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Two-Stage Multivariable Antiwindup Design for Internal Model Control ⋆

Ambrose A. Adegbege* William P. Heath*

* *Control System Center, School of Electrical and Electronics Engineering, The University of Manchester, P.O.Box 88, Sackville Street, Manchester, M60 1QD, UK (e-mail: ambrose.a.adegbege@postgrad.manchester.ac.uk, william.heath@manchester.ac.uk)*

Abstract: Multivariable plants under input constraints such as actuator saturation are liable to performance deterioration due to problems of control windup and directional change in control action. In this paper, we propose a two-stage internal model control (IMC) antiwindup design which guarantees optimal closed loop performance both at transient and at steady state. The two-stage IMC is based on the solution of two quadratic programs (QP). The first QP addresses the transient behaviour of the system and ensures that the constrained closed-loop response is as close as possible to the unconstrained closed-loop response. The second QP guarantees optimal steady-state behaviour of the system. Simulated examples show that the two-stage IMC has superior performance when compared to other existing optimization-based antiwindup methods. We consider a scenario where the proposed two-stage IMC competes favourably with a long prediction horizon model predictive control (MPC).

Keywords: Antiwindup, Directionality, Internal model control, Predictive control, Input constraints, Quadratic programming.

1. INTRODUCTION

Control design for processes under input constraints such as actuator saturation nonlinearities has been widely studied either from antiwindup design approach (Campo and Morari, 1990; Zheng et al., 1994; Kendi and Doyle, 1997; Edwards and Postlethwaite, 1998; Horla, 2009; Tarbouriech and Turner, 2009) or a model predictive control (MPC) perspective (Muske and Rawlings, 1993; Rawlings and Chien, 1996; Rao and Rawlings, 1999; Maciejowski, 2002). For multivariable or multi-input multi-output (MIMO) systems, the presence of control input saturation introduces additional problems due to directional change in control action also known as process directionality (Soroush and Valluri, 1999) alongside the widely known controller windup phenomenon. These two problems, control windup and process directionality, can result in substantial closed-loop performance degradation if not separately accounted for during the controller design (Doyle et al., 1987).

In the design of analytical dynamic controllers such as internal model control (IMC), a common approach is to first design a linear controller neglecting the saturation and then a saturation compensation scheme is added to provide a graceful closed-loop performance degradation in the presence of saturation Campo and Morari (1990). This ad-hoc saturation compensation scheme is termed antiwindup. A specific example of antiwindup design is the modified internal model control structure which may be interpreted as solving instantaneously an optimization problem at each time step (Zheng et al., 1994).

Antiwindup designs must be augmented with dynamic compensators to account for process directionality in MIMO systems. One approach is to scale down all the control inputs in such a way the control input direction is maintained (Doyle et al., 1987; Campo and Morari, 1990). This approach is restricted to a class of control problems and may not necessarily be optimal. Other optimization based schemes have been suggested (Hanus and Kinnaert, 1989; Walgama and Sternby, 1993; Peng et al., 1998; Chen and Perng, 1998; Soroush and Valluri, 1999). While these schemes offer optimal dynamic behaviour, their performances deteriorate significantly in steady state especially when the constraints are active. Schemes that guarantee optimal steady state behaviour such as (Heath and Wills, 2004), may have poor transient characteristics. The focus of this paper is an IMC antiwindup scheme which optimizes the transient performance of the system and also guarantees steady-state optimal behaviour.

The paper is structured as follows. Section 2 describes the problem set-up and some notations. We introduce the concept of internal model control for antiwindup design in section 3. In section 4, we discuss directionality compensation schemes within the framework of internal model control structure. Section 5 contains the main contributions of the paper where we present the two-stage IMC anti-windup for not only dealing with the performance degradation associated with control windup and directionality but also for ensuring steady state performance in input constrained multivariable problem. In terms of nominal

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performance, the two-stage IMC compares favourably with a long prediction horizon MPC while its computational requirement is equivalent to that of a single-horizon MPC. The performance of the proposed approach is illustrated via simulation examples in section 6.

2. PROBLEM SETUP

We consider a class of stable and linear systems described by

$$y = Gu + d \tag{1}$$

where G is a rational, strictly proper transfer function matrix and $y, d \in \mathcal{L}^p$, $u \in \mathcal{L}^m$ (or $y, d \in l^p$, $u \in l^m$) are the Laplace transforms (or discrete equivalent, Z-transforms) of the output signal $y(t)$, the manipulated input signal $u(t)$ and the output disturbance $d(t)$ signal respectively. The input signal u is constrained such that

$$u_i^{min} \leq u_i(t) \leq u_i^{max} \quad i = 1, \ldots, m \tag{2}$$

This can be represented by following saturation function. If we define the function sat(.) as

$$\text{sat}(u_i(t)) = \begin{cases} u_i(t) & |u_i(t)| \leq 1 \\ \text{sgn}(u_i(t)) & |u_i(t)| > 1 \end{cases} \tag{3}$$

where $\text{sat}(u_i(t))$ represents the normalized saturation non-linearity associated with each of the manipulated input $u_i(t)$, then the constrained input becomes

$$\text{sat}(u(t)) = [\, \text{sat}(u_1(t)), \; \cdots \;, \; \text{sat}(u_m(t)) \,]^T \tag{4}$$

The system characteristic matrix $\mathcal{C}$ which describes the transient behaviour of the system (1) is defined for a square system as

$$\mathcal{C} = \lim_{s(orz) \to \infty} [\text{diag}\{s(orz)^{r_m}\}G] \tag{5}$$

where $r_i = \min(r_{i1}, r_{i2}, \ldots, r_{im})$ and $r_{i,j}$ is the relative order of output y_i with respect to manipulated input u_j.

3. THE INTERNAL MODEL CONTROL STRUCTURE EVOLUTION

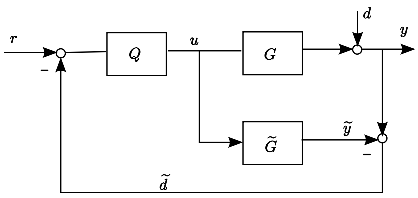

Fig. 1. The Standard IMC Structure

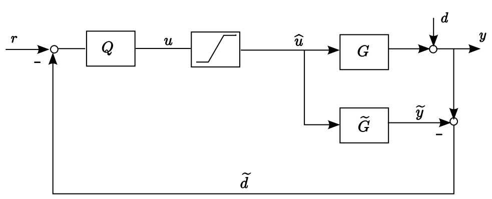

Fig. 2. The Conventional IMC Antiwindup Structure

The standard internal model control (IMC) structure introduced in Garcia and Morari (1982) is illustrated in figure 1 where $G, \widetilde{G}$ and Q denote the plant, the model of the plant and the IMC controller respectively. The design of Q for optimal performance and robustness is

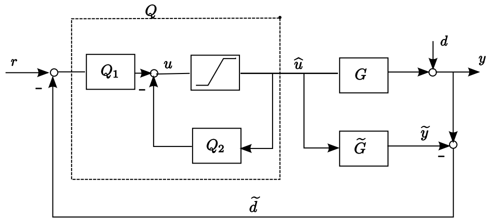

Fig. 3. The Modified IMC Antiwindup Structure

well discussed in the literature (Morari and Zafiriou, 1989; Zheng et al., 1994). With the assumption of perfect model i.e. $G = \widetilde{G}$, the stability of G and Q guarantees nominal stability of the unsaturated closed loop system (Morari and Zafiriou, 1989).

However, for saturating system with $\hat{u}(t) = \text{sat}(u(t))$, the standard IMC implementation can lead to instability. In this case, the plant and the model are driven by different inputs. The resultant model/plant mismatch is shown in the closed loop equation (6).

$$u = Q\,(r - d) + QG(u - \hat{u}) \tag{6}$$

A first step towards avoiding the state mismatch between the plant and the model is the conventional IMC anti-windup structure of figure 2 (Morari and Zafiriou, 1989; Zheng et al., 1994). Although closed loop nominal stability is guaranteed when there is no model mismatch, the nonlinear performance may be excessively sluggish. The closed loop equation (7) shows that the saturation effect on the plant output is not fed back to the controller. The controller only acts on the error between the reference signal r and the output disturbance d.

$$u = Q\,(r - d) \tag{7}$$

$$y = G\hat{u} + d \tag{8}$$

The modified IMC structure shown in figure 3 was proposed as an antiwindup scheme to deal with the pronounced performance deterioration associated with the standard IMC structure (Zheng et al., 1994). Assuming no plant-model mismatch, the closed loop equations are given by

$$u = Q_1\,(r - d) - Q_2\hat{u} \tag{9}$$

$$y = G\hat{u} + d \tag{10}$$

where $Q = (I + Q_2)^{-1}Q_1$. Here, the controller not only acts on the error between the reference signal and the output disturbance but it is also fed directly with information on the saturating control actions. When the system is away from saturation (i.e. $\hat{u} = u$), equations (9) and (10) reduces to the closed loop equations for the implementation in figure 1. For a given Q there are different ways of assigning Q_1 and Q_2. It is imperative that appropriate choices are made to achieve a good non-linear performance while ensuring stability. One factorization option is

$$Q_1 = \Lambda Q + (I - \Lambda)Q\,(\infty) \tag{11}$$

where $\Lambda = \lambda I$ is a diagonal weighting matrix and $\lambda \in [0, 1]$. The choice of $\lambda = 1$ results in the conventional IMC structure which chops off the control input resulting in performance deterioration (sluggish response) but nominal stability is guaranteed. On the other hand, the choice of $\lambda = 0$ corresponds to the factorization proposed in (Goodwin et al., 1993). The performance in this case is greatly improved, but nominal stability of the closed-loop system is not guaranteed. Trade off between performance and stability can therefore be achieved by appropriate choice of λ, provided Q is minimum phase (Zheng et al., 1994).

4. DIRECTIONALITY COMPENSATORS FOR MULTIVARIABLE INTERNAL MODEL CONTROL

Performance deterioration in multivariable plant with input saturation can be attributed to two major factors. These are the problems resulting from control windup and that of directional change in control. To ensure a graceful performance degradation in the presence of input saturation, a particular choice of Q_1 and possibly an additional nonlinear element in form of directionality compensator is often incorporated into the controller as shown in figure 4 (Zheng et al., 1994; Doyle et al., 1987; Peng et al., 1998; Soroush and Valluri, 1999; Campo and Morari, 1990; Chen and Perng, 1998; Heath and Wills, 2004; Heath et al., 2005; Kendi and Doyle, 1997). In this section, a brief review of some of the existing directionality compensator designs is presented within the IMC framework.

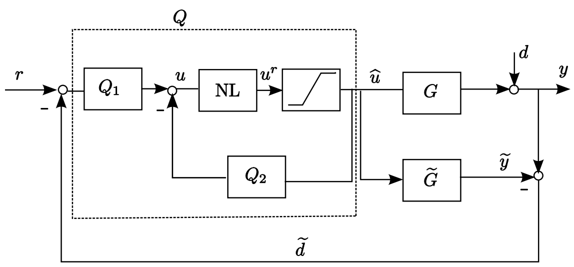

Fig. 4. The Modified IMC Structure with Directionality Compensation

The Modified IMC Antiwindup (MIA) structure of figure 3 can be considered as a special case of the scheme in figure 4 where the directionality compensator NL = I. However, in order to preserve the output direction, Zheng et al. (1994) recommend the choice

$$Q_1 = f_A G Q \tag{12}$$

where f_A is a non-causal filter that must be chosen such that $f_A G|_{s=\infty} = I$ or $f_A G|_{z=\infty} = I$ and Q_1 is of minimum phase. These conditions ensure that Q_2 is strictly proper which guarantees that there is no algebraic loop in the interconnection of figure 4.

It should be noted that the choice $f_A = G^{-1}$ is equivalent to choosing $\lambda = 1$ in (11) above.

In the Direction Preservation (DP) approach of Campo and Morari (1990), the constrained control action is obtained by scaling down the controller outputs so that the u and $\hat{u}$ have same direction in the event of saturation. The non-linearity block NL in figure 4 is defined as

$$u^r = \mathrm{NL}(u) = \begin{cases} u & \text{if } u \text{ is in linear region} \\ \min\left\{\dfrac{sat(u_i)}{u_i}\right\} u & \text{if } u \text{ enters saturation} \end{cases} \tag{13}$$

In this case, subsequent saturation will have no effect since its input u^r always remains in the linear region. The concept of directional preservation has been shown to be beneficial for some class of constrained multivariable control problems (Doyle et al., 1987; Campo and Morari, 1990; Kendi and Doyle, 1997).

A number of Optimization based Conditioning Techniques (OCT) have been proposed in the literature (Walgama and Sternby, 1993; Chen and Perng, 1998; Peng et al., 1998). All these are extensions of the conditioning techniques originally discussed in Hanus and Kinnaert (1989) which is based on the concept of realizable reference w^r. When a controller output is infeasible, a realizable control input u^r is obtained by solving an online optimization problem such that the realizable reference w^r is as close as possible to the actual process set-point r. Following the development in Peng et al. (1998), the NL is defined as

$$u^r = \arg\min_{u^r} \|D_k^{-1}u^r - D_k^{-1}u\|_{\mathcal{Q}}^2 \tag{14}$$

subject to the constraints

$$u_{min} \le u_i^r \le u_{max} \quad i = 1, \ldots, m$$

where D_k is the feedthrough matrix of controller Q i.e. $D_k = Q(\infty)$ and $\mathcal{Q}$ is a positive definite matrix which takes into account the relative importance of achieving the objectives represented by each component of r. In order to address the optimality questions associated with both the modified IMC and direction preservation schemes, Soroush and Valluri (1999) have suggested the optimal dynamic compensator (ODC) by solving the following optimization problem

$$\min_{u^r} \|P\mathcal{C}u^r - P\mathcal{C}u\|_{\mathcal{Q}}^2 \tag{15}$$

subject to the constraints

$$u_{min} \le u_i^r \le u_{max} \quad i = 1, \ldots, m$$

where P is a diagonal matrix whose diagonal elements depend on the relative orders of each of the controlled output, $\mathcal{C}$ is the characteristic matrix of the plant and $\mathcal{Q}$ is a positive definite weighting matrix. In this approach, the characteristic matrix $\mathcal{C}$ contains information about the directional nature of the plant; thus the constrained optimization of (15) is such that the components of $u^r - u$ in the high gain plant direction are minimized. However, since the characteristic matrix only characterizes the sensitivity of plant to input changes over a very short horizon, the optimality of the solution is only guaranteed over a very short time horizon (Soroush and Valluri, 1999). Heath and Wills (2004) have argued the use of steady state structural properties such as the steady state gain but only in the context of cross directional control. The scheme in Heath and Wills (2004) guarantees optimal steady state performance by solving the following optimization problem.

$$\min_{u^r} \|\mathcal{K}_p u^r - \mathcal{K}_p u\|_{\mathcal{Q}}^2 \tag{16}$$

subject to the constraints

$$u_{min} \le u_i^r \le u_{max} \quad i = 1, \ldots, m \tag{17}$$

where $\mathcal{Q}$ is a positive definite symmetric matrix and $\mathcal{K}_p$ is the steady state gain $G(0)$ (or $G(1)$ for discrete systems). This scheme may lead to a degraded transient performance especially when $\mathcal{K}_p$ is significantly different from $\mathcal{C}$.

In table 1, we categorize these antiwindup schemes according to their performance characteristics during transient stage and steady state when one or more of the input constraints are active. None of the schemes guarantees optimal performance at both phases.

5. TWO-STAGE MULTIVARIABLE INTERNAL MODEL CONTROL ANTIWINDUP STRUCTURE

We now introduce the two-stage IMC antiwindup scheme. This approach is based on the solution of two quadratic programs (QP) termed the dynamic QP and the steady-state QP. While the former addresses the transient behaviour of the plant and ensures that the constrained plant response is as close as possible to the unconstrained plant response, the latter ensures optimal steady state performance and it is based on steady state properties of the plant. The idea of using a separate QP to calculate steady state set-points has been previously introduced in MPC formulations (Muske and Rawlings, 1993; Rawlings and Chien, 1996; Muske, 1997; Rao and Rawlings, 1999).

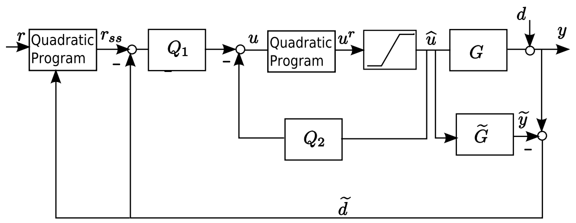

Fig. 5. The two-stage IMC Antiwindup

Because of the presence of saturation nonlinearities in the system, the output of the constrained system y differs from y', the output of the unconstrained system. In general, the control objective is to keep every output y of the constrained system as close as possible to those of the unconstrained system y'. We define the mapping

$$y' = \mathcal{P}u \quad \text{and} \quad y = \mathcal{P}\hat{u} \tag{18}$$

where $\mathcal{P}$ represent the plant operator. $\hat{u}$ and u are the constrained and unconstrained control inputs respectively.

Mathematically, we seek a feasible control input u^* that is a solution to the following constrained optimization problem.

$$u^* = \arg\min_{\hat{u}} \|\mathcal{P}\hat{u} - \mathcal{P}u\|^2_{\mathcal{Q}} \tag{19}$$

subject to the constraints

$$u_{min} \leq \hat{u}_i \leq u_{max} \quad i = 1, \ldots, m \tag{20}$$

$\mathcal{Q}$ is assumed to be positive definite symmetric matrix.

For the system (1) where $p = m$, the initial response of the system output to step change in the input vector depends on the characteristics matrix $\mathcal{C}$ (Daoutidis and Kravaris, 1992; Soroush and Valluri, 1999). Therefore, the plant operator $\mathcal{P}$ can be chosen as the characteristic matrix $\mathcal{C}$ of the plant. Making this substitution in (19) results in the following optimization problem.

$$u^r = \arg\min_{u^r} \|\mathcal{C}u^r - \mathcal{C}u\|^2_{\mathcal{Q}} \tag{21}$$

subject to the constraints

$$u_{min} \leq u^r_i \leq u_{max} \quad i = 1, \ldots, m$$

In steady state, the output response of the system can be expressed as

$$y_{ss} = \mathcal{K}_p u_{ss} + \hat{d} \tag{22}$$

where u_{ss} is the steady state control input that makes the controlled variable achieve y_{ss} and $\hat{d}$ is the estimate of the disturbance. $\mathcal{K}_p$ is the steady state gain of the plant which can be obtained from the plant's state space matrices as $G(0) = -CA^{-1}B$ for $G(s)$ or $G(1) = C(I - A)^{-1}B$ for $G(z)$ provided A is non-singular. If the input constraints are active in steady state, then y_{ss} may not attain the target prescribed by the reference signal r. The objective is to make y_{ss} as close as possible to r in some sense and within the limit imposed by the input constraints.

The solution of the following quadratic program can be used to determine a feasible steady state set-point r_{ss} that should be applied as shown in figure 5 instead of r such that system closed loop response in steady state y_{ss} is as close as possible to r.

$$r_{ss} = \arg\min_{u_{ss}} \|r - y_{ss}\|^2_{\mathcal{Q}_{ss}} \tag{23}$$

subject to the constraints

$$u_{min} \leq u_{ss} \leq u_{max} \quad i = 1, \ldots, m \tag{24}$$

$$y_{ss} = \mathcal{K}_p u_{ss} + \hat{d} \tag{25}$$

where $\mathcal{Q}_{ss}$ is a positive definite symmetric matrix for penalizing deviations in each of the controlled variables and their relative importance. From figure 5, we write the controller as

$$\begin{aligned} \tilde{d} &= y - \tilde{G}u^r \\ u &= Q_1(r_{ss} - \tilde{d}) - Q_2 u^r \\ u^r &= \phi_1(u) \\ r_{ss} &= \phi_2(r, \tilde{d}) \end{aligned} \tag{26}$$

where ϕ_1 and ϕ_2 are non-linear functions representing the quadratic programs (21) and (23) respectively. Without the constraints, the control law reduces to the unconstrained standard IMC control equation

$$\begin{aligned} u &= (I + Q_2)^{-1} Q_1 (r - \tilde{d}) \\ \tilde{d} &= y - \tilde{G}u \end{aligned} \tag{27}$$

Correct steady state behaviour is ensured by designing Q such that $Q(0) = G(0)^{-1}$ for $G(s)$ or $Q(1) = G(1)^{-1}$ for $G(z)$. If constraints are active in steady state, then optimal steady state behaviour is guaranteed by solving (23) subject to the constraints.

Remark 1. For the class of systems whose characteristics matrix $\mathcal{C}$ and steady state gain $\mathcal{K}_p$ are similar, the optimization problem in (21) effectively meets the steady state requirement of (23). Hence only (21) need be solved to achieve both optimal transient and steady state responses in the presence of control input saturation.

Table 1. Performance Comparison of Multivariable Antiwindup Schemes

	Modified IMC	Direction Preservation	Optimization Conditioning Technique	Optimal Dynamic Compensation	Optimal Steady State
Transient performance	Optimal	Poor	Optimal	Optimal	Poor
Steady State performance	Poor	Good	Poor	Poor	Optimal

6. SIMULATION EXAMPLE

Example 1. Consider the following example taken from Zheng et al. (1994) where

$$G(s) = \frac{10}{100s + 1} \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} \tag{28}$$

with $|u_i| \le 1$, $i = 1,2$ and a step reference input of $[0.63 \quad 0.79]^T$.
The classical IMC controller design for a step input is

$$Q(s) = \frac{100s+1}{10(20s+1)} \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} \tag{29}$$

and the corresponding unity feedback controller is

$$K(s) = \frac{100s+1}{200s} \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} \tag{30}$$

Following the development in Zheng et al. (1994), the plant model is slightly modified as

$$\widetilde{G}(s) = \frac{10}{100s+1} \begin{bmatrix} 4 & \frac{-5}{0.1s+1} \\ \frac{-3}{0.1s+1} & 4 \end{bmatrix} \tag{31}$$

in order to satisfy the requirement of $f_A G(s)|_{s=\infty} = I$. The non-causal filter f_A is then designed for $\widetilde{G}(s)$ such that $f_A \widetilde{G}(s)|_{s=\infty} = I$ where $f_A = 2.5(s+1)I$. The factorization of $Q(s)$ is obtained as $Q_1 = f_A \widetilde{G} Q$. In this example,

$$\mathcal{K}_p = \begin{bmatrix} 40 & -50 \\ -30 & 40 \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 0.4 & -0.5 \\ -0.3 & 0.4 \end{bmatrix} \text{ and } D_k^{-1} = \begin{bmatrix} 8 & -10 \\ -6 & 8 \end{bmatrix}$$

Observe that

$$\mathcal{C} = \frac{1}{100}\mathcal{K}_p, \quad \text{and} \quad D_k^{-1} = \frac{1}{5}\mathcal{K}_p$$

so the directionality compensators of (14) and (15) are equivalent and effectively meet the criterion for optimal nominal steady state performance of (16). This is depicted in figure 6 where the closed loop responses for DP, OCT, ODC and OSS schemes are the same.

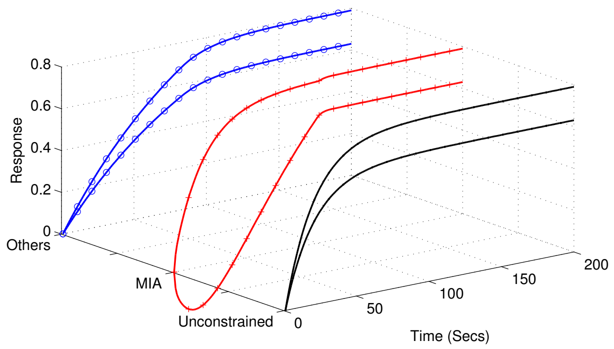

Fig. 6. Example 1: The Modified IMC (MIA,'+') yields a faster reponse for the first output at the expense of a poor transient response in the second out while the other schemes (DP, OCT, ODC, OSS and two-stage IMC,'o') all yield same response.

Example 2. This example is taken from Soroush and Valluri (1999) where the state space model of the process is given as

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -0.01 & -0.0002 \\ -0.5 & -0.03 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.25 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \tag{32}$$

with $|u_1| \le 0.12$ and $|u_2| \le 0.12$ and a set point change of $[0.85 \quad 2.2]^T$. The plant can be represented in continuous-time transfer function as

$$G(s) = \begin{bmatrix} \frac{0.25(s+0.003)}{s^2+0.04s+0.0002} & \frac{-0.0008}{s^2+0.04s+0.0002} \\ \frac{-0.125}{s^2+0.04s+0.0002} & \frac{4(s+0.01)}{s^2+0.04s+0.0002} \end{bmatrix} \tag{33}$$

The classical IMC controller design for a step input is

$$Q(s) = \begin{bmatrix} \frac{4s+0.04}{5s+1} & \frac{0.0008}{2s+1} \\ \frac{0.125}{5s+1} & \frac{0.25s+0.0075}{2s+1} \end{bmatrix} \tag{34}$$

and the corresponding unity feedback controller is

$$K(s) = \begin{bmatrix} \frac{4s+0.04}{5s} & \frac{0.0008}{2s} \\ \frac{0.125}{5s} & \frac{0.25s+0.0075}{2s} \end{bmatrix} \tag{35}$$

The controller $Q(s)$ is factorized based on the modified IMC approach as $Q_1 = f_A G Q$ and $Q_2 = f_A G - I$ with

$$f_A = \begin{bmatrix} 4(s+1) & 0 \\ 0 & 0.25(s+1) \end{bmatrix} \tag{36}$$

With relative orders $r_1 = 1, r_2 = 1$, the plant structural matrices are given as

$$\mathcal{K}_p = \begin{bmatrix} 37.5 & -4 \\ -625 & 200 \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 0.25 & 0 \\ 0 & 4 \end{bmatrix} \text{ and } D_k^{-1} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.125 \end{bmatrix}$$

The plant has a diagonal characteristic matrix which is significantly different from the steady state gain matrix and hence the ODC will result in a different steady state performance compared to the OSS scheme. Figure 7 shows the two-stage IMC results in the closest closed loop performance to the unconstrained case as compared to the other antiwindup schemes.

We also compare the performance of the two-stage IMC with a particular MPC formulation Maciejowski (2002). We consider two MPC cases; a single horizon MPC (prediction horizon $N_p = 1$ and control horizon $N_c = 1$) and a long horizon MPC (prediction horizon $N_p = 100$ and control horizon $N_c = 50$). The closed loop responses in Figures 8 and 9 show that the two-stage IMC competes favourably with a long horizon MPC while only requiring the computation equivalent to that of a single horizon MPC. It is however, envisaged that a long horizon MPC will outperform the two-stage IMC especially when there is high-order unmodeled dynamics in the system. The two-stage IMC does not require the receding horizon computation of MPC and may serve as a less computationally intensive and more transparent (in terms of tuning for robustness) alternative to MPC.

7. CONCLUSION

We have demonstrated the effectiveness of the two-stage internal model control antiwindup in dealing with the performance degradation associated with control windup and process directionality in input constrained multivariable systems. While MPC algorithms are known to handle such problems, their implementation require extensive online computation and tuning for robustness is achieved in an obscure fashion. The distinguishing feature of the proposed two-stage IMC is that it lacks the horizon of MPC. It is based on the solution of two low-order QPs.

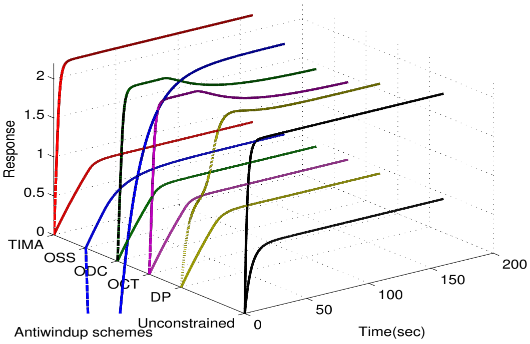

Fig. 7. Example 2: Two-stage IMC (TIMA) yields the closest performance to the unconstrained case. DP and OSS schemes have improved steady state behaviours but poor transient characteristics as opposed to the OCT and ODC schemes both of which have optimal transient behaviours but degraded steady state performances.

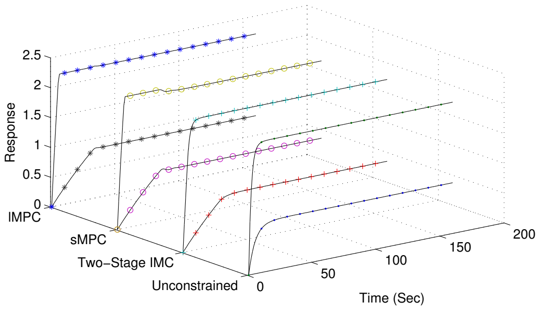

Fig. 8. Two-stage IMC ('+') outperforms the single horizon MPC (sMPC, 'o') and yields similar reponse to long horizon MPC (lMPC, '*')

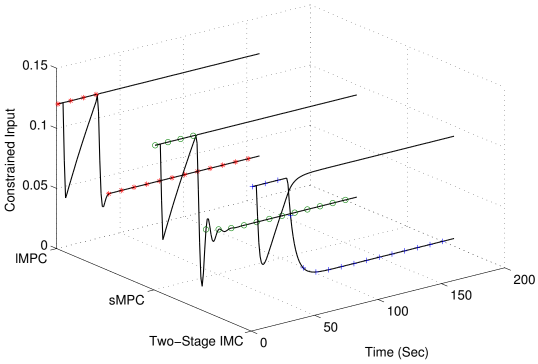

Fig. 9. Constrained input: Two-stage IMC ('+'), single horizon MPC (sMPC, 'o') and long horizon MPC (lMPC, '*')

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