

Model reduction of the ASM3 extended for two-step nitrification

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The Activated Sludge Model No. 3 (ASM3) extended for two-step nitrification is analyzed and reduced to enable fast and accurate simulations under typical process conditions. Due to the addition of energy storage effects, ASM3 enables the description of substrate consumption as well as Oxygen uptake with a higher precision than the older versions of the family of ASM. In addition the extension of the model to describe the two step nitrification enables the calculation of the NO_2 concentration as an independent variable. An exhaustive analysis of the model results in a number of modifications which reduce the model effectively, while keeping its accuracy.

Keywords: model reduction, activated sludge, ASM3, two-step nitrification-denitrification, bypassing nitrate generation

1. INTRODUCTION

The most commonly applied process for biological treatment of waste water is the Activated Sludge Process (ASP) (Gernaey, van Loosdrecht et al. 2004). The family of Activated Sludge Models (ASM) (Henze 2000) represents the state of the art model framework for the simulation of ASP. The model ASM1 (Henze, Grady et al. 1986) is the most widely used in the industry (Petersen, Gernaey et al. 2002), ASM2 (Gujer, Henze et al. 1995) is applied to simulate processes with phosphorus removal (Kim, Hao et al. 2001) and the latest version, ASM3 (Gujer, Henze et al. 1999), includes the quantification of energy storage in order to describe substrate and oxygen uptake with higher accuracy. Finally, a new version of ASM3, referred to in this contribution as extended ASM3, where nitrification and denitrification are considered as two-step processes, (taking into account nitrite as an intermediate) has been presented (Kaelin, Manser et al. 2009). This work focuses on the reduction of this extended ASM3 version to a tractable form.

The extended ASM3 has proven to be a very accurate model to describe the ASP. However, modeling confronts as always the disjunctive between model accuracy and model simplicity. It is then convenient, to reduce the model, in order to minimize the computation costs, trying to preserve the original dynamics

“The goal of model order reduction is to transform the system of differential equations to one of lower order and still retain the key dynamic information.” (Okino and Mavrouniotis 1998).

The reduction of a particular model maintaining the model characteristics is a common task in all fields of engineering.

Examples are: Lumping (Aris and Gavalas 1966), Sensitivity Analysis (Liao and Delgado 1993) and Time-Scale Analysis (Stamatelatos, Syrou et al. 2009).

2. ASM3 EXTENDED FOR TWO-STEP NITRIFICATION

The division of the nitrification-denitrification reaction in a two step reaction (Fig. 1) is essential when trying to describe the bypassing nitrate generation process in ASP (Katsogiannis, Kornaros et al. 2002). If the change from aerobic to anoxic phase in a sequencing batch reactor (SBR) is adequate, ammonia is converted to nitrite in the presence oxygen (nitritation), which is instantly converted into nitrogen under anoxic conditions before the second oxidation producing nitrate (nitrataion) can take place.

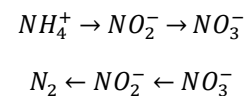


Fig. 1. Nitrification-denitrification process as a two -step reaction.

It should be noted, that the nitrate bypassing process can only be described by a model which divides the nitrification-denitrification process in a reaction with at least two steps. On the other hand the extended ASM3 demands significant computational time, which represents an obstacle for efficient optimization and model-based control. Furthermore, the extended ASM3 describes many states, which are not reached within conventional ASPs. If the process runs under optimal conditions, in other words, if the correct aeration strategy is selected and the initial concentrations are in the desired range, many of the process conditions described by the extended ASM3 can be avoided and therefore, no mathematical description is needed. Consequently, in order to allow a fast process simulation, and thus, an efficient model-

based optimization, a model reduction is considered necessary.

3. A PROPOSED 8-STATE MODEL

The model proposed in this work is named 8-state model, in relation to the number of state variables it contains. Eight differential equations describe the basic variables (concentrations, namely: 1.- substrate, 2.- heterotrophic bacteria, 3.- ammonia oxidizers, 4.- nitrite oxidizers, 5.- dissolved oxygen, 6.- ammonia, 7.- nitrite and 8.- nitrate. In addition an algebraic equation is added so as to describe the effects of energy storage on biomass growth and on the substrate as well as oxygen uptake such as described by the extended ASM3.

The reduction of this model is based on the principle that an activated sludge process should stop as soon as all the concentrations satisfy the regulations. Furthermore, these regulations should be reached in the shortest time possible. Consequently lower output concentrations than required are an indicator of a suboptimal operation.

Another important assumption is that the bacteria never exhaust their stored energy. Except for the recycle process, bacteria are always in a medium rich in substrate. Therefore, the value of the stored energy should be high at any moment during the process and never limit bacterial growth.

In addition, the death rates of the bacteria are considered constant. The growth rate has a major impact on substrate and oxygen concentrations. Nevertheless, changes on biomass concentration during one cycle in an SBR are relatively small (less than 10%). As a result, the biomass growth rate denotes a very important aspect of the model, not so the change on the biomass concentration, and therefore it is viable to consider a constant death rate without affecting the model dynamics significantly.

4. MODEL REDUCTION APPROACH

4.1 Storage

The implementation of energy storage represents the main improvement of ASM3 in comparison with older versions. This concept caused the addition of three new state equations and more than 10 new parameters to the preceding model. Eliminating these equations would impede a proper description of the process and result in a retrogressive model reduction. For this reason, the energy storage and its effects in substrate and oxygen concentration cannot be simply neglected. Instead, the 8-state model includes some adaptations to the substrate and oxygen uptake equations and the addition of an algebraic one. The new set of equations is presented in such a way that, both, the substrate uptake increment and the oxygen uptake increment caused by the storage, are included in the original substrate and oxygen differential equations. By this means, and as long as the substrate concentration is above zero, the algebraic equation, describing the behavior of energy stored can be eliminated without affecting the behavior of the substrate or oxygen concentration. This appears to be an inconsistent assumption, though it is almost certain that the process continues after substrate elimination to enable the ammonia degradation. Nevertheless previous model versions (ASM1 and ASM2) fit

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the data although they lack also of a storage variable. In other words, the 8-state model responds to the substrate limitation similar to how ASM1 does but describes also the substrate and oxygen uptake as precise as ASM3.

4.2 Substitution of the storage equation

The system of equations related to the extended ASM3 is too large to be presented in this contribution. For this reason, in this work the 15 ordinary differential equations of the extended ASM3 include only the process rate variables without its explicit equation. The process rate equations have been numbered in the same order as previously presented in (Kaelin, Manser et al. 2009):

Heterotrophic Organisms

r_1 : Hydrolysis, r_2 : Aerobic Storage, r_3 : Anoxic Storage, r_4 : Anoxic Storage of SS NO₂-N₂, r_5 : Aerobic Growth of XH, r_6 : Anoxic Growth NO₃-NO₂, r_7 : Anoxic Growth NO₂-N₂, r_8 : Aerobic Endog. Resp. of HET, r_9 : Anoxic Endog. Resp. NO₃-NO₂, r_{10} : Anoxic End. Resp. NO₂-N₂, r_{11} : Aerobic Resp. of XSTO, r_{12} : Anoxic Resp. of SXTO NO₃-NO₂, r_{13} : Anoxic Resp. of XSTO NO₂-N₂,

AOB

r_{14} : Aerobic Growth, Nitrification, r_{15} : Aerobic End. Resp., r_{16} : Anoxic End. Resp.,

NOB:

r_{17} : Aerobic Growth, Nitrification. r_{18} : Aerobic End. Resp., r_{19} : Anoxic Endog. Resp., r_{20} : Aeration

We first analyze the three state equations of the extended ASM3 directly involved in the energy storage.

Readily biodegradable substrate:

$$\frac{dS_s}{dt} = -r_2 - r_3 - r_4 + (1 - f_{SI}) * r_1 \quad (1)$$

where the term $(1 - f_{SI}) * r_1$ represents the conversion of slow biodegradable substrate into readily biodegradable substrate and can be eliminated if it is considered that all substrate is readily biodegradable.

Energy Storage:

$$\begin{aligned} \frac{dX_{STO}}{dt} &= Y_{STOaer} * r_2 + Y_{STONO3} * r_3 + Y_{STONO2} * r_4 \\ &- \frac{1}{Y_{Haer}} * r_5 - \frac{1}{Y_{HNO3}} * r_6 - \frac{1}{Y_{HNO2}} * r_7 - r_{11} - r_{12} \\ &- r_{13} \end{aligned} \quad (2)$$

where the terms $-r_{11} - r_{12} - r_{13}$ represent the respiration reactions and are neglected in the 8-state model.

Heterotrophic Biomass:

$$\frac{dX_H}{dt} = r_5 + r_6 + r_7 - r_8 - r_9 - r_{10} \quad (3)$$

where the terms $-r_8 - r_9 - r_{10}$ represent the death rates of the heterotrophic bacteria and are substituted by death constants. A closer look at the rate equations (4) - (9) shows that all of them are dependent on X_H and, considering that the

switching functions have a constant value (meaning that the concentrations are considerably larger than their saturation constants), there is a linear relation $\frac{k_{sto}}{\mu_H}$ between (4) - (6) and (7) - (9):

$$r_2 = k_{sto} * \frac{S_O}{S_O + K_{H,O2}} * \frac{S_S}{S_S + K_S} * X_H \quad (4)$$

$$r_3 = k_{sto} * \eta_{H,NO3} * \frac{K_{H,O2inh}}{K_{H,O2inh} + S_O} * \frac{S_S}{S_S + K_S} * \frac{S_{NO3}}{S_{NO3} + K_{H,NO3}} * X_H \quad (5)$$

$$r_4 = k_{sto} * \eta_{H,NO2} * \frac{K_{H,O2inh}}{K_{H,O2inh} + S_O} * \frac{S_S}{S_S + K_S} * \frac{S_{NO2}}{S_{NO2} + K_{H,NO2}} * X_H \quad (6)$$

$$r_5 = \mu_H * \frac{S_{Alk}}{S_{Alk} + K_{Alk}} * \frac{S_O}{S_O + K_{O1}} * \frac{S_{NH4}}{S_{NH4} + K_{NH}} * \frac{X_{STO}/X_H}{K_{H,STO} + X_{STO}/X_H} * X_H \quad (7)$$

$$r_6 = \mu_H * \eta_{H,NO3} * \frac{K_{H,O2inh}}{K_{H,O2inh} + S_O} * \frac{S_{Alk}}{S_{Alk} + K_{Alk}} * \frac{X_{STO}/X_H}{K_{H,STO} + X_{STO}/X_H} * X_H \quad (8)$$

$$r_7 = \mu_H * \eta_{H,NO2} * \frac{K_{H,O2inh}}{K_{H,O2inh} + S_O} * \frac{S_S}{S_S + K_S} * \frac{X_{STO}/X_H}{K_{H,STO} + X_{STO}/X_H} * X_H \quad (9)$$

We now bound our process conditions, such that neither ammonia, nor alkalinity, nor substrate, nor storage are limiting to the process. In other words (10):

$$k * \frac{S_S}{S_S + K_S} = \frac{S_{Alk}}{S_{Alk} + K_{Alk}} * \frac{S_{NH4}}{S_{NH4} + K_{NH}} * \frac{X_{STO}/X_H}{K_{H,STO} + X_{STO}/X_H} \quad (10)$$

where k is an arbitrary constant.

The following equations (11) can now be implemented:

For $i = 2,3,4$

$$k * Y_{STOaer} * \frac{r_i}{\frac{k_{sto}}{\mu_H}} = \frac{1}{Y_{Haer}} * r_{i+4} = a_i \quad (11)$$

Substitution in the three state equations (1) – (3) gives:

$$\frac{dS_S}{dt} = \left(\frac{1}{Y_{STOaer}} * a_2 + \frac{1}{Y_{STON03}} * a_3 + \frac{1}{Y_{STON02}} * a_4 \right) * \frac{k_{sto}}{k * \mu_H} + \frac{1}{Y} \quad (12)$$

$$\frac{dX_{STO}}{dt} = (a_2 + a_3 + a_4) * \frac{k_{sto}}{k * \mu_H} - a_2 - a_3 - a_4 = (a_2 + a_3 + a_4) * \left(\frac{k_{sto}}{k * \mu_H} - 1 \right) \quad (13)$$

$$\frac{dX_H}{dt} = Y_{Haer} * a_2 + Y_{HNO3} * a_3 + Y_{HNO2} * a_4 \quad (14)$$

If we add a new constant $St_S = \frac{k_{sto}}{k * \mu_H} - 1$:

and consider that:

$Y_{Haer} = Y_{STOaer}$ and $Y_{Hanox} = Y_{STON03} = Y_{STON02} = Y_{HNO3} = Y_{HNO2}$ (which is true for the values shown in (Kaelin, Manser et al. 2009) by replacing them in the simplified state equations (12) – (14), we obtain:

$$\frac{dS_S}{dt} = \left(-\frac{1}{Y_{Haer}} * a_2 - \frac{1}{Y_{Hanox}} * (a_3 + a_4) \right) * (1 + St_S) \quad (15)$$

$$\frac{dX_{STO}}{dt} = (-a_2 - a_3 - a_4) * (-St_S) \quad (16)$$

$$\frac{dX_H}{dt} = Y_{Haer} * a_2 + Y_{HNO3} * a_3 + Y_{HNO2} * a_4 \quad (17)$$

The most important characteristic of this new set of equations is given in (18)

$$\frac{dS_S}{dX_{STO}} = \frac{dX_H}{dX_{STO}} = 0 \quad (18)$$

This means that we can now eliminate the ninth state equation responsible for energy storage.

$$\frac{dS_S}{dt} = \left(-\frac{1}{Y_{Haer}} * r_{aae} - \frac{1}{Y_{Hanox}} * (r_{aNO3} + r_{aNO2}) \right) * (1 + St_S) \quad (19)$$

Finally, a rearrangement of the equations in order to substitute a_i for new rate constants in (15) – (17), results in a model with 9 state equations. The energy storage equation

can be further reduced to an algebraic equation to obtain the 8-state model.

$$Sto = \left(C_{Sto} - \frac{S_s}{1 + St_s} \right) * St_s \quad (20)$$

The 8-state model can also be adapted to describe the substrate limited phase. Because of (20) the information about the amount of energy stored is not lost. To enable the biomass growth beyond substrate limitation, an inhibition function and new slightly modified rate equations are to be added. This would increment the size of the reduced model and is not studied in this work.

It should be noted, that the reduced equation system will not react to all the limitations exactly like the original model. For example, in the extended ASM3 the energy storage is not limited by the ammonia concentration, but only the growth of the biomass. Having both storage and growth in the same equation, the 8-state model is forced to have the same switching functions for both cases.

4.3 Parameter fit

The main objective of this work is to achieve a reduced version of the extended ASM3 which presents the same behavior as the extended ASM3. Consequently, the new parameters of the 8-state model were compared and fitted against the extended ASM3.

Saturation constants:

The only possible way to assure, that the model will respond to the concentration limitations as similar as possible to the extended ASM3, is to apply identical saturation constants. This was the case in this work. All the constants $K.x$ have the same value as published in the extended version of ASM3 (Kaelin, Manser et al. 2009).

5. MATHEMATICAL REPRESENTATION OF THE 8-STATE MODEL

5.1 ordinary differential equations

The 8 ordinary differential equations are shown in (21) – (28), and their belonging rate equations are described in (30) – (34). The algebraic equation describing the storage is shown in (29).

$$\frac{dS_s}{dt} = \left(-\frac{1}{Y_{Haer}} * r_{aae} - \frac{1}{Y_{Hanox}} * (r_{aNO3} + r_{aNO2}) \right) * (1 + St_s) \quad (21)$$

$$\frac{dX_H}{dt} = r_{aae} + r_{aNO3} + r_{aNO2} \quad (22)$$

$$\frac{dX_{Ns}}{dt} = r_{aaNs} \quad (23)$$

$$\frac{dX_{Nb}}{dt} = r_{aaNb} \quad (24)$$

$$\begin{aligned} \frac{dS_O}{dt} &= K_{La}(S_O^* - S_O) - \frac{1 - Y_{Haer}}{Y_{Haer}} r_{aae} - \left(\frac{3.43}{Y_{A1}} - 1 \right) r_{aaNs} \\ &\quad - \left(\frac{1.14}{Y_{A2}} - 1 \right) r_{aaNb} \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{dS_{NH4}}{dt} &= - \left(-\frac{i_{NSS}}{Y_{Haer}} + i_{NB} \right) r_{aae} - \left(\frac{1}{Y_{A1}} + i_{NB} \right) r_{aaNs} \\ &\quad - i_{NB} r_{aaNb} - \left(-\frac{i_{NSS}}{Y_{Hanox}} + i_{NB} \right) r_{aNO3} \\ &\quad - \left(-\frac{i_{NSS}}{Y_{Hanox}} + i_{NB} \right) r_{aNO2} \end{aligned} \quad (26)$$

$$\begin{aligned} \frac{dS_{NO2}}{dt} &= \frac{1}{Y_{A1}} r_{aaNs} - \frac{1}{Y_{A2}} r_{aaNb} + \frac{1 - Y_{Hanox}}{1.14 Y_{Hanox}} (r_{aNO3} \\ &\quad - r_{aNO2}) \end{aligned} \quad (27)$$

$$\frac{dS_{NO3}}{dt} = \frac{1}{Y_{A3}} r_{aaNb} - \frac{1 - Y_{Hanox}}{1.14 Y_{Hanox}} r_{aNO3} \quad (28)$$

$$Sto = \left(C_{Sto} - \frac{S_s}{1 + St_s} \right) * St_s \quad (29)$$

5.2 reaction rates

$$r_{aae} = \mu_H * \frac{S_s}{S_s + K_S} * \frac{S_O}{S_O + K_{O1}} * \frac{S_{NH4}}{S_{NH4} + K_{NH}} * X_H \quad (30)$$

$$r_{aaNs} = \mu_{A1} * \frac{S_O}{S_O + K_O} * \frac{S_{NH4}}{S_{NH4} + K_{NH}} * X_{Ns} \quad (31)$$

$$\begin{aligned} r_{aaNb} &= \mu_{A2} * \frac{S_{NO2}}{S_{NO2} + K_{NO21}} * \frac{S_O}{S_O + K_O} * \frac{S_{NH4}}{S_{NH4} + K_{NH}} \\ &\quad * X_{Nb} \end{aligned} \quad (32)$$

$$\begin{aligned} r_{aNO3} &= \mu_{H1} * \frac{S_s}{S_s + K_S} * \frac{S_{NO3}}{S_{NO3} + K_{NO3}} * \frac{K_{O21}}{K_{O21} + S_O} \\ &\quad * \frac{S_{NH4}}{S_{NH4} + K_{NH}} * X_H \end{aligned} \quad (33)$$

$$\begin{aligned} r_{aNO2} &= \mu_{H2} * \frac{S_s}{S_s + K_S} * \frac{S_{NO2}}{S_{NO2} + K_{NO2}} * \frac{K_{O22}}{K_{O22} + S_O} \\ &\quad * \frac{S_{NH4}}{S_{NH4} + K_{NH}} * X_H \end{aligned} \quad (34)$$

The values of the constants used for the simulations are presented in table 1.

Table 1: 8-state model constants and its values

SOSTar =	7;	[mgO ₂ /l]	process
K_La =	1000;	[d-1]	process
i_NB =	0.086;	[gN/gCOD]	fitted
mou_H =	0.6021;	[d-1]	fitted
mou_A1 =	0.6552;	[d-1]	fitted
mou_A2 =	0.3468;	[d-1]	fitted
Y_Haer =	0.1302;	[gCOD/gCOD]	fitted
Y_A1 =	0.1327;	[gCOD/gN]	fitted
Y_A2 =	0.0985;	[gCOD/gN]	fitted
Y_A3 =	0.0331;	[gCOD/gN]	fitted
i_NSS =	0.01;	[gN/gCOD]	ASM3
Y_Hanox =	0.0632;	[gCOD/gCOD]	fitted
mou_H1 =	0.0511;	[d-1]	fitted
mou_H2 =	0.0362;	[d-1]	fitted
K_NH1 =	0.01;	[mgCOD/l]	ASM3
K_NH2 =	0.1;	[mgCOD/l]	ASM3
K_S =	10;	[mgCOD/l]	ASM3
K_S1 =	0.1;	[mgCOD/l]	ASM3
K_S2 =	0.1;	[mgCOD/l]	ASM3
K_NHH =	0.05;	[mgN/l]	ASM3
K_O1 =	0.2;	[mgO ₂ /l]	ASM3
K_NH =	0.1;	[mgN/l]	ASM3
K_O =	0.8;	[mgO ₂ /l]	ASM3
K_NO21 =	0.5;	[mgO ₂ /l]	ASM3
K_NO3 =	0.5;	[mgN/l]	ASM3
K_O21 =	0.2;	[mgO ₂ /l]	ASM3
K_NO2 =	0.25;	[mgN/l]	ASM3
K_O22 =	0.2;	[mgO ₂ /l]	ASM3
stS =	1.7;	[]	fitted
stO =	0.08;	[]	fitted

6. RESULTS

6.1 Simulations

The 8-state model was set to various conditions so as to confirm its stability and accuracy. The most representative results are present in Fig.2-5.

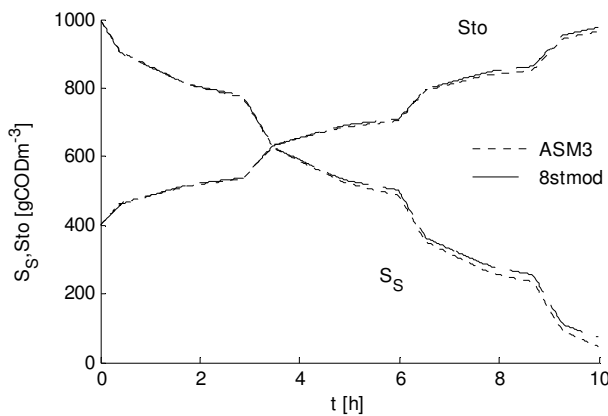


Fig. 2. Substrate concentration S_S and stored energy Sto against time. The initial value of the storage was set to 400 gCOD/m³ considering that the time between loads is short.

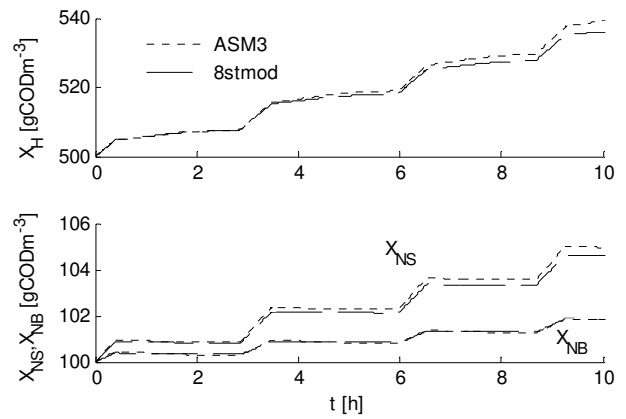


Fig. 3. Biomass against time. Changes in the biomass are very small (less than 10%). Nevertheless the influence of the growth rate plays a significant role in the load characteristics.

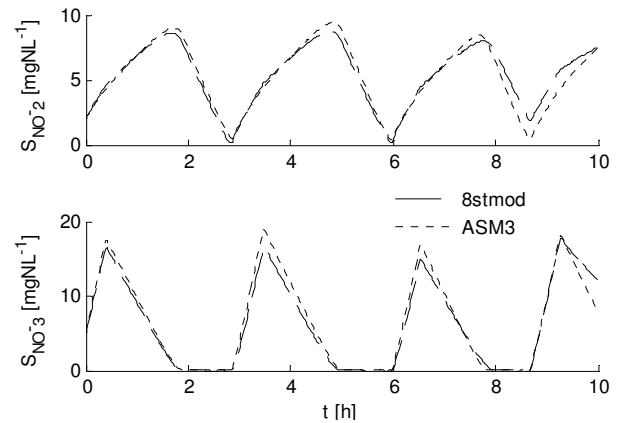


Fig. 4. NO_x concentration against time. The concentration of NO₂ and NO₃ are kept under 20 mgN/L. The control variable is the aeration of the tank.

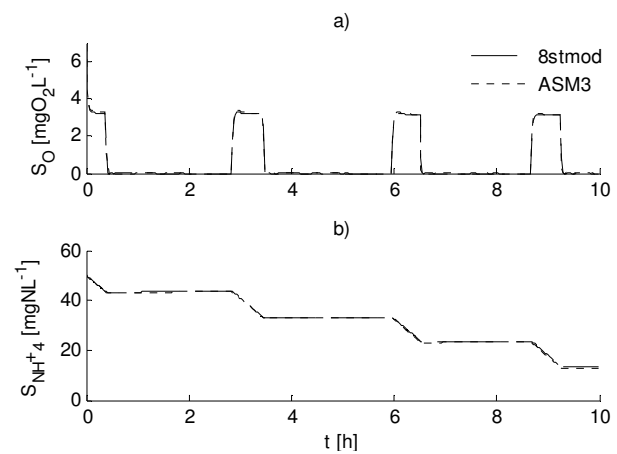


Fig. 5. a) Oxygen concentration in the medium against time. The aeration is turned off when the concentration of NO₂ or NO₃ reaches 20 mgN/L and turned on when the concentration of NO₂ is under 0.3 mgN/L. b) Ammonia concentration against time.

The simulation represents a tank ideally mixed, where the aeration can be turned on and off to induce either aerobic or anoxic condition. The main purpose is to set both models to drastic changes and various limitations. The concentrations of NO₂ and NO₃ are limited by turning the aeration off, if any of these two concentrations rises above 20 gCOD/m³, and on, if NO₂ reaches values smaller than 0.3 mgN/L. The initial value of the storage is set to 400 gCOD/m³. This is explained by considering that the bacteria have been in the process for some time and have already stored energy. As a result, a constantly changing process is obtained, which makes it very difficult to be described identically by two models with different characteristics. The process conditions prove that the 8-state model accurately describes limitations of dissolved O₂, NO₂ and NO₃.

6.2 Limitations of the 8-state model

It is worth reminding that the 8-state model is not valid along the same range as extended the ASM3. Some limitations are to put up with in order to reduce the model and speed up the simulation in the region of interest. Moreover, because of the new storage equation, the bacteria can store energy, but cannot use it when there is no more substrate available. For this reason, the reduced version describes the process as long as substrate is present in the medium.

In the extended ASM3, the ammonia concentration does not limit the storage of energy. This results in a consumption of substrate even under NH₄ limitations. Once again, because of the coupled equations, the 8-state model predicts a substrate consumption only as long as ammonia is present in the medium. Finally, the growth of heterotrophic biomass can be mathematically described as a second order differential equation. For this reason, if the energy stored by the bacteria is low, a time delay can be seen in the growth curve. This time delay is not represented in the 8-state model. Considering that the storage has a value at least larger than 100 gCOD/m³, both growth curves match.

7. CONCLUSIONS

The extended version of ASM3 has been successfully reduced to a simpler model. The 8-state model mimics the behavior of the extended ASM3 in a broad operation range. The results obtained in this work suggest that the reduced model can be one order of magnitude faster (table 2) and be applied to both, continuous and to SBR ASP.

Table 2. Comparison of the computation time. Models simulated with Matlab® R2008b

Model	AMD Processor (1.81GHz) CPU time: 1000 func. eval.
Extended ASM3	2.225 sec
8-state	0.157 sec

Furthermore, the 8-state model enables the process simulation for optimization and for model based control.

REFERENCES

- Aris, R. and G. R. Gavalas (1966). "On the theory of reactions in continuous mixtures." Philosophical Transactions for the Royal Society of London. Series A, Mathematical and Physical Sciences: 351-393.
- Gernaey, K. V., M. C. M. van Loosdrecht, et al. (2004). "Activated sludge wastewater treatment plant modelling and simulation: state of the art." Environmental Modelling and Software **19**(9): 763-783.
- Gujer, W., M. Henze, et al. (1995). "The activated sludge model no. 2: biological phosphorus removal." Water science and technology **31**(2): 1-11.
- Gujer, W., M. Henze, et al. (1999). "Activated sludge model No. 3." Water Science & Technology **39**(1): 183-193.
- Henze, M. (2000). Activated sludge models ASM1, ASM2, ASM2d and ASM3.
- Henze, M., C. Grady, et al. (1986). "Activated sludge model no. 1, IAWPRC task group on mathematical modelling for design and operation of biological wastewater treatment." Report completed.
- Kaelin, D., R. Manser, et al. (2009). "Extension of ASM3 for two-step nitrification and denitrification and its calibration and validation with batch tests and pilot scale data." Water Research **43**(6): 1680-1692.
- Katsogiannis, A. N., M. Kornaros, et al. (2002). "Enhanced nitrogen removal in SBRs bypassing nitrate generation accomplished by multiple aerobic/anoxic phase pairs." Water Science and Technology **47**(11): 53-59.
- Kim, H., O. J. Hao, et al. (2001). "SBR system for phosphorus removal: ASM2 and simplified linear model." Journal of environmental engineering **127**: 98.
- Liao, J. C. and J. Delgado (1993). "Advances in metabolic control analysis." Biotechnology Progress **9**(3): 221-233.
- Okino, M. S. and M. L. Mavrovouniotis (1998). "Simplification of mathematical models of chemical reaction systems." Chem. Rev **98**(2): 391-408.
- Petersen, B., K. Gernaey, et al. (2002). "Evaluation of an ASM 1 model calibration procedure on a municipal-industrial wastewater treatment plant." Journal of Hydroinformatics **4**(1): 15-38.
- Stamatelatos, K., L. Syrou, et al. (2009). "An invariant manifold approach for CSTR model reduction in the presence of multi-step biochemical reaction schemes. Application to anaerobic digestion." Chemical Engineering Journal **150**(2-3): 462-475.