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Fault Detection and Isolation for Multimode Processes with Recursive Principal Component Analysis

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Abstract: Contribution plots of the monitored statistics, Q and T^2, are investigated to locate faulty variables when the statistics are out of their control limits. It is a popular method for fault isolation; however, it is well known that the smearing out of contributions leads to misdiagnose the faulty variables. Alternatively, the reconstruction-based contribution approach is claimed to guarantee correct diagnosis. It has been examined in this paper that the approach fails to precisely locate faulty variables when encountering multiple sensor faults. A fault isolation chart on principal component (PC) subspace is provided to locate faulty variables for a process with multiple operating regions. The results of the quadruple-tank process simulation show the proposed approach successfully locate faulty variables in a case of multiple sensor faults, as long as the process behavior can be depicted by the scores on the PC subspace.

1. INTRODUCTION

Nowadays, most complex processes are equipped with measurement sensors for process control or indicators. Massive amounts of process data become accessible in a real-time manner, whereas, most of them are stored in historical databases. Therefore, it is more practical to develop methods of fault detection and isolation (FDI) for process monitoring based on data-driven approaches comparing to other methods based on rigorous process models or knowledge-based approaches. In chemical processes, there are a large number of measured and controlled variables which are highly correlated. Principal component analysis (PCA) is a popular method to decompose the variable space into principal component (PC) subspace and residual subspace. Two statistics with their control limits are defined from historical data for the detection of process abnormalities. The T^2 statistic is used to monitor data variations on the PC subspace; on the other hand, the deviations on the residual subspace can be captured by using the Q statistic. However, for a multimode process, the T^2 statistic is inadequate to monitor the systematic parts of PCA, since the data variations will not follow a Gaussian distribution on the PC subspace. In contrast, the Q statistic is still available for monitoring the residual parts of PCA for a process with multiple operating regions, as long as, the retained PCs are capable of capturing the common-cause variability of the process, and the residual subspace only contains measured noise.

Gaussian mixture model (GMM) with Bayes rule is a popular tool for unsupervised pattern recognition. Based on the assumption that the data variations follow a Gaussian distribution within a steady-state operating mode, GMM has been utilized to extract multiple operating regions from historical process data. For example, Wang and McGreavy (1998) applied Bayesian automatic classification (AutoClass) developed by NASA to cluster data from a FCCU into classes corresponding to various operating modes. Yu and Qin (2008) used the Figueiredo-Jain (F-J) algorithm to determine the cluster parameters of Bayesian classification. In their approach, a fault detection index was derived based on Mahalanobis distance and the posterior probability of each cluster; however, faulty variables were not isolated when the index exceeded its control limits.

After a fault is detected, the faulty variables need to be isolated in order to diagnose the root cause. The contribution plots are the most popular tools to identify which variables push the statistics out of their control limits. However, Alcala and Qin (2009) have proven that the contribution plot approaches will not guarantee to isolate the faulty variables, precisely. Besides, they proposed a reconstruction-based contribution (RBC) approach to locate faulty variables without fault smearing effect. However, the RBC approach may not guarantee to isolate the correct faulty variables when encountering an abnormal event with multiple fault directions. It has been detailed in Appendix.

In this paper, a local T^2 statistic with its control limits is provided to monitor the systematic parts of PCA for each operating mode. The differences from observations to cluster center on the PC subspace are measured, when an abnormal event is detected, for isolating faulty variables. The control limits of the differences are derived in this paper. When the PC subspace fails to portray process behavior during abnormal event, the subspace needs to be adapted by using recursive PCA (RPCA, Li *et al.*, 2000) with new event data. After that, the parameters of known event clusters are transferred onto the updated subspace (Liu, 2008). It is much easier to interpret the revealed information when the process behavior can be captured by the PC subspace. The proposed approach is designed to locate the faulty variables for post analysis. It is not necessary to be implemented in a real-time manner. The root causes of the abnormal events need to be diagnosed through locating the faulty variables in the first place to prevent the faults reoccurring. These tasks are accomplished by post analysis with the process knowledge involved, i.e., the process engineers have to justify the revealed information by the proposed approach.

The remainder of this paper is organized as follows. Section 2 gives a preliminary of PCA and GMM with Bayes rule. The proposed approach of monitoring local operating region and creating fault isolation charts is detailed in section 3. In section 4, the utility of the proposed monitoring approach is demonstrated with a simulated quadruple-tank process example. Finally, conclusions are given.

2. BASIC THEORY

2.1 Principal Component Analysis

Consider the data matrix $\mathbf{X} \in R^{m \times n}$ with m rows of observations and n columns of variables. Each column is normalized to zero mean and unit variance. The covariance of the reference data can be estimated as:

$$\mathbf{S} \approx \frac{1}{(m-1)} \mathbf{X}^{\mathrm{T}}\mathbf{X} = \mathbf{P}\mathbf{\Lambda}\mathbf{P}^{\mathrm{T}} + \tilde{\mathbf{P}}\tilde{\mathbf{\Lambda}}\tilde{\mathbf{P}}^{\mathrm{T}} \tag{1}$$

where $\mathbf{\Lambda}$ is a diagonal matrix with the first K terms of the significant eigenvalues, and $\mathbf{P}$ contains the respective eigenvectors. The $\tilde{\mathbf{\Lambda}}$ and $\tilde{\mathbf{P}}$ are the residual eigenvalues and eigenvectors respectively. The data matrix $\mathbf{X}$ can be decomposed as:

$$\mathbf{X} = \mathbf{X}\mathbf{P}\mathbf{P}^{\mathrm{T}} + \mathbf{X}\tilde{\mathbf{P}}\tilde{\mathbf{P}}^{\mathrm{T}} = \hat{\mathbf{X}} + \mathbf{E} \tag{2}$$

with $\hat{\mathbf{X}}$ being the projection of the data matrix $\mathbf{X}$ onto the subspace formed by the first K eigenvectors, named principal component (PC) subspace, and $\mathbf{E}$ being the remainder of $\mathbf{X}$ that is orthogonal to the subspace.

Statistic Q is defined as a measure of the variations of residual parts of data.

$$Q = \left(\mathbf{x} - \hat{\mathbf{x}}\right)\left(\mathbf{x} - \hat{\mathbf{x}}\right)^{\mathrm{T}} = \mathbf{x}\tilde{\mathbf{P}}\tilde{\mathbf{P}}^{\mathrm{T}}\mathbf{x}^{\mathrm{T}} \tag{3}$$

In addition, another measure for the variations of systematic parts on the PC subspace is the statistic T^2.

$$T^2 = \mathbf{x}\mathbf{P}\mathbf{\Lambda}^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{x}^{\mathrm{T}} = \mathbf{t}\mathbf{\Lambda}^{-1}\mathbf{t}^{\mathrm{T}} \tag{4}$$

where $\mathbf{t}$ are the first K term scores. It is Mahalanobis distance from the origin of the subspace to the projection of the observations. The confidence limits of Q and T^2 can be found in Jackson (1991).

2.2 Gaussian Mixture Model with Bayes rule

For a process with c operating regions, it can be assumed that the data spread follows a Gaussian distribution within a steady-state region. The probability density function (pdf) of the training data on the PC subspace can be estimated by using Gaussian mixture model:

$$p(\mathbf{t}) = \sum_{k=1}^{c} p\left(\mathbf{t} \middle| \boldsymbol{\theta}_k\right) P_k \tag{5}$$

where P_k and $p\left(\mathbf{t} \middle| \boldsymbol{\theta}_k\right)$ are the priori probability and the conditional pdf, which is a Gaussian distribution with parameters $\boldsymbol{\theta}_k$, of the k^{th} operating region. If P_k and $p\left(\mathbf{t} \middle| \boldsymbol{\theta}_k\right)$ for all classes are known, the posteriori probability of the j^{th} event can be found from Bayes rule.

$$P\left(j \middle| \mathbf{t}\right) = p\left(\mathbf{t} \middle| \boldsymbol{\theta}_j\right) P_j \Big/ \sum_{k=1}^{c} p\left(\mathbf{t} \middle| \boldsymbol{\theta}_k\right) P_k \tag{6}$$

Those cluster parameters, including priori probabilities $\mathbf{P} = \left[P_1, P_2, \ldots P_c\right]$, cluster centers $\boldsymbol{\mu} = \left[\boldsymbol{\mu}_1, \ldots, \boldsymbol{\mu}_c\right]$, and covariances $\boldsymbol{\Sigma} = \left[\boldsymbol{\Sigma}_1, \ldots, \boldsymbol{\Sigma}_c\right]$, can be iterated from the Expectation-Maximization (EM) algorithm as follows:

E-Step:

Calculate the posteriori probabilities at the t^{th} iteration:

$$P\left(j \middle| \mathbf{t}; t\right) = \frac{p\left(\mathbf{t} \middle| \boldsymbol{\mu}_j(t), \boldsymbol{\Sigma}_j(t)\right) P_j(t)}{\sum_{k=1}^{c} p\left(\mathbf{t} \middle| \boldsymbol{\mu}_k(t), \boldsymbol{\Sigma}_k(t)\right) P_k(t)}, \quad j = 1 \ldots c \tag{7}$$

M-Step:

Compute the next estimated parameters by:

$$\boldsymbol{\mu}_j(t+1) = \frac{\sum_{k=1}^{m} P\left(j \middle| \mathbf{t}_k; t\right) \mathbf{t}_k}{\sum_{k=1}^{m} P\left(j \middle| \mathbf{t}_k; t\right)} \tag{8}$$

$$\boldsymbol{\Sigma}_j(t+1) = \frac{\sum_{k=1}^{m} P\left(j \middle| \mathbf{t}_k; t\right) \left(\mathbf{t}_k - \boldsymbol{\mu}_j(t)\right)^T \left(\mathbf{t}_k - \boldsymbol{\mu}_j(t)\right)}{\sum_{k=1}^{m} P\left(j \middle| \mathbf{t}_k; t\right)} \tag{9}$$

$$P_j(t+1) = \frac{1}{m} \sum_{k=1}^{m} P\left(j \middle| \mathbf{t}_k; t\right) \tag{10}$$

in which m is the number of observations. The solution of maximum log-likelihood function is found by repeating E and M steps until every parameter has converged to within a tolerance criterion ε.

3. PROPOSED APPROACH

3.1 Local Statistic T^2 and Fault Isolation Charts

The control limits of traditional statistic T^2 are derived based on the assumption that the normal operational data spread following a Gaussian distribution; therefore, it is inadequate to monitor systematic variations of a process with multiple operating modes by using conventional T^2 statistic. In this paper, a local statistic T^2 is utilized to detect the variations of systematic parts of PCA. After converging the EM steps, perform singular value decomposition (SVD) on each cluster covariance.

$$\mathbf{\Sigma}_i = \mathbf{U}_i \mathbf{\Lambda}_i \mathbf{U}_i^{\mathrm{T}}, \quad i = 1...c \tag{11}$$

where $\mathbf{\Lambda}_i$ is a diagonal matrix with eigenvalues of the i^{th} cluster covariance, and $\mathbf{U}_i$ is a full matrix with corresponding eigenvectors. The local T^2 statistic for the i^{th} cluster can be obtained from the Mahalanobis distance to the cluster center:

$$T_i^2 = (\mathbf{t} - \boldsymbol{\mu}_i)\mathbf{\Sigma}_i^{-1}(\mathbf{t} - \boldsymbol{\mu}_i)^{\mathrm{T}} = (\mathbf{t} - \boldsymbol{\mu}_i)\mathbf{U}_i\mathbf{\Lambda}_i^{-1}\mathbf{U}_i^{\mathrm{T}}(\mathbf{t} - \boldsymbol{\mu}_i)^{\mathrm{T}} \tag{12}$$

where $\mathbf{t}$ is a score vector on the PC subspace, and $\boldsymbol{\mu}_i$ is the i^{th} cluster center. Since the conditional pdf is a Gaussian distribution, the confidence limits of the i^{th} cluster can be defined as:

$$T_{i,\alpha}^2 = \frac{K(m_i - 1)}{m_i - K} F_{K,m_i-K,\alpha} \tag{13}$$

where m_i is the number of observations belonging to the i^{th} cluster, K is the dimension of the subspace, and $F_{K,m_i-K,\alpha}$ is an F distribution with degrees of freedom K and m_i-K within (1-α) confidence limits.

Alcala and Qin (2009) have proven that the contribution plot approaches will not guarantee to isolate the correct faulty variables. It can be expected that the contribution plots of local T^2 suffer the same difficulty. A new fault isolation chart is provided in this subsection. Assuming that the process operates in the i^{th} operating mode, it can be detected from local T^2 with its control limits when a sample leaving the steady-state region. The faulty variables are isolated by comparing the differences between measurements and cluster centers, i.e. $\mathbf{x} - \boldsymbol{\mu}_i\mathbf{P}^{\mathrm{T}}$, with their confidence limits. The confidence limits of the differences for each variable need to be derived for locating faulty variables, it is because that the data variations on each variable direction may not be identical within a steady-state region.

In the case of the process behavior that can be captured by the scores on the PC subspace during abnormalities, which is the statistic Q still under its control limits, the differences can be rewritten as $(\mathbf{t} - \boldsymbol{\mu}_i)\mathbf{P}^{\mathrm{T}}$. Based on the assumption that the data variations follow a Gaussian distribution within an operating mode, the (1-α) confidence limits for the differences within the i^{th} operating region can be written as (Conlin *et al.*, 2000):

$$\pm z_{\alpha/2}\sigma_i(\mathbf{x} - \boldsymbol{\mu}_i\mathbf{P}^{\mathrm{T}}) \approx \pm z_{\alpha/2}\sigma_i((\mathbf{t} - \boldsymbol{\mu}_i)\mathbf{P}^{\mathrm{T}}) \tag{14}$$

in which $z_{\alpha/2}$ is the corresponding standard normal deviate, and σ_i is the standard deviation for each variable difference calculated from the data belonging to the i^{th} cluster. The standard deviation can be calculated from the covariance of the i^{th} cluster.

$$\sigma_i((\mathbf{t} - \boldsymbol{\mu}_i)\mathbf{P}^{\mathrm{T}}) = diag(\mathbf{P}\mathbf{\Sigma}_i\mathbf{P}^{\mathrm{T}})^{0.5} \tag{15}$$

The faulty variables can be isolated by plotting the variable differences with their control limits. It should be noted that the necessary condition of the proposed method is that the PC subspace is capable of describing the process abnormal behavior, i.e., the Q statistics under its control limits. When the statistic Q is out of its control limits, the PC subspace needs to be adapted by using new event data for describing the process's new behavior.

3.2 *Recursive PCA and Adapted Clusters*

Assuming the data of new events with m' rows of observations, the data matrix is denoted as $\mathbf{W}' \in R^{m' \times n}$. The mean vector ($\overline{\mathbf{W}'}$) and the diagonal matrix of standard deviations ($\mathbf{S}'$) of the new event data have to be prepared to normalize the data matrix $\mathbf{X}'_{m'} = (\mathbf{W}' - \mathbf{1}\overline{\mathbf{W}'})\mathbf{S}'^{-1}$ with zero means and unit variances, where $\mathbf{1}$ is a column vector in which all elements are one. The covariance matrix of the new dataset can be obtained from the normalized data matrix: $\mathbf{\Sigma}'_{m'} = \mathbf{X}'^{\mathrm{T}}_{m'}\mathbf{X}'_{m'}/(m' - 1)$.

The mean vector and the standard deviations of combining the reference and the new datasets can be derived as follows:

$$\overline{\mathbf{W}}^* = \frac{m}{m*}\overline{\mathbf{W}} + \frac{m'}{m*}\overline{\mathbf{W}'} \tag{16}$$

$$\sigma_i^* = \sqrt{\frac{(m-1)\sigma_i^2 + m\,\bar{w}_i^2 + (m'-1)\sigma_i'^2 + m'\,\bar{w}_i'^2 - m*\bar{w}_i^{*2}}{m*-1}} \tag{17}$$

$$\mathbf{S}^* = diag\left[\sigma_1^* \quad \sigma_2^* \quad \cdots \quad \sigma_n^*\right]$$

in which $\overline{\mathbf{W}}$ and σ_i respectively are the mean vector and the i^{th} standard deviation in the old dataset, and the $m*$ is the total number of observations in the combined dataset, i.e. $m* = m + m'$. Based on the updated means and standard deviations, the covariance matrix of the combined dataset is written as:

$$\mathbf{\Sigma}_{m*}^* = (m-1)/(m*-1)\mathbf{\Sigma}_m^* + (m'-1)/(m*-1)\mathbf{\Sigma}_{m'}^* \tag{18}$$

where $\mathbf{\Sigma}_m^*$ and $\mathbf{\Sigma}_{m'}^*$ respectively are the covariances of old and new event datasets based on means and standard deviations of combining dataset. The eigenvectors ($\mathbf{P}^*$) can be obtained to span the new PC subspace.

$$\mathbf{\Sigma}_{m*}^* = \mathbf{P}^{*\mathrm{T}}\mathbf{\Lambda}^*\mathbf{P}^* \tag{19}$$

where $\mathbf{\Lambda}^*$ is the diagonal matrix of the eigenvalues.

After updating the PC subspace, the known event clusters need to be transferred onto the new subspace. The center of the i^{th} cluster can be obtained from the previous subspace through coordinate rotating ($\mathbf{C}_{k,k*}$) and shifting ($\mathbf{1}\Delta\overline{\mathbf{W}}\mathbf{S}^{*-1}\mathbf{P}^*$).

$$\boldsymbol{\mu}_i^* = \boldsymbol{\mu}_i\mathbf{C} + \mathbf{1}\Delta\overline{\mathbf{W}}\mathbf{S}^{*-1}\mathbf{P}^*, \ \mathbf{C} \equiv \mathbf{P}^{\mathrm{T}}\mathbf{S}\mathbf{S}^{*-1}\mathbf{P}^* \tag{20}$$

The corresponding covariance is written by:

$$\mathbf{\Sigma}_i^* = \mathbf{C}^{\mathrm{T}}\mathbf{\Sigma}_i\mathbf{C} + \tilde{\mathbf{C}}^{\mathrm{T}}\tilde{\mathbf{\Sigma}}_i\tilde{\mathbf{C}} \tag{21}$$

in which $\tilde{\mathbf{\Sigma}}_i \equiv \tilde{\mathbf{T}}_i^{\mathrm{T}}\tilde{\mathbf{T}}_i/(m_i - 1)$ is the covariance of the $(k+1)^{\text{th}}$ to n^{th} term score vectors for the i^{th} cluster, and its coordinate rotating term is $\tilde{\mathbf{C}} \equiv \tilde{\mathbf{P}}^{\mathrm{T}}\mathbf{S}\mathbf{S}^{*-1}\mathbf{P}^*$.

4. ILLUSTRATIVE EXAMPLE

The quadruple-tank process was developed by Johansson (2000) as a multivariable laboratory process with an adjustable zero. The process consists of four interconnected water tanks, two pumps, and associated valves. The schematic diagram is shown in Fig. 1. A nonlinear model is derived based on mass balances and Bernoulli's law as follows:

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{f_1}{A_1}, \quad f_1 = \gamma_1 k_1 v_1 \tag{22a}$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{f_2}{A_2}, \quad f_2 = \gamma_2 k_2 v_2 \tag{22b}$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{f_3}{A_3}, \quad f_3 = (1-\gamma_2) k_2 v_2 \tag{22c}$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{f_4}{A_4}, \quad f_4 = (1-\gamma_1) k_1 v_1 \tag{22d}$$

where A_i is the cross section of the Tank i, a_i is the cross section of the outlet and h_i is the water level. The voltage applied to Pump i is v_i and the corresponding flow is k_iv_i. The parameters $\gamma_1, \gamma_2 \in (0,1)$ are determined from the valves set before the experiment. The flow to Tank 1 is $\gamma_1 k_1 v_1$ and the flow to Tank 4 is $(1-\gamma_1)k_1v_1$ and similarly for Tank 2 and Tank 3. The acceleration of gravity is denoted as g. The quadruple-tank process has been studied at two operating modes. The parameter values and the initial water levels are listed in Table 1. The normal operational data with two modes were generated by using (22a) - (22d) with respective parameters listed in Table 1, where the pump voltages and the tank levels were corrupted by Gaussian white noise with zero mean and standard deviation of 0.05. Tank levels h_1 - h_4 and flow rates f_1 - f_4 were observed every 10 seconds.

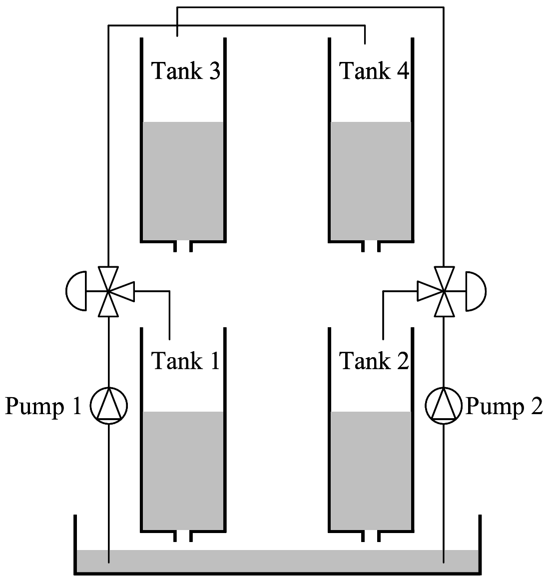

Fig. 1. Schematic diagram of the quadruple-tank process.

For each mode, 100 observations were collected. PCA was applied to the normal operational data and 2PCs were retained by using cross-validation. The PCA model captured about 93% of the total variance. The two scores have been plotted in Fig. 2, in which the dash line is 99% confidence limit of conventional T^2. The data were clustered into two groups, labeled with C1 and C2, representing two operating modes, Mode 1 and 2 respectively. The solid lines are 99% confidence limits of local T^2 for each mode. It is obvious that the local T^2 statistic is more suitable to monitor systematic variations of PCA than the conventional T^2 for a multimode process.

Table 1. Simulation parameters for the quadruple-tank process with two operating modes.

Parameter	Unit	Mode 1	Mode 2
A_1, A_3	cm^2	28	
A_2, A_4	cm^2	32	
a_1, a_3	cm^2	0.071	
a_2, a_4	cm^2	0.057	
g	cm/s^2	981	
h_1, h_2	cm	12.4, 12.7	12.6, 13.0
h_3, h_4	cm	1.8, 1.4	4.8, 4.9
v_1, v_2	V	3.00, 3.00	3.15, 3.15
k_1, k_2	cm^3/Vs	3.33, 3.35	3.14, 3.29
γ_1, γ_2		0.7, 0.6	0.43, 0.34

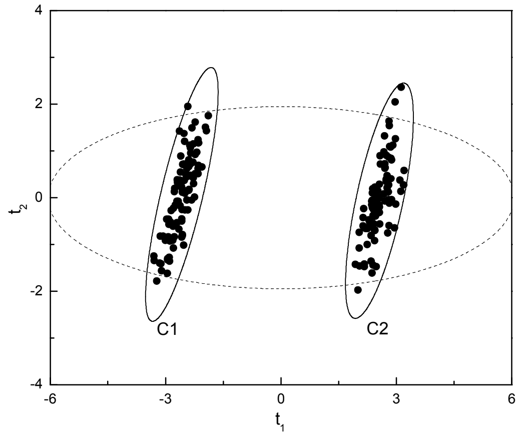

Fig. 2. The two scores of the normal operational data with 99% confidence limits of conventional T^2, dash line, and local T^2, solid lines.

An abnormal situation with multiple sensor faults was studied in this work. Before inducing the abnormal event, 100 normal operational data were generated by using (22a) - (22d) with parameters of Mode 1. Then, 100 abnormal event data were generated by modifying the case study from He *et al.* (2005). They assumed that there was a small hole at the bottom of Tank 1. The mass balance equation for Tank 1 was rewritten as:

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{f_1}{A_1} - \frac{a_{leak}}{A_1}\sqrt{2gh_1} \tag{23}$$

in which the cross section a_{leak} = 0.005 cm^2. In this case, a small hole at the bottom of Tank 2 was assumed at the same time for generating multiple sensor faults. The mass balance equation for Tank 2 was rewritten as:

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{f_2}{A_2} - \frac{a_{leak}}{A_2}\sqrt{2gh_2} \quad (24)$$

The other mass balance equations were same as the normal operating condition. In Fig. 3, it indicates both statistics Q and local T^2 are out of their control limits after the 100^{th} observation. The normalized RBC of Q has been plotted in Fig. 4, in which each contribution has been normalized with corresponding 99% confidence limit. It is obvious that the multiple sensor faults smeared out over all variables. Results show the RBC approach fails to guarantee to isolate faulty variables precisely in a case of multiple faulty variables.

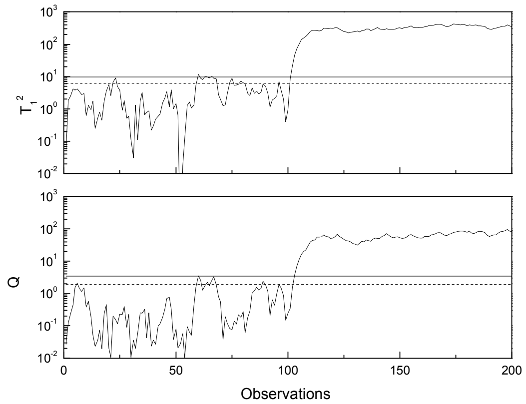

Fig. 3. Process monitoring with statistic Q and local T^2.

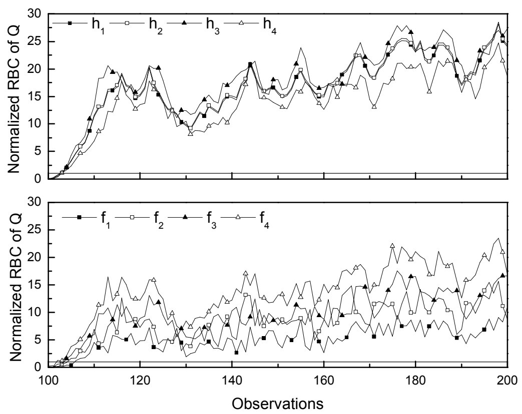

Fig. 4. Fault isolation with normalized RBC of Q.

The PC subspace was adapted by using RPCA with new event data. After adapting the PC subspace, the statistic Q of new event data were under the control limits, as Fig. 5 shows. It should be noted that the local T^2 were still out of the control limits after the 100^{th} sample. It demonstrates that the drawback of RPCA, which the monitoring model would be misled by blindly updating, can be eliminated by introducing local T^2 statistic. After transferring the cluster parameters onto the updated PC subspace, the proposed fault isolation charts have been plotted in Fig. 6, in which each difference from the cluster center has been normalized by using the corresponding 99% confidence limit. It shows that the levels of Tank 1 and 2, h_1 and h_2, were under their lower control limits after the 100^{th} sample, whereas the other variables were within their control limits. Results show the proposed method is capable of isolating the faulty variables precisely, as long as the process behavior can be captured by the scores on the PC subspace, i.e., the statistic Q staying within its control limits.

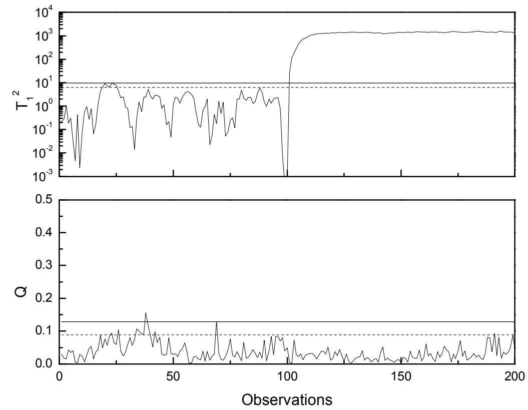

Fig. 5. Statistic Q and local T^2 on the adapted PC subspace.

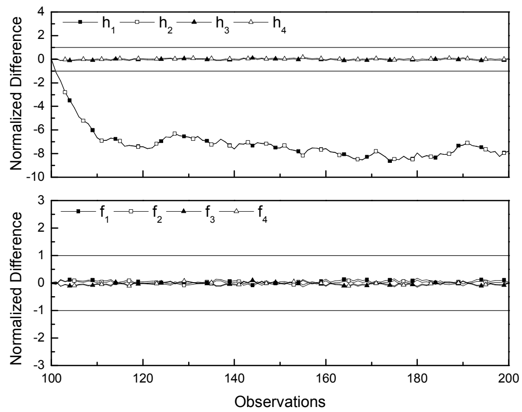

Fig. 6. Fault isolation charts on the adapted PC subspace.

5. CONCLUSIONS

In chemical processes, multiple normal operations are common. The conventional T^2 statistic is inadequate to monitor the systematic parts of PCA because of that the distribution of normal operational data is not a Gaussian distribution. The multiple operating regions are extracted from historical data by using Gaussian mixture model with Bayes rule in this paper. A local T^2 statistic with its control limits is provided to monitor data variations on the PC subspace for each operating region. It is much sounder than conventional T^2 from the perspective of statistical process monitoring (SPM). Fault isolation charts with their control limits are also provided to locate faulty variables in this paper. Since traditional contribution plots and RBC approach suffer fault smearing effect when encountering multiple sensor faults, they fail to guarantee correct diagnosis results. The results of quadruple-tank process simulation show the faulty variables are located precisely in a case of multiple sensor faults, as long as the PC subspace is capable of capturing process behavior during abnormalities. Besides, it has been demonstrated that the drawback of RPCA, which the monitoring model would be misled by blindly updating, is eliminated by introducing local T^2 statistic.

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APPENDIX. ILLUSTRATE THE RBC APPROACH ENCOUNTERING MULTIPLE SENSOR FAULTS

In this appendix, the RBC of Q is examined and the other types of RBC suffer the similar situations. According to the work of Alcala and Qin (2009), the RBC of Q for the i^{th} variable is defined as:

$$RBC_i^Q \equiv \frac{\left(\xi_i^{\mathrm{T}}\tilde{\mathbf{C}}\mathbf{x}\right)^2}{\xi_i^{\mathrm{T}}\tilde{\mathbf{C}}\xi_i} \tag{A.1}$$

where ξ_i is a column vector in which the i^{th} element is one and the others are zero, and $\tilde{\mathbf{C}} = \tilde{\mathbf{P}}\tilde{\mathbf{P}}^{\mathrm{T}}$. The measurements $\mathbf{x}$ contain two sensor faults by following the assumption of Alcala and Qin (2009):

$$\mathbf{x} = \xi_j f_1 + \xi_k f_2 \tag{A.2}$$

in which ξ_j and ξ_k are the directions of faulty variables and f_1 and f_2 are the respective fault magnitudes. The RBC of Q for the i^{th} and j^{th} variables can be obtained by substituting the fault in (A.2) into (A.1).

$$RBC_i^Q = \frac{\tilde{c}_{ij}^2}{\tilde{c}_{ii}} f_1^2 + \frac{\tilde{c}_{ik}^2}{\tilde{c}_{ii}} f_2^2 + 2\frac{\tilde{c}_{ij}\tilde{c}_{ik}}{\tilde{c}_{ii}} f_1 f_2 \tag{A.3}$$

$$RBC_j^Q = \frac{\tilde{c}_{jj}^2}{\tilde{c}_{jj}} f_1^2 + \frac{\tilde{c}_{jk}^2}{\tilde{c}_{jj}} f_2^2 + 2\frac{\tilde{c}_{jj}\tilde{c}_{jk}}{\tilde{c}_{jj}} f_1 f_2 \tag{A.4}$$

where $\tilde{c}_{ij} = \xi_i^{\mathrm{T}}\tilde{\mathbf{C}}\xi_j$. Correct diagnosis is guaranteed only when the RBC value of the non-faulty variable is less than or equal to the RBC value of the faulty one, i.e., $RBC_j^Q - RBC_i^Q \geq 0$ in this case. Rearrange (A.3) and (A.4):

$$\begin{aligned} RBC_j^Q - RBC_i^Q = {} & \left(\frac{\tilde{c}_{jj}^2}{\tilde{c}_{jj}} - \frac{\tilde{c}_{ij}^2}{\tilde{c}_{ii}}\right) f_1^2 + \left(\frac{\tilde{c}_{jk}^2}{\tilde{c}_{jj}} - \frac{\tilde{c}_{ik}^2}{\tilde{c}_{ii}}\right) f_2^2 \\ & + 2\left(\frac{\tilde{c}_{jj}\tilde{c}_{jk}}{\tilde{c}_{jj}} - \frac{\tilde{c}_{ij}\tilde{c}_{ik}}{\tilde{c}_{ii}}\right) f_1 f_2 \end{aligned} \tag{A.5}$$

Since the fault magnitudes of f_1 and f_2 are arbitrary values, above equation is larger than or equal to zero only when the coefficients of the right-hand terms are larger than or equal to zero. The coefficient of the first term is sustained by the work of Alcala and Qin (2009), but the other ones may not hold. For example, the coefficient of the second term represents the smearing effect of the k^{th} faulty variable over the i^{th} and the j^{th} variables. There is no particular reason that the smearing effect of the j^{th} variable is larger than or equal to the i^{th} variable from the k^{th} faulty variable. It leads that the RBC approach does not guarantee to isolate the correct faulty variables when encountering multiple sensor faults.