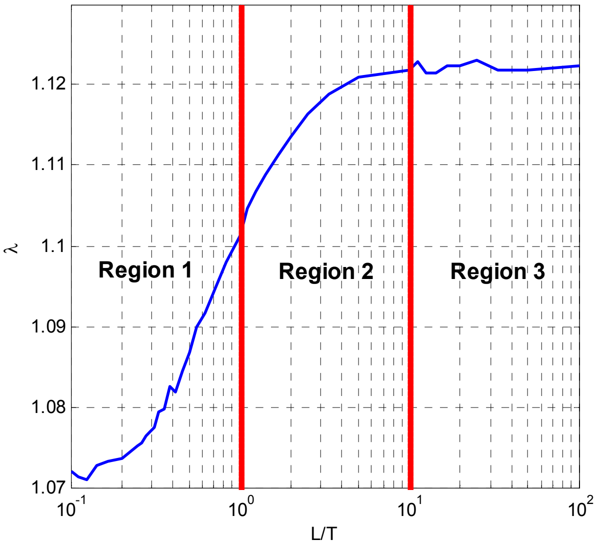

Fig. 9. The fractional order of λ obtained for different values of L/T.

Figure 9 depicts the optimal fractional order of λ obtained for different values of L/T. Some interesting conclusions can be drawn from this figure, which can also be divided into three regions. For delay dominant processes, λ is a constant equal to 1.12. For lag-dominant processes, the behaviour is similar to that of a PI controller, due to the low value of λ and is very close to unity. For balanced processes, the value of λ linearly increases as a function of increasing L/T.

The behaviour of λ in regions 2 and 3 is completely different for FOPDT processes and for models typically found in industry, which has been remarked in Gude and Kahoraho (2009b). The constant value of λ in Region 3 supports the idea of developing tuning rules for fractional PI controllers with fixed λ equal to 1.12, as introduced in Gude and Kahoraho (2009c, 2009d).

6. CONCLUSIONS

This paper compared the performance of PI, fractional PI, and PID controllers within the context of a typical process control environment. The primary objective of this study was to determine the appropriate use conditions for each controller type. J_v performance was obtained as a function of the ratio between the dead time and the time constant L/T for a collection of FOPDT models.

The primary result of this study is that for values of L/T in the interval of $[10^0, 10^2]$, J_v substantially decreases when a PI^λ controller is used instead of a PI controller. Another result is that for an L/T in the interval of $[10^1, 10^2]$, J_v moderately decreases when a PI^λ controller is used instead of a PID controller. This range also increases when the control effort generated by the controllers is considered. The influence of robustness constraints on controller performance was also considered, and several comments in the context of industrial practice were offered therein.

Given that a substantial improvement is obtained by using PI^λ instead of PI controllers, future work should focus on obtaining new tuning rules for fractional PID controllers.

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