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Spline Wavelets for System Identification

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Abstract: The paper introduces spline wavelets as a modelling tool for system identification and proposes the technique of consistent output prediction using wavelets for estimating system parameters. It suggests that direct weighted summation of projections in approximation space could be used for deriving consistent output prediction in case model structure is built with spline wavelets. This can be viewed as identification using prefiltered input and output. The prefiltering is motivated to decorrelate samples such that local fit can be considered as a possible solution. An iterative algorithm, alternately projecting the solution in time and wavelet domain for penalized minimization of local error in wavelet coefficients could be designed for estimating system parameters. The algorithm is computationally efficient and exhibits excellent performance in cross validation. As a case study, the paper addresses the problem of modelling Liquid Zone Control System (LZCS) in a large Pressurized Heavy Water Reactor (PHWR). In this work, an identification scheme of a single input single output (SISO) linear time invariant (LTI) model of the LZCS system is studied. Excellent approximation is achieved by modelling with Biorthogonal spline wavelets used for deriving consistent output prediction of the LZCS process.

Keywords: Spline wavelets, identification, consistent, predictions, heavy water reactor.

1. INTRODUCTION

In recent times, identification of partially linear and non-linear systems using wavelet basis has attracted very active research interests (Chang and Qu, 2004; Juditsky et al., 1995; Sjöberg et al., 1995; Ljung, 1999; Doroslovacki and Fan, 1996; Zhao and Bentsman, 2000) because of the ability of wavelets to provide good approximations of the system function. A popular class of wavelets for this purpose have been the orthogonal family of wavelets using which the model parameters are estimated by minimizing errors in the wavelet domain in a least squares sense (Chang and Qu, 2004). This paper presents the method of consistent output prediction for identification of complex systems from the time-frequency evolution of input and output, using spline wavelet basis, which are not necessarily orthogonal. This opens up the solution space of identification to a new class of wavelet basis. It is important to note at the outset that consistency in system identification generally refers to an asymptotically unbiased estimate of system parameters (Ljung, 1999). In this work, the the usage of the term consistent output prediction refers to the signal which has the same representation in wavelet domain as the original output, inspired by the notion of consistent estimate in signal processing literature (Cvetković and Vetterli, 1995). The proposed method primarily checks the consistency in output signal in the context of prediction. In this sense, the method is termed as one of consistent output prediction.

System parameters are estimated in the approximation space to where both input and output are mapped. Spline wavelets span the approximation space and could be very effectively used for system identification because of their short support and excellent approximation properties. The spline functions, however, are not orthogonal except for those of degree 0. When the basis functions are orthogonal, minimization of sum of squared errors could be done in either time or wavelet domain. On the other hand, when the basis is not strictly orthogonal, the issue of stability of reconstruction in time needs to be addressed. Providentially, when splines are used as generalized basis, direct weighted addition of projections in approximation space could be used for consistent output predictions and it can be shown that the solution seeking local fit in approximation space does not necessarily require the assumption of strict orthogonality. Hence higher order spline wavelet basis is admissible for modelling. In general, the method of estimation of model parameters, with wavelet basis, could be cast as a penalized least squares problem. For a given strictly positive threshold, the solution is arrived at by soft thresholding of wavelet coefficients (Chang and Qu, 2004).

The efficacy of the technique has been demonstrated by modelling a complex process in a large Pressurized Heavy Water Reactor (PHWR) namely the Liquid Zone Control System (LZCS) employed for providing continuous fine control of reactor power level and power distribution in the core (Reddy et al., 2007). Response of the system with the associated nonlinearities could be very appropriately and effectively captured by biorthogonal spline wavelet basis. An identified linear time invariant (LTI) model with wavelet basis is cross validated using a new set of input-output data. An excellent match is observed between the model output and the actual output, using a small number of basis.

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The rest of the paper is organized as follows. Section 2.1 reviews the theory of spline wavelets basis and presents the essential fundamentals of system identification with wavelet basis functions. The technique of consistent output prediction is presented in Section 3. A simple modelling example and the application of the proposed technique to the LZCS using *biorthogonal* spline wavelets is presented in Section 4. The accuracy of the model so obtained is verified by comparing its output with the actual output signals of the LZCS from experiments. Section 5 concludes the paper indicating main contributions.

2. PRELIMINARIES

2.1 Time-varying model and wavelet representation

A regression model in terms of input $u(t)$ and output $y(t)$ can be given by

$$y(t) = h(t, \tau) \star u(t) + e(t) \tag{1}$$

where $h(t, \tau)$ is an unknown function in Hilbert space L_2 (space of all functions that are square-integrable in Lebesgue's sense) and $e(t)$ is random noise assumed to be *i.i.d.* distributed. Define the shift invariant sub-space V of Hilbert space L_2 for $h(t, \tau)$

$$V(\psi) = \left\{ h(t, \tau) = \sum_i \alpha_i(t)\varphi_i(\tau)\ ;\ \alpha_i \in l_2 \right\} \tag{2}$$

such that $h(t, \tau)$ denotes the system response in time-domain. Shift invariant basis functions φ_i and time varying coefficients $\alpha_i(t)$, together constitute the discrete-continuous parametric model of the time varying system. For φ_i to qualify as a basis of V, it is necessary that three conditions are satisfied i.e. the sequence of coefficients must be square summable, the family of basis functions should form a Riesz basis of V and the basis functions satisfy the partition of unity condition (Unser, 2000).

Associate the discrete wavelet transform (DWT) with an operator W. Applying W to the noisy observation we obtain the DWT of $y(t)$ as

$$Wy(t) = w(t) \tag{3}$$

The Riesz basis condition ensures a stable reconstruction by the inverse DWT operator because the energy in discrete and continuous domain satisfies the following condition.

$$\left\| \sum_i \alpha_i(t)(\varphi_i \star u)(t) \right\|^2 \leq C\, \|w(t)\|^2 \quad 0 < C < \infty \tag{4}$$

By virtue of (4), a solution of the parametric identification problem could be obtained in wavelet domain as well by minimizing the total energy (in least squares sense) as discussed in section 3. Here, α_is are the parameters which are estimated satisfying the least squares criterion.

2.2 Spline Wavelets as Basis

A system can be described in terms of a linear time varying response function expressed as weighted sum of a finite number of integer indexed basis functions with compact support. Let $\theta_i, i = 1, 2, \ldots P$ and $\gamma_i, i = 1, 2, \ldots Q$ denote respectively the basis functions for inputs and outputs. The estimated one-step-ahead output, $\hat{y}_{k+1}$ of a dynamical system can be obtained by linear filtering of projections (which are given by the convolutions) of past input and past outputs onto θ_i and γ_i respectively. It may be noted here, that subscripts on signals denote the sample number and the same on the basis functions denote the index of the basis. An estimate of the one-step-ahead measured output $y_{k+1}$of the system having input u can be written in general as a linear combination of u and y and the model for approximation of the measurement $y_{k+1}$is expressed as

$$\hat{y}_{k+1} = \sum_i a_{ik}(\theta_i \star y)_k + \sum_i b_{ik}(\gamma_i \star u)_k \tag{5}$$

When θ_i and γ_i are sinc functions, the convolutions in (5) pick-up time samples of input and output and the model reduces to the classical Auto Regressive with eX-ogenous input (ARX) type model. In general, θ_i and γ_i could be considered as shift invariant generating functions. $(\theta_i \star y)(t)$ and $(\gamma_i \star u)(t)$ are convolution of output and input with respective ith basis and samples at t = k, $(\theta_i \star y)_k$ and $(\gamma_i \star u)_k$ could be called generalized samples of output and input respectively. In (5), the approximation of the measurement y_{k+1} is no longer based on input and output but on the basis of filtered version of input and output. In general input and output are vectors belonging to different spaces. Hence a mapping is necessary for linearly transforming a vector from input or output space to the approximation space where the parameters $a_{ik}s$ and $b_{ik}s$ are estimated.

The primary objective of prefiltering is to decorrelate samples such that direct addition of projections for local fit can be considered as a possible solution. Moreover, thresholding decorrelated generalized samples obtained by prefiltering makes the identified model insensitive to noise. Without any loss of generality it can be assumed that $I = P = Q$ and in vector-matrix notation, (5) can be restated as

$$\hat{y}_{k+1} = \begin{bmatrix} (\theta_1 \star y)_k \\ \vdots \\ (\theta_l \star y)_k \\ (\gamma_1 \star u)_k \\ \vdots \\ (\gamma_l \star u)_k \end{bmatrix}^T \begin{bmatrix} a_{1k} \\ \vdots \\ a_{lk} \\ b_{1k} \\ \vdots \\ b_{lk} \end{bmatrix} \quad (6)$$

For approximating the measurement y_{k+1}, projections onto the basis functions θ_i andγ_i could be weighted and directly added for every index i, if eventually both input and output are mapped into the space spanned by the same set of basis. For example, if $\theta_i = \beta^m \star \beta_i^r$ and $\gamma_i = \beta^n \star \beta_i^r$ the weighted projections on θ_i and γ_i can be directly added because finally both input and output are projected onto the same basis β_i^r. Direct addition of projections is a useful technique that could be used in consistent output prediction discussed later in the paper. Moreover, the formulation allows use of two different set of wavelet basis for output and input. A better match with the wavelet basis shall yield fewer coefficients of the modelled signal in the transform domain. The structures of θ_i and γ_i suggest use of spline wavelets as basis for system identification because higher order spline functions are formed by successive convolution of the spline function of order zero.

$$\beta^n(t) = \beta^{n-1} \star \beta^0(t) \quad (7)$$

where $\beta^0(t)$ is the box function spline of degree 0.

$$\beta^0(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & |t| \geq 1/2 \end{cases} \quad (8)$$

It can be seen that the structure of β^n is same as that of θ_i and γ_i. Scaling functions (generated from spline functions) which also satisfy the two-scale relation are admissible as a generating function.

$$\phi(\frac{t}{2}) = \sqrt{2} \sum_{k \in Z} f_k \phi(t-k) \quad (9)$$

where f_k is refinement filter. Again, to admit wavelet basis functions instead of a single space $V(\phi) = V_0$, a ladder of rescaled subspaces are considered. These subspaces are indexed by scale number j and are given by

$$V_j = \text{span}(\phi_{j,k})_{k \in Z} \text{ with } \phi_{j,k} = 2^{-\frac{j}{2}} \phi(\frac{t}{2^j} - k).$$

If ϕ satisfies (9)then these spaces are nested and form a multi-resolution analysis (MRA) of L_2. Defining difference spaces $W_j = V_{j-1} - V_j$, wavelet basis functions $\psi(t)$ given by

$$\psi(\frac{t}{2}) = \sqrt{2} \sum_{k \in Z} g_k \psi(t-k) \quad (10)$$

can be designed for the discrete-continuous models, such that they form the Riesz basis of difference spaces W_j, $i.e. W_j = span\,(\psi_{jk})_{k \in Z}$, and satisfy partition of unity condition. The underlying scaling function for designing spline biorthogonal wavelets is a box spline. Since $\psi(\frac{t}{2})$ is a linear combination of box splines $\psi(t-k)$, it is a compactly supported polynomial spline of same degree.

3. CONSISTENT OUTPUT PREDICTION

When orthogonal wavelets are used, the energy of the signal is preserved in both time and wavelet domain by virtue of Parseval's relation and hence error minimization in least squares sense in either domain would give the same solution. The proposed method of estimation of model parameters based on the idea of consistent output prediction (Mukhopadhyay and Tiwari, 2010), however, does not necessarily need the assumption of strict orthogonality.

Definition 1. A consistent prediction is such that the actual output signal and its prediction have the same signal representation in the wavelet domain.

An estimate of the predicted output in time residing in the reconstruction set of the representation is obtained by inverse wavelet transform W^{-1} of the weighted sum of wavelet coefficients of input and output. The method of consistent output estimate uses penalized local error minimization in wavelet domain. The technique forces error at every significant sample point in wavelet domain locally to go to zero.

Let us denote one-step-ahead prediction at the k^{th} time instant as $\hat{y}_{k+1}$ and shifted version of measurement $y(t+T)$ as $y_s(t)$, where T is the sampling time. Measurement y_{k+1} can be expressed in terms of projections of shifted version of the measurement $y_s(t)$ onto θ_i

$$y_{k+1} = (y_s)_k = \sum_i (\theta_i \star y_s)_k \quad (11)$$

Under the assumption of shift invariance of wavelet basis, minimum error solution for α_is in least squares sense is obtained by minimizing the error functional,

$$J = \sum_k (y_{k+1} - \hat{y}_{k+1})^2 = \sum_k e_k^2 \quad (12)$$

where,

$$e_k = \sum_i (\theta_i \star y_s)_k - \sum_i a_{ik} (\theta_i \star y)_k - \sum_i b_{ik} (\gamma_i \star u)_k.$$

$(\theta_i \star y_s)_k$ are wavelet coefficients of the shifted output and a_{ik}, b_{ik} are the parameters associated with wavelet coefficients of $(\theta_i \star y)$ and $(\gamma_i \star u)_k$ to be estimated. As direct summation is allowed in the space spanned by θ_i, we can write

$$e_k = \sum_i [(\theta_i \star y_s)_k - a_{ik} (\theta_i \star y)_k - b_{ik} (\gamma_i \star u)_k].$$

The parameters are estimated as usual by setting the partial derivatives to zero. For instance,

$$\frac{\partial J}{\partial a_{ik}} = 2 \sum_k \sum_i e_k (\theta_i \star y)_k = 0 \quad (13)$$

It can be seen from (13) that for a linear system, a solution is obtained by setting the coefficients of wavelet expansion of the shifted measurement at every time instant k, equal to the weighted sum of coefficients of wavelet expansion of output $(\theta_i \star y)_k$ and input $(\gamma_i \star u)_k$. Soft thresholding in wavelet domain is used to reduce noise and is a solution to the penalized least squares problem. Let λ_u and λ_y be two strictly positive values. In penalized minimization, only those wavelet coefficients of input and output are used which have modulus values more than λ_u and λ_y respectively. Let us define those as significant wavelet coefficients. Proposed method sets the local error i.e. the error at the location of each significant wavelet coefficient of the shifted output to zero.

$$(\theta_i \star y_s) - a_{ik}(\theta_i \star y)_k - b_{ik}(\gamma_i \star u)_k = 0$$
$$\forall i,k \ \in \left\{ I_u : |(\gamma_i \star u)_k| \geq \lambda_u \bigcap \ I_y : |(\theta_i \star y)_k| \geq \lambda_y \right\}$$
$$a_{ik} = b_{ik} = 0 \ \forall i,k \notin I_u \text{ and } \forall i,k \notin I_y \quad (14)$$

The solution in (14), although suboptimal, precisely states the solution of consistent output estimate. It may be noted that in the above derivation, no assumption of strict orthogonality has been made and hence biorthogonal spline basis of order higher than 0 are admissible. It is however, necessary to ensure stability of reconstruction which readily follows from the assumption of Riesz basis (refer (4)).

The correlation functions in wavelet domain are known to decay faster than the correlation functions of the original signal in time Tewfik and Kim (1992). As the generalized samples of the transformed system in wavelet domain are likely to have less memory compared to the time samples of the original system, the solution of consistent output prediction using local error minimization will work better with generalized samples. The memory however is not lost. In fact it remains embedded in the properly designed wavelet basis. Moreover, a residual dependency structure still remains between magnitudes of wavelet coefficients both across the scale and at neighbouring temporal locations. In this context a completely memory-less system can be defined (Mukhopadhyay and Tiwari, 2010), which is recalled below

Definition 2. A system for which output estimate at any instant is completely decided by the weighted sum of the actual input and output at that instant is called a completely memory-less system.

It may be noted that the output estimate could be a prediction . Extending the arguments, the following conjecture may be made.

Conjecture 1. Any dynamical system can be transformed into a completely memory-less system by proper design of wavelet basis to represent it.

The conjecture above leads us to a proposition stated in Mukhopadhyay and Tiwari (2010)

Proposition 1. The consistent output prediction, obtained by penalized local error minimization in wavelet domain, approaches the optimum solution as the transformed system given by wavelet based representation tends to become completely memory-less.

Proof of proposition 1 can be constructed by arguing that the optimum solution for a completely memory less system shall have no dependence on the measurements at any time instant other than the ones at present time instant. If conjecture 1 is true *i.e.*, if it is possible to decorrelate wavelet coefficients of the signal to the extent that the estimated prediction at any time instant is solely dependent on the input and output of a single (present) time instant, consistent prediction by local error minimization in wavelet domain shall give the optimum solution.

In case it is known apriori that the process is LTI, it can be assumed that in the identified time varying model, variation in the system parameters are only due to residual noise in the de-noised output (obtained by employing soft thresholding). Now for an LTI model, substituting $a_{ik} = k_1\alpha_{ik}$ and $b_{ik} = k_2\alpha_{ik}$ in (6)and using direct addition of projections as suggested earlier, prediction can be written as

$$\hat{y}_{k+1} = \begin{bmatrix} k_1(\theta_1 \star u)_k + k_2(\gamma_1 \star y)_k \\ \vdots \\ k_1(\theta_I \star u)_k + k_2(\gamma_I \star y)_k \end{bmatrix}^T \begin{bmatrix} \alpha_{1k} \\ \vdots \\ \alpha_{Ik} \end{bmatrix} \quad (15)$$

It may be noted that for an LTI model, k_1, k_2 and α_{ik} are assumed to remain constant over time. In the light of proposition 1, the following theorem can be proved (Mukhopadhyay and Tiwari, 2010) from the solution of consistent estimate given by (14).

Theorem 1. Assuming that the noise in the estimate is stationary, iid $N(0, \sigma^2)$ distributed, a_{ik} and b_{ik} are given by $a_{ik} = k_1\alpha_{ik}$, $b_{ik} = k_2\alpha_{ik}$ where k_1 and k_2 are two real valued constants independent of time, then the first order estimate of the model parameters based on the consistent output estimate using penalized local error minimization in wavelet domain is given by

$$\hat{\alpha}_i = \frac{1}{K}\sum_{k=1}^{K}\left[\frac{(\theta_i \star y_s)_k}{k_1(\theta_i \star y)_k + k_2(\gamma_i \star u)_k}\right] \quad (16)$$

The structure of (16) however, suggests that an iterative scheme can be formulated to find the solution. The algorithm seeks the solution to have minimum local error in wavelet coefficients by alternately projecting the solution in time and wavelet domain (Mukhopadhyay and Tiwari, 2010). At every iteration, the solution is estimated once using the projections of the output followed by using those from the input. The alternate projection algorithm gives better results compared to that from exact implementation of (16) as restrictive constraints $a_{ik} = k_1\alpha_{ik}$ and $b_{ik} = k_2\alpha_{ik}$ are avoided in the implementation. In step 1 of each iteration, intermediate values of parameters or weights are computed using contribution from significant wavelet coefficients of input only. In step 2 the solution is projected in time domain and again projected back onto wavelet basis. In step 3 final values of parameters for the iteration is computed using contribution from significant wavelet coefficients of the output. The algorithm continues till the mean squared errors between two consecutive iterations is less than a predefined error threshold. Initial values of parameters are all set to zero.

4. DEMONSTRATION OF RESULTS

The model of an LTI system in the wavelet domain can be written in the form

$$h(\tau) = \sum_k c_J\phi_{Jk}(\tau) + \sum_k \sum_{0\leq j\leq J} d_j\psi_{jk}(\tau) \quad (17)$$

The structure of the model is clearly decided by the choice of the wavelet. To draw a correspondence with the traditional method of identification, a simple system can be rigged up by connecting a zero order hold (the box function, spline of degree 0), parallel to its dilations. The right choice for modelling such system would be with Haar Wavelet. When the method of consistent prediction is applied, the system parameters $\{c_J, d_0, \cdots, d_J\}$ are found to correspond to the number of zero order holds used for simulation. Although this result is interesting in the sense that these parameters correlate to assumed structure, it is not essential that parameters of the wavelet based model correspond to the poles and zeros of a traditional model. It is sufficient to cross validate a model with different inputs for testing the consistency in the output.

The technique of parameter estimation is demonstrated by modeling the LZCS in a large PHWR. Control of the reactor power level and the core power distribution is achieved by LZCS through variation of light water levels in the Zone Control Compartments (ZCCs). In the experiments the water level in each ZCC was regulated by its level controller. Figures 1(a) and 1(b) depict two sets of input output data collected from the LZCS test set-up at 50 ms uniform interval. Input signal is shown as the equivalent desired position of the control valve (CV) in terms of percentage opening (%OPN). The output signal is the level of water expressed as percentage of full scale (%FS). Full scale level means that the height of the water column is equal to the full height of the ZCC.

Identification of LZCS with classical models is attempted first. A simple first order model required for design of reactor regulating system can be developed from first principles considering the ZCC as a tank in which the water level variation is caused due to variations in inflow that occurs when position of control valve changes due to variations in input. Although the first order model is adequate for the initial design of control system, simulation needs rigorous models of LZCS. A model yielding better results than the first order model, would require knowledge of valve design data including the characteristics of its different accessories. In view of such difficulties, developing the model for ZCC water level dynamics employing a suitable method of identification from measurement of input and output is preferred.

Initially, identification with *sinc* basis (the classical) ARX model and its variants were attempted. ARX type low order models showed unacceptable prediction errors between the output of the model and actual output in both the experiments. Simulated Box-Jenkins (BJ) model showed best result where order of all four filters was found to be nine. The mismatch does not reduce even by increasing the filter orders or fine tuning other parameters. Such a high order model, in any case is not fit for the purpose of control system analysis. It was also observed from the pole-zero plot, that the identified system is almost always unstable, having poles outside the boundary of the unit circle. This is not surprising since open-loop LZCS system is essentially an integrator with nonlinearities due to control valve and flow characteristic (Mukhopadhyay and Tiwari, 2010). The instability to the step input is due to the response of the integrator and associated nonlinearity which could not be captured appropriately by sinc basis used in the ARX type models. In conclusion, as experienced by authors, the modeling exercise with *sinc* basis ARX model and its variants was not satisfactory. Hence, modeling based on wavelet basis was explored. Wavelet based models are expected to perform better due to their excellent local approximation property.

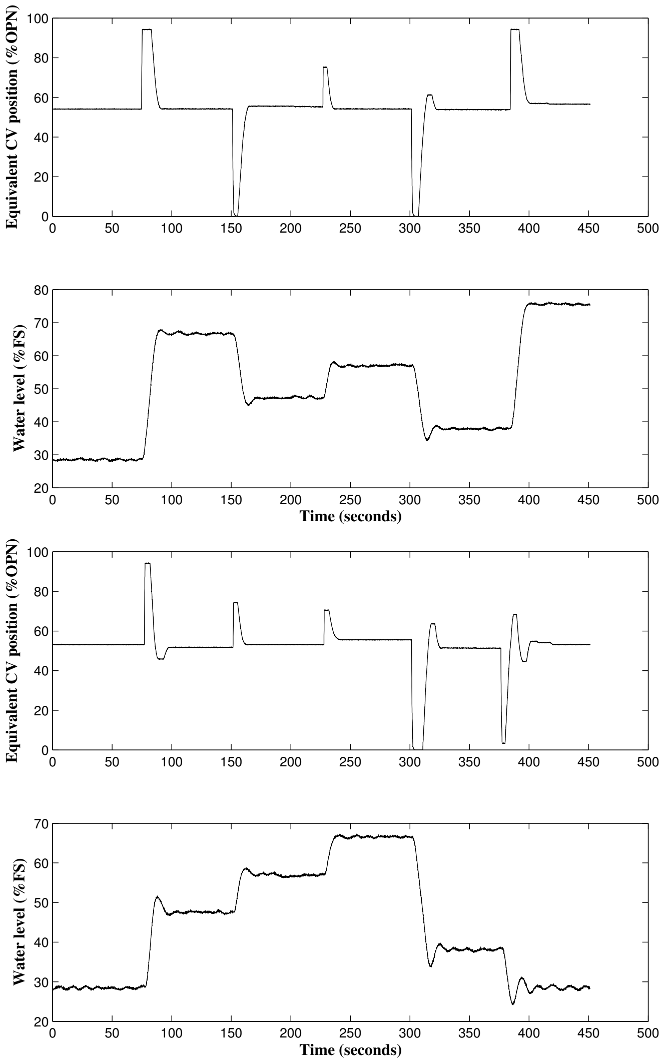

Fig. 1. (a) Input-output profile for training (b) Input-output profile for validation

For modelling with wavelet basis as given in (17), the time response of the LZCS is implemented using spline basis. Two biorthogonal spline wavelets of different order are used, one for projecting the input (Biorthogonal 1.5) and the other for projecting the output (Biorthogonal 2.4). The wavelet used for projecting step input is of lower order compared to the wavelet used for projecting the output reflecting dynamic modes of the system. Data of Figure 1(a) is used for identification of the model and data from the second experiment is used for validation of the model. The proposed iterative algorithm estimates the time invariant parameters at each scale. The reconstructed water level output signal after 7 iterations and actual water level output signal are compared in Figure 2. An excellent match is observed between the consistent prediction and the actual output. The identified LTI model based on the input output data given in Figure 1(a), thus obtained, is

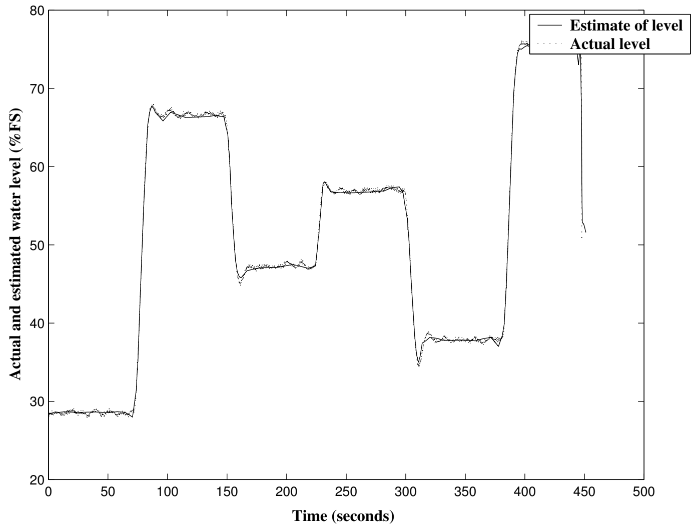

Fig. 2. Actual vs. predicted levels on training data set

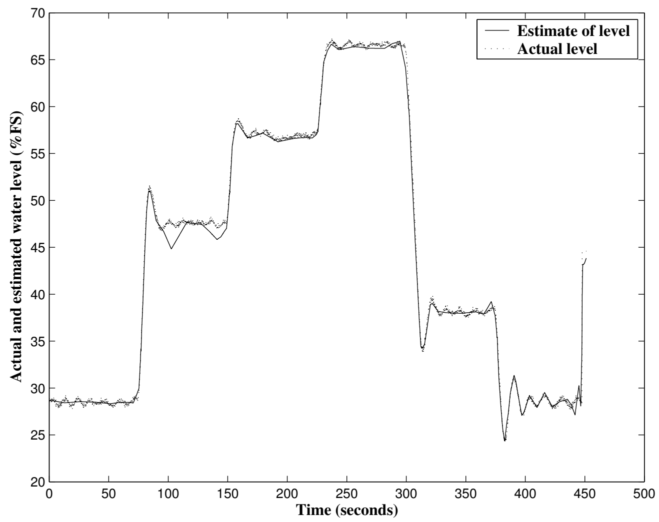

Fig. 3. Actual vs. predicted levels on validation data set

now tested to check if actual output can be predicted, also for the input output data shown in Figure 1 (b). The output in this case, is again measured by exciting the control valve with a different sequence of steps. The cross validation result is shown in Figure 3. Note that an excellent match is observed in both the transient and steady state responses, between the model output and the actual output level of the ZCC. The result conclusively proves the validity of proposed method of parameter estimation based on consistent output prediction.

5. CONCLUSIONS

The paper introduced spline wavelets for consistent output prediction in wavelet domain as an algorithmic solution to the classical least squares minimization problem. Penalized minimization of local errors in wavelet domain is used to obtain estimate of system parameters. The algorithm is computationally efficient and exhibits excellent performance in cross validation. With the use of spline wavelets, direct weighted summation of projections are permitted and the assumption of strict orthogonality is not required. The proposed algorithm has been demonstrated on a case study involving identification of the LZCS in a large PHWR. Results of the iterative alternate projection algorithm for estimating process parameters show excellent match with the experimental data in cross validation.

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