Proceedings of the 9th International Symposium on
Dynamics and Control of Process Systems (DYCOPS 2010),
Leuven, Belgium, July 5-7, 2010
Mayuresh Kothare, Moses Tade, Alain Vande Wouwer, Ilse Smets (Eds.)

Handling Communication Disruptions in Distributed Model Predictive Control

Mohsen Heidarinejad [*] **Jinfeng Liu** [**]
David Muñoz de la Peña [***] **Panagiotis D. Christofides** [*,**,1]

[*] *Department of Electrical Engineering, University of California, Los Angeles, CA, 90095-1592, USA.*
[**] *Department of Chemical and Biomolecular Engineering, University of California, Los Angeles, CA, 90095-1592, USA.*
[***] *Departamento de Ingeniería de Sistemas y Automática Universidad de Sevilla, Sevilla 41092, Spain.*

Abstract: In this work, we deal with distributed model predictive control (DMPC) of nonlinear systems with communication disruptions between the distributed controllers. Specifically, we focus on the design of DMPC systems that take into account communication channel noise and data losses between the distributed controllers explicitly. In contrast to most of the existing DMPC methods which assume flawless communication, we employ a specific channel model to consider a number of realistic data transmission scenarios. In order to ensure the stability of the closed-loop system under communication disruptions, each model predictive controller utilizes a stability constraint which is based on a suitable Lyapunov-based controller. The theoretical results are demonstrated through a nonlinear chemical process example.

Keywords: Distributed model predictive control; Nonlinear systems; Networked control.

1. INTRODUCTION

With rapid growth in the area of networking and networked control systems, augmentation of local control systems with additional networked sensors and actuators becomes a subject of increasing importance. These augmentations which are known as networked control systems (NCS), can significantly improve the efficiency, flexibility, robustness and fault tolerance of an industrial control system while reducing the installation, reconfiguration and maintenance costs at the cost of coordination and design/redesign of different control systems in the new architecture (Christofides et al. (2007)). Model predictive control (MPC) is a natural framework to deal with the design and coordination of distributed control systems because of its ability to handle input and state constraints. MPC is based on incorporating a model to predict the future evolution of the plant at each sampling time according to the current state along a given prediction horizon. These predictions are used to obtain an optimal input trajectory which minimizes a given performance index. To reduce the computational burden of the optimization problem, MPC optimizes over the family of piecewise constant trajectories with fixed sampling time and finite prediction horizon. Once the optimization problem is solved, only the first manipulated input value is implemented, discarding the rest of the trajectory and repeating the optimization in the next sampling step (Rawlings (2000)). In a centralized MPC design, all the manipulated inputs of a given control system are coupled in a single optimization problem to obtain the optimal input trajectory. In the case of large number of state variables and manipulated inputs for a given control system, the computational complexity of the centralized MPC may increase significantly and consequently degrade closed-loop system performance, especially in the case of employing a nonlinear model in MPC. A computationally-effective approach to overcome the above mentioned drawbacks of centralized MPC is to employ distributed MPC (DMPC) (e.g., Camponogara et al. (2002)) in which the optimal trajectory is obtained through solving a number of optimization problems with lower dimensionality compared to the centralized design.

In our previous work (Liu et al. (2009), Liu et al. (2010)), we proposed a DMPC architecture in which two distributed MPCs are designed via Lyapunov-based MPC (LMPC) to coordinate their control actions using one-directional communication. In this previous work, the communication between the distributed controllers was assumed to be flawless (perfect) which is reasonable in the applications where point-to-point-communication links are utilized. Recently, wireless networks have received significant attention (Tabbara et al. (2007)) and play an important role in multi-agent systems. In chemical process systems (Christofides et al. (2007)), there is an increasing trend toward developing industrial DMPC designs where individual MPCs operate through a shared wireless/wired communication network. However, the design of networked-based DMPC system has to deal with the dynamics introduced by the communication network, which may include communication disruptions such as communication channel noise, data losses, bandwidth limitations, time-varying delays, and data quantization (Muñoz de la Peña and Christofides (2008)).

[1] Corresponding author: Panagiotis D. Christofides. Tel.:+1 310 794 1014; fax: +1 310 206 4107; e-mail: pdc@seas.ucla.edu.

Motivated by the lack of available methodologies to deal with communication disruptions in DMPC architectures, in the present work, we deal with DMPC of nonlinear systems with communication disruptions between the distributed controllers. Specifically, we focus on the design of DMPC architectures that take explicitly into account communication channel noise and data losses between the distributed controllers. We employ a communication channel model to consider communication disruptions and an additional feasibility problem is formulated to determine the reliability of the information transmitted through the communication channel. Based on the feasibility of this problem, the distributed controller receiving the information decides whether to implement the received information or discard it. The distributed controllers are designed via LMPC based on a suitable Lyapunov-based controller. The proposed DMPC has an explicit characterization of the closed-loop stability region of the plant and guarantees that the closed-loop system is ultimately bounded in an invariant set which contains the origin. The theoretical results are illustrated using a chemical process example.

2. PRELIMINARIES

2.1 Problem formulation

We consider a nonlinear system described by the following state-space model

$$\dot{x}(t) = f(x(t)) + \sum_{i=1}^{2} g_i(x(t))u_i(t) + k(x(t))w(t) \qquad (1)$$

where $x(t) \in R^{n_x}$ denotes the vector of state variables, $u_i(t) \in R^{n_{u_i}}$ $(i = 1, 2)$ are two sets of control (manipulated) inputs and $w(t) \in R^{n_w}$ denotes the vector of disturbance variables. The two inputs are restricted to be in two nonempty convex sets $U_i \subseteq R^{n_{u_i}}$ where $U_i := \{u_i \in R^{n_{u_i}} : |u_i| \leq u_i^{\max}\}$ $(i = 1, 2)$ and the disturbance vector is bounded, i.e., $w(t) \in W$ where $W := \{w \in R^{n_w} : |w| \leq \theta_w, \theta_w > 0\}$. We assume that the vector functions $f(x), g_1(x), g_2(x)$ and $k(x)$ are locally Lipschitz and the origin is an equilibrium point of the nominal system (system of Eq. 1 with $w(t) = 0$ for all t) with $u_1 = 0$ and $u_2 = 0$ which implies that $f(0) = 0$. We further assume that the system state measurements are available and sampled at synchronous time instants $t_k = t_0 + k\Delta$ where t_0 is the initial time and Δ is the sampling time. In this work, we use the operator $|\cdot|$ to denote Euclidean norm of a vector, and a continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$. We use Ω_r to denote the set $\Omega_r := \{x \in R^{n_x} : V(x) \leq r\}$, and the operator '/' denotes set subtraction, i.e., $A/B := \{x \in R^{n_x} : x \in A, x \notin B\}$.

2.2 Model of the communication channel

For a given input $r \in R^{n_{u_2}}$ to the communication channel, the output $\tilde{r} \in R^{n_{u_2}}$ is characterized as

$$\tilde{r} = lr + n \qquad (2)$$

where l is a Bernoulli random variable with parameter α and $n \in R^{n_{u_2}}$ is a vector whose elements are white gaussian noise with zero mean and the same variance σ^2. The random variable l is used to model data losses in the communication channel. The white noise, n, is used to model channel noise, quantization error or any other error to the transmitted signal, and it is independent of the data losses in a probabilistic sense. If the receiver determines that a successful transmission is made, then $l = 1$, otherwise $l = 0$. Furthermore, in order to get deterministic stability results, we assume that, when a successful transmission is made, the noise, n, attached to the input signal, r, is bounded by θ (that is $|n| \leq \theta$). Both assumptions are meaningful from a practical standpoint; please see the example in Section 5.

2.3 Lyapunov-based controller

We assume that there exists a Lyapunov-based controller $u_1(t) = h(x(t))$ which satisfies the input constraint on u_1 for all x inside a certain stability region and renders the origin of the nominal closed-loop system asymptotically stable with $u_2(t) = 0$. Using converse Lyapunov theorems, this assumption implies that there exist class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ and a continuous differentiable Lyapunov function V for the nominal closed-loop system that satisfy the following inequalities

$$\begin{aligned} \alpha_1(|x|) \leq V(x) &\leq \alpha_2(|x|) \\ \frac{\partial V(x)}{\partial x}(f(x) + g_1(x)h(x)) &\leq -\alpha_3(|x|) \\ \left|\frac{\partial V(x)}{\partial x}\right| \leq \alpha_4(|x|),\ & h(x) \in U_1 \end{aligned} \qquad (3)$$

for all $x \in D \subseteq R^{n_x}$ where D is an open neighborhood of the origin. We denote the region Ω_ρ as the stability region of the closed-loop system under the control inputs $u_1 = h(x)$ and $u_2 = 0$. By continuity, the local Lipschitz property assumed for the vector functions $f(x), g_1(x), g_2(x)$ and $k(x)$ and the fact that the manipulated inputs u_1 and u_2 belong to the convex sets U_1 and U_2, it can be concluded that there exists a positive constant M such that

$$|f(x(t)) + \sum_{i=1}^{2} g_i(x(t))u_i(t) + k(x(t))w(t)| \leq M \qquad (4)$$

for all $x \in \Omega_\rho$, $u_1 \in U_1$, $u_2 \in U_2$ and $w \in W$. In addition, by the continuous differentiable property of the Lyapunov function V and the Lipschitz property assumed for the vector functions $f(x), g_1(x), g_2(x)$ and $k(x)$, there exist positive constants L_x, L_{u1}, L_{u2}, and L_w such that

$$\begin{aligned} \left|\frac{\partial V(x)}{\partial x}f(x) - \frac{\partial V(x')}{\partial x}f(x')\right| &\leq L_x|x - x'| \\ \left|\frac{\partial V(x)}{\partial x}g_i(x) - \frac{\partial V(x')}{\partial x}g_i(x')\right| &\leq L_{ui}|x - x'|, i = 1, 2 \\ \left|\frac{\partial V(x)}{\partial x}k(x)\right| &\leq L_w \end{aligned} \qquad (5)$$

for all $x, x' \in \Omega_\rho$, $u_1 \in U_1$, $u_2, u_2' \in U_2$ and $w \in W$. These constants will be employed in the proof of the stability of the closed-loop system (Theorem 1 in Section 4).

Remark 1. Note that while there are currently no general methods for constructing Lyapunov functions for general nonlinear systems, for broad classes of nonlinear systems arising in the context of chemical process control applications, quadratic Lyapunov functions are widely used and provide very good estimates of closed-loop stability regions; please see example in Section 5.

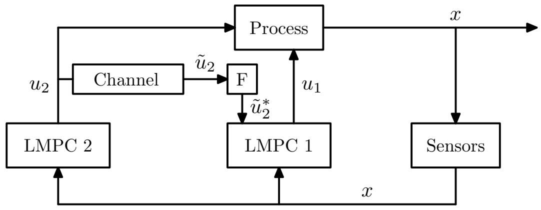

Fig. 1. Distributed LMPC control architecture (F means solving a feasibility problem).

3. DMPC WITH COMMUNICATION DISRUPTIONS

In our previous work (Liu et al. (2009)), a DMPC architecture with flawless communication between controllers was introduced. In practice, however, there is communication disruption including channel noise and data loss between distributed controllers. The objective of this work is to propose a DMPC framework which deals with communication disruptions while maintaining closed-loop stability and improving closed-loop performance. In the sequel, we design two LMPCs, namely LMPC 1 and LMPC 2, to calculate input trajectories of u_1 and u_2, respectively. A schematic diagram of the proposed DMPC design for systems subject to communication disruptions is depicted in Fig. 1.

We propose to use the following implementation strategy:

1. Both LMPC 1 and LMPC 2 receive the sensor measurements $x(t_k)$ at sampling time t_k.
2. LMPC 2 evaluates the optimal input trajectory of u_2 based on the $x(t_k)$ and sends the first step input value to its corresponding actuators and transmits the entire optimal input trajectory through a communication channel to LMPC 1.
3. LMPC 1 solves a feasibility problem to accept or reject the trajectory it received from LMPC 2.
4. LMPC 1 evaluates the future input trajectory of u_1 based on the $x(t_k)$ and the result of the feasibility problem.
5. LMPC 1 sends the first step input value of u_1 to its corresponding actuators.

Upon receiving the sensor measurement $x(t_k)$, LMPC 2 obtains its optimal input trajectory by solving the following optimization problem:

$$\min_{u_{d2}\in S(\Delta)} \int_0^{N\Delta} [\tilde{x}^T(\tau)Q_c\tilde{x}(\tau) + \textstyle\sum_{i=1}^2 u_{di}^T(\tau)R_{ci}u_{di}(\tau)]d\tau \quad (6a)$$

$$\dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau)) + \textstyle\sum_{i=1}^2 g_i(\tilde{x}(\tau))u_{di}(\tau) \quad (6b)$$

$$u_{d1}(\tau) = h(\tilde{x}(j\Delta)),\ \ \forall\, \tau \in [j\Delta, (j+1)\Delta) \quad (6c)$$

$$\tilde{x}(0) = x(t_k) \quad (6d)$$

$$u_{d2}(\tau) \in U_2 \quad (6e)$$

$$\frac{\partial V(x(t_k))}{\partial x} g_2(x(t_k))u_{d2}(0) \leq 0 \quad (6f)$$

where $S(\Delta)$ is the family of piece-wise constant functions with sampling period Δ, Q_c, R_{c1} and R_{c2} are positive definite weight matrices that define the cost, $j = 0, ..., N-1$, $x(t_k)$ is the state measurement obtained at t_k, $\tilde{x}$ is the predicted trajectory of the nominal system for the input trajectory computed by the LMPC 2, and N is the prediction horizon.

Let $u_{d2}^*(\tau|t_k)$ denote the optimal solution of the optimization problem of Eq. 6. LMPC 2 sends the first step value of $u_{d2}^*(\tau|t_k)$ to its actuators and transmits the whole optimal trajectory through the communication channel to LMPC 1. LMPC 1 receives a corrupted version of $u_{d2}^*(\tau|t_k)$ which can be formulated as:

$$\tilde{u}_{d2}(\tau|t_k) = l u_{d2}^*(\tau|t_k) + n \quad (7)$$

Note that a power level comparison can be employed to specify whether data loss has occurred at the receiver side (LMPC 1) of the communication channel.

Upon receiving $\tilde{u}_{d2}(\tau|t_k)$ and assuming that data loss has not happened, to make sure that LMPC 2 inherits the stability from the Lyapunov-based controller $h(x)$, LMPC 1 first solves the following feasibility problem

$$\text{find } z \in S(\Delta)$$

$$\tilde{u}_{d2}(\tau|t_k) - \theta \leq z(\tau) \leq \tilde{u}_{d2}(\tau|t_k) + \theta \quad (8a)$$

$$z(\tau) \in U_2 \quad (8b)$$

$$\frac{\partial V(x(t_k))}{\partial x} g_2(x(t_k))z(0) > 0 \quad (8c)$$

According to the bounded noise value and the received signal from the communication channel, LMPC 1 considers all the possibilities of noise effect on the optimal trajectory of LMPC 2 (i.e., constraint of Eq. 8a) and checks whether in these cases LMPC 2 still satisfies the contractive constraint (8c). Note that when the optimization problem of Eq. 8 is not feasible, it is guaranteed that the original signal $u_{d2}^*(\tau|t_k)$ after transmission through the channel still satisfies the stability constraint of Eq. 6f. We also note that there is no requirement that θ is sufficient small, however, larger values of θ increase the range of $z(\tau)$ and influence the feasibility of the problem of Eq. 8.

If the optimization problem of Eq. 8 is not feasible, then the trajectory information received by LMPC 1 (i.e., $\tilde{u}_{d2}(\tau|t_k)$) is used in the evaluation of LMPC 1; and if the optimization problem of Eq. 8 is feasible, then $\tilde{u}_{d2}(\tau|t_k)$ is discarded and a zero trajectory for u_2 will be used in the evaluation of LMPC 1. If we define the trajectory of u_2 that is used in the evaluation of LMPC 1 as $\tilde{u}_{d2}^*(\tau|t_k)$, then it is defined as follows:

$$\tilde{u}_{d2}^*(\tau|t_k) = \begin{cases} \tilde{u}_{d2}(\tau|t_k) & \text{if (8) is not feasible and there is no data loss} \\ \mathbf{0} & \text{if (8) is feasible or there exists data loss} \end{cases}$$

where $\mathbf{0} \in R^{n_{u2}}$. Note that when data loss in the communication channel occurs, a zero trajectory of u_2 is also used in the evaluation of LMPC 1. Note also that the above strategy on the use of the corrupted communication information is just one of many possible options to handle communication disruptions in the DMPC architecture.

Employing $\tilde{u}_{d2}^*$, LMPC 1 obtains its optimal trajectory according to the following optimization problem:

$$\min_{u_{d1}\in S(\Delta)} \int_0^{N\Delta} [\tilde{x}^T(\tau)Q_c\tilde{x}(\tau) + u_{d1}^T(\tau)R_{c1}u_{d1}(\tau) + \tilde{u}_{d2}^{*T}(\tau|t_k)R_{c2}\tilde{u}_{d2}^*(\tau|t_k)]d\tau \quad (9a)$$

$$\dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau)) + g_1(\tilde{x}(\tau))u_{d1}(\tau) + g_2(\tilde{x}(\tau))\tilde{u}_{d2}^*(\tau|t_k) \quad (9b)$$

$$u_{d1}(\tau) \in U_1 \quad (9c)$$

$$\tilde{x}(0) = x(t_k) \quad (9d)$$

$$\frac{\partial V(x(t_k))}{\partial x} g_1(x(t_k))u_{d1}(0) \leq \frac{\partial V(x(t_k))}{\partial x} g_1(x(t_k))h(x(t_k)) \quad (9e)$$

Once both LMPCs solve their optimization problems, the manipulated inputs of the proposed DMPC design are defined as follows:

$$\begin{aligned} u_1(t) &= u^*_{d1}(t - t_k|t_k),\ \forall t \in [t_k, t_{k+1}) \\ u_2(t) &= u^*_{d2}(t - t_k|t_k),\ \forall t \in [t_k, t_{k+1}). \end{aligned} \quad (10)$$

4. DMPC STABILITY

As it will be proved in Theorem 1 below, the proposed DMPC framework takes advantage of the contractive constraints of Eqs. 6f and 9e to compute the optimal trajectories u_1 and u_2 such that the Lyapunov function value $V(x(t_k))$ is a decreasing sequence with a lower bound and achieves the closed-loop stability of the system.

Theorem 1 *Consider the system of Eq. 1 in closed-loop under the DMPC design of Eqs. 9-10 based on a controller $u_1 = h(x)$ that satisfies the conditions of Eq. 3. Let $\epsilon_w > 0$, $\Delta > 0$ and $\rho > \rho_s > 0$ satisfy the following constraint:*

$$-\alpha_3(\alpha_2^{-1}(\rho_s))+(L_x+\sum_{i=1}^{2} L_{ui}u_i^{\max})M\Delta+L_w\theta_w \leq -\epsilon_w/\Delta. \quad (11)$$

If $x(t_0) \in \Omega_\rho$ and if $\rho^* \leq \rho$ where $\rho^* = \max\{V(x(t+\Delta)) : V(x(t)) \leq \rho_s\}$, then the state $x(t)$ of the closed-loop system is ultimately bounded in Ω_{ρ^*}.

Proof: The proof consists of two parts. We first prove that the optimization problems of Eqs. 6 and 9 are feasible for all states $x \in \Omega_\rho$. Subsequently, we prove that, under the DMPC design of Eqs. 9-10, the state of the system of Eq. 1 is ultimately bounded in a region that contains the origin.

Part 1: First, we consider the feasibility of LMPC 2 and then focus on the feasibility of LMPC 1. All input trajectories of $u_2(\tau)$ such that $u_2(\tau) = 0$, $\forall \tau \in [0, \Delta)$ satisfy all the constraints (including the input constraint of Eq. 6e and contractive constraint of Eq. 6f) of LMPC 2, thus the feasibility of LMPC 2 is obtained. The feasibility of LMPC 1 follows because all input trajectories $u_1(\tau)$ such that $u_1(\tau) = h(x(t_k))$, $\forall \tau \in [0, \Delta)$ are feasible solutions to the optimization problem of LMPC 1 since all such trajectories satisfy the input constraint of Eq. 9c; this is guaranteed by the closed-loop stability property of the Lyapunov-based controller h and the contractive constraint of Eq. 9e.

Part 2: Let $x(t_k) \in \Omega_\rho$. Considering the inequalities of Eq. 3, addition of inequalities of Eqs. 6f and 9e implies that

$$\begin{aligned} &\frac{\partial V(x(t_k))}{\partial x}(f(x(t_k)) + \sum_{i=1}^{2} g_i(x(t_k))u^*_{di}(0|t_k)) \\ &\leq \frac{\partial V(x(t_k))}{\partial x}(f(x(t_k)) + g_1(x(t_k))h(x(t_k))) \\ &\leq -\alpha_3(|x(t_k)|). \end{aligned} \quad (12)$$

The time derivative of the Lyapunov function along the actual state trajectory $x(t)$ of system of Eq. 1 in $t \in [t_k, t_{k+1})$ is given by:

$$\begin{aligned} \dot{V}(x(t)) = &\frac{\partial V(x)}{\partial x}(f(x(t)) + \sum_{i=1}^{2} g_i(x(t))u^*_{di}(0|t_k) \\ &+k(x(t))w(t)). \end{aligned}$$

Adding and subtracting $\frac{\partial V(x(t_k))}{\partial x}(f(x(t_k)) + g_1(x(t_k))$ $u^*_{d1}(0|t_k) + g_2(x(t_k))u^*_{d2}(0|t_k))$ and taking Eq. 12 into account, we obtain the following inequality:

$$\begin{aligned} \dot{V}(x(t)) \leq &-\alpha_3(|x(t_k)|) + \frac{\partial V(x)}{\partial x}(f(x(t)) \\ &+\sum_{i=1}^{2} g_i(x(t))u^*_{di}(0|t_k) + k(x(t))w(t)) \\ &-\frac{\partial V(x(t_k))}{\partial x}(f(x(t_k)) + \sum_{i=1}^{2} g_i(x(t_k))u^*_{di}(0|t_k)). \end{aligned}$$

From Eq. 3 and the above inequality, the following inequality is obtained for all $x(t_k) \in \Omega_\rho/\Omega_{\rho_s}$:

$$\begin{aligned} \dot{V}(x(t)) \leq &-\alpha_3(\alpha_2^{-1}(\rho_s)) + L_w|w(t)| \\ &+(L_x + \sum_{i=1}^{2} L_{ui}u^*_{di}(0|t_k))|x(t) - x(t_k)|. \end{aligned}$$

Taking into account Eq. 4 and the continuity of $x(t)$, the following bound can be written for all $t \in [t_k, t_{k+1})$, $|x(t) - x(t_k)| \leq M\Delta$. Using this expression, we obtain the following bound on the time derivative of the Lyapunov function for $t \in [t_k, t_{k+1})$, for all initial states $x(t_k) \in \Omega_\rho/\Omega_{\rho_s}$:

$$\dot{V}(x(t)) \leq -\alpha_3(\alpha_2^{-1}(\rho_s))+(L_x+\sum_{i=1}^{2} L_{ui}u_i^{max})M\Delta+L_w\theta_w.$$

If the condition of Eq. 11 is satisfied, then there exists $\epsilon_w > 0$ such that the following inequality holds for $x(t_k) \in \Omega_\rho/\Omega_{\rho_s}$:

$$\dot{V}(x(t)) \leq -\epsilon_w/\Delta,\ \forall t = [t_k, t_{k+1}).$$

Integrating this bound on $t \in [t_k, t_{k+1})$, we obtain that:

$$\begin{aligned} V(x(t_{k+1})) &\leq V(x(t_k)) - \epsilon_w \\ V(x(t)) &\leq V(x(t_k)),\ \forall t \in [t_k, t_{k+1}) \end{aligned} \quad (13)$$

for all $x(t_k) \in \Omega_\rho/\Omega_{\rho_s}$. Using Eq. 13 recursively, it is proved that, if $x(t_0) \in \Omega_\rho/\Omega_{\rho_s}$, the state converges to Ω_{ρ_s} in a finite number of sampling times without leaving the stability region. Once the state converges to $\Omega_{\rho_s} \subseteq \Omega_{\rho^*}$, it remains inside Ω_{ρ^*} for all times. This statement holds because of the definition of ρ^*. This proves that the closed-loop system under the distributed LMPC design is ultimately bounded in Ω_{ρ^*}.

Remark 2. The condition of Eq. 11 guarantees that if the state of the closed-loop system at a sampling time t_k is outside the level set $V(x(t_k)) = \rho_s$ but inside the level set $V(x(t_k)) = \rho$, the derivative of the Lyapunov function of the state of the closed-loop system is negative under the proposed design.

Remark 3. For nonlinear systems under continuous control implementation, a sufficient condition for invariance is that the Lyapunov function is decreasing on the boundary of a set. For systems with continuous-time dynamics and sample-and-hold control implementation, this condition is

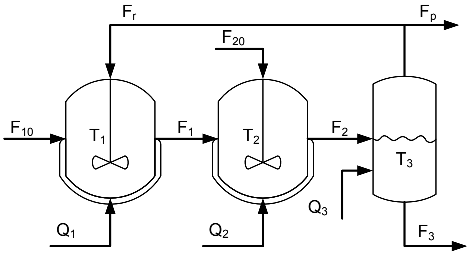

Fig. 2. Two CSTRs and a flash tank with recycle stream.

not sufficient because the derivative may become positive during the sampling period and the system may leave the set before a new sample is obtained. Based on Theorem 1, ρ^* is the maximum value that the Lyapunov function can achieve in a time period of length Δ when $x(t_k) \in \Omega_{\rho_s}$. Ω_{ρ^*} defines an invariant set for the state $x(t)$ under sample-and-hold implementation of the control action.

Remark 4. Note that the feasibility of the optimization problems of Eqs. 6 and 9 are guaranteed by the constraints of Eqs. 6f and 9e. The use of the corrupted input trajectory information of u_2 (i.e., $\tilde{u}_{d2}$) does not affect the feasibility of the optimization problems of Eqs. 6 and 9 as well as the stability of the closed-loop system; however, it does affect the closed-loop system performance. This is the reason for the introduction of the feasibility problem of Eq. 8 which is used to decide whether the corrupted information can be used to improve the closed-loop performance.

5. APPLICATION TO A CHEMICAL PLANT

The process considered in this study is a three vessel, reactor-separator system consisting of two continuously stirred tank reactors (CSTRs) and a flash tank separator as shown in Figure 2. A detailed process description, the process model and the value of the process parameters can be found in (Chilin et al. (in press)).

We assume that the state measurements which include the temperatures and species concentrations in the three vessels are available synchronously and continuously at time instants $\{t_{k\geq 0}\}$ with $t_k = t_0 + k\Delta$, $k = 0, 1, \ldots$ where t_0 is the initial time and Δ is the sampling time. For the simulations carried out in this section, we pick the initial time to be $t_0 = 0$ and the sampling time to be $\Delta = 0.01\ hr = 36\ sec$.

The first set of manipulated inputs is the heat injected to or removed from the three vessels, that is $u_1 = [Q_1 - Q_{1s}\ Q_2 - Q_{2s}\ Q_3 - Q_{3s}]^T$; the second set of manipulated inputs is the deviated inlet flow rate to vessel 2, that is $u_2 = \Delta F_{20} = F_{20} - F_{20s}$. The open-loop system has one unstable and two stable steady states. The control objective is to regulate the system to the unstable steady-state x_s corresponding to the operating point defined by Q_{1s}, Q_{2s}, Q_{3s} and F_{20s}. The steady-state values for u_1 and u_2 are zero. Taking this control objective into account, the process model belongs to the following class of nonlinear systems: $\dot{x}(t) = f(x(t)) + g_1(x(t))u_1(t) + g_2(x(t))u_2(t) + w(t)$ where $x^T = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8\ x_9\ x_{10}\ x_{11}\ x_{12}] = [T_1 - T_{1s}\ C_{A1} - C_{A1s}\ C_{B1} - C_{B1s}\ C_{C1} - C_{C1s}\ T_2 - T_{2s}\ C_{A2} - C_{A2s}\ C_{B2} - C_{B2s}\ C_{C2} - C_{C2s}\ T_3 - T_{3s}\ C_{A3} - C_{A3s}\ C_{B3} - C_{B3s}\ C_{C3} - C_{C3s}]$ is the state, $u_1^T = [u_{11}\ u_{12}\ u_{13}] = [Q_1 - Q_{1s}\ Q_2 - Q_{2s}\ Q_3 - Q_{3s}]$ and $u_2 = \Delta F_{20} = F_{20} - F_{20s}$ are the

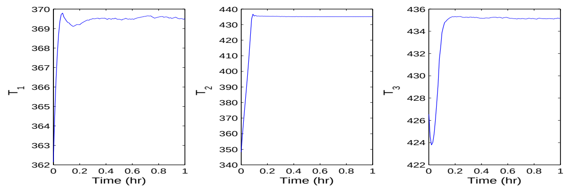

Fig. 3. Temperature trajectories of the process under the proposed DMPC design.

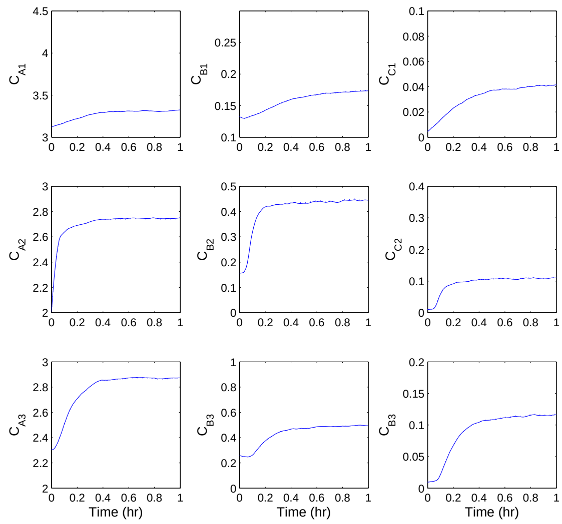

Fig. 4. Mass fraction trajectories of the process under the proposed DMPC design.

manipulated inputs which are deviation variables and are subject to the constraints $|u_{1i}| \leq 10^4\ KJ/hr$ $(i = 1, 2, 3)$ and $|u_2| \leq 5\ m^3/hr$, and w is a bounded noise.

We consider a quadratic Lyapunov function $V(x) = x^T P x$ with $P = diag([10\ 10^4\ 10^4\ 10^4\ 10\ 10^4\ 10^4\ 10^4\ 10\ 10^4\ 10^4\ 10^4])$ and design the controller $h(x)$ as three PI controllers with proportional gains $K_{p1} = K_{p2} = K_{p3} = 8000$ and integral time constants $\tau_{I1} = \tau_{I2} = \tau_{I3} = 10$ based on the measurements of T_1, T_2 and T_3, respectively. The values of the weights in P have been chosen in a way such that the Lyapunov-based controller $h(x)$ satisfies the input constraints, stabilizes the closed-loop system and provides good closed-loop performance. Note that, in the absence of process and measurement noise, this design of $h(x)$ manipulating $u_1 = [Q_1\ Q_2\ Q_3]$ can stabilize the closed-loop system asymptotically without the help of u_2. Based on $h(x)$ and $V(x)$, we design LMPC 1 to determine u_1 and LMPC 2 to determine u_2 following the forms given in Eqs. 6 and 9, respectively. In the design of the LMPC controllers, the weighting matrices are chosen to be $Q_c = diag([10\ 10^4\ 10^4\ 10^4\ 9\ 10^4\ 10^4\ 10^4\ 10\ 10^4\ 10^4\ 10^4])$, $R_1 = diag([(5\ 5\ 5) \cdot 10^{-4}])$ and $R_2 = 10^4$. The prediction horizon for the optimization problem is $N = 5$ with a time step of $\Delta = 0.01\ hr$. The initial condition which is utilized to carry out the simulations is $x(0)^T = [362.14\ 3.1191\ 0.13\ 0.01\ 348.21\ 2.01\ 0.16\ 0.01\ 462.55\ 2.31\ 0.26\ 0.01]$. We set the communication channel noise power (σ^2), the data loss probability α and the noise bound θ to 0.01, 0.1 and 0.25, respectively.

The state trajectory of the process under the proposed DMPC design from the initial state are shown in Figs. 3 and 4. These figures show that the proposed control design drive the temperatures and the mass fractions in the closed-loop system close to the desired steady-state and achieves closed-loop stability.

To emphasize the importance of solving the feasibility problem in LMPC 1 during obtaining its optimal input trajectory, we have carried out a set of simulations to compare the proposed design with our previous control scheme (in Liu et al. (2009)) in which LMPC 1 incorporates the received channel signal in its optimization problem without any pre-processing. In other words, in this case LMPC 1 ignores the fact that whether communication channel noise and data loss effects violate the feasibility constraints of LMPC 2 optimization problem. We have carried out a number of simulations to compare the proposed DMPC design with our previous DMPC design with the same parameters and initial condition from a performance index point of view. Table 1 shows the total cost computed for 10 different closed-loop simulations under the proposed DMPC design and our previous control scheme. To carry out this comparison, we have computed the total cost of each simulation with different operating conditions (different initial states and process disturbances) based on the index of the following form

$$J = \sum_{i=0}^{M} x(t_i)^T Q_c x(t_i) + u_1(t_i)^T R_{c1} u_1(t_i) + u_2(t_i)^T R_{c2} u_2(t_i)$$

where t_0 is the initial time of the simulations and $t_M = 1\ hr$ is the final time of the simulations. As we can see in Table 1, the proposed distributed LMPC design has a cost lower than the previous DMPC design in all 10 simulations. This illustrates that in this example, the proposed distributed LMPC design improves our previous design from a closed-loop performance point of view.

Finally, we have carried out a set of simulations to evaluate the performance of the proposed DMPC design over the one in (Liu et al. (2009)) from a closed-loop performance index point of view under different communication channel noise powers and data loss probabilities. Tables 2 and 3 show the total cost computed for 10 different data loss probabilities and noise powers compared to our previous DMPC design, respectively. As it can be seen from these tables, the proposed DMPC design is superior from a closed-loop performance point of view for different noise power and data loss probability values.

Remark 5. Note that the DMPC design in Liu et al. (2009) can still guarantee the closed-loop system stability in the presence of communication disruptions; however, the closed-loop performance may be degraded. In this work, we propose a practical approach to deal with communication disruptions to improve the closed-loop performance while maintaining the stability properties of the closed-loop system. In all simulations, the proposed DMPC design accounting for disruptions yields reduced performance costs compared to the previous DMPC design, even though this benefit cannot be proved to hold in general.

Table 1. Total performance cost ($*10^7$) along the closed-loop system trajectories.

sim.	Prop.	Prev.	sim.	Prop.	Prev.
1	5.486	5.488	6	2.549	2.559
2	2.497	2.519	7	1.691	1.697
3	1.771	1.785	8	6.688	6.695
4	1.203	1.215	9	6.632	6.633
5	3.163	3.181	10	2.498	2.515

Table 2. Total performance cost ($*10^7$) along the closed-loop system trajectories for different data loss probabilities and $\sigma^2 = 0.01$.

α	Prop.	Prev.	α	Prop.	Prev.
0.05	6.803	6.900	0.30	6.808	6.901
0.10	6.779	6.908	0.35	6.818	6.906
0.15	6.821	6.897	0.40	6.779	6.901
0.20	6.821	6.905	0.45	6.793	6.893
0.25	6.801	6.899	0.50	6.744	6.895

Table 3. Total performance cost ($*10^7$) along the closed-loop system trajectories for different channel noise power values and $\alpha = 0.1$.

σ^2	Prop.	Prev.	σ^2	Prop.	Prev.
0.005	6.787	6.907	0.030	6.802	6.899
0.010	6.762	6.894	0.035	6.809	6.894
0.015	6.820	6.895	0.040	6.769	6.939
0.020	6.744	6.898	0.045	6.835	6.909
0.025	6.841	6.893	0.050	6.756	6.892

REFERENCES

Camponogara, E., Jia, D., Krogh, B.H., and Talukdar, S. (2002). Distributed model predictive control. *IEEE Control System Magazine*, 22(1), 44–52.

Chilin, D., Liu, J., Muñoz de la Peña, D., and Christofides, P.D. (in press). Detection, isolation and handling of actuator faults in distributed model predictive control systems. *Journal of Process Control*. DOI:10.1016/j.jprocont.2010.05.001.

Christofides, P.D., Davis, J.F., El-Farra, N.H., Clark, D., and Gipson, K.H.J. (2007). Smart plant operations: Vision, progress and challenges. *AIChE Journal*, 53(11), 2734–2741.

Liu, J., Muñoz de la Peña, D., and Christofides, P.D. (2009). Distributed model predictive control of nonlinear process systems. *AIChE Journal*, 55, 1171–1184.

Liu, J., Muñoz de la Peña, D., and Christofides, P.D.(2010). Distributed model predictive control of nonlinear systems subject to asynchronous and delayed measurements. *Automatica*, 46, 52–61.

Muñoz de la Peña, D. and Christofides, P.D. (2008). Lyapunov-based model predictive control of nonlinear systems subject to data losses. *IEEE Transactions Automatic Control*, 53(9), 2076–2089.

Rawlings, J.B. (2000). Tutorial overview of model predictive control. *IEEE Control System Magazine*, 20, 38 – 52.

Tabbara, M., Nesic, D., and Teel, A. (2007). Stability of wireless and wireline networked control systems. *IEEE Transactions on Automatic Control*, 52(9), 1615–1630.