A Receding Horizon Approach to Input Design for Closed Loop Identification

Rohit S. Patwardhan and R. Bhushan Gopaluni

***Abstract*— The Identification of high fidelity models is a critical element in the implementation of high performance model predictive control (MPC) applications in the industry. These controllers are large in size with input-ouput dimensions ranging from 5×10 to 50×100. Identifying models of this scale accurately is a time consuming and demanding exercise. In this work, we present a novel approach wherein an information rich test signal is generated in closed loop by maximizing the MPC objective, as opposed to minimization that is done in the standard controller. We show that the proposed input design approach is T-optimal (trace optimal) for autoregressive exogenous input models. Our approach automatically accounts for the input and output constraints and is implemented in a receding horizon manner. It is demonstrated through simulation examples on both well and ill-conditioned processes.**

I. INTRODUCTION

Model predictive control has become the norm for multivariable constrained processes where regulatory control techniques are faced with severe limitations. Standard interfacing protocols now allow for faster deployment of advanced control applications in a plant setting. Unfortunately, the approach to step testing and modeling has remained largely unchanged and performed by moving one variable at a time. This is primarily due to: (a) lack of appropriate tools, and (b) lack of industrial awareness of closed loop identification theory. Surveys of state-of-the-art in MPC technology are available in Qin and Badgewell (2003), Bauer and Craig (2008).

Dynamic models play a central role in MPC technology. Industrial experience has shown that the most challenging and time-consuming task in an MPC commissioning project is that of step testing and model identification. A traditional approach to step testing would involve a control engineer who spends many shifts in the control room operating the plant in open loop. Additionally, during MPC maintenance phase, the main task is often model re-identification. A traditional model identification test on a refinery unit, such as the crude unit, can take several weeks. The quality of collected data depends primarily on the experience of the control engineer. After the test, it can take significant time to analyze the data and to identify appropriate models. Currently available modeling tools involve significant amounts of trial and error to make the models conform to the industrial data. At the end of the modeling exercise, the control engineer is left with, at best, an intuitive feel for the fidelity of the individual models. Since the models form the heart of any MPC application, it is critical that the project team has confidence in the models before deploying them online.

There is a growing demand for more efficient model identification methods that reduce duration of plant tests, the time needed for model identification and the disturbances to optimal operation of the plant during the test. The quality of models estimated and the efficiency of these identification methods depend on the choice of input during the plant test and whether the test is done under open or closed loop conditions.

There is extensive literature on designing inputs for linear processes (Goodwin and Payne, 1977; Ljung, 1999; Hjalmarsson, 2005; Jansson and Hjalmarsson, 2005; Qin, 2006). While a large portion of the literature focusses on input design under open loop, there have also been some attempts to articulate the need for closed loop identification and its relevance to high performance controllers in general (Van Den Hof and Schrama, 1995; Gopaluni *et al.*, 2003; Gopaluni *et al.*, 2002; Forssell and Ljung, 1999; Forssell and Ljung, 2000). These traditional approaches to input design are based on finding an optimal input sequence that minimizes a function of the parameter covariance matrix. Consequently, there are a few common challenges to implementation of these input design algorithms: (a) the optimization problem involved is often nonconvex, (b) the optimal input depends on the "true" process model, and (c) the input and output constraints are not explicitly accounted. To the best knowledge of the authors, Cooley and Lee (2001) and Jansson and Hjalmarsson (2005) are some of the few articles that attempt to formulate a convex input design optimization problem and account for constraints.

In this work, we explore a novel approach to the generation of an information rich test signal relevant to MPC applications. The idea of using a model predictive framework based on the current controller model is formulated to calculate a set of moves that maximize the output (controlled variable) variability to the extent allowed by the process constraints. The test moves are then implemented in receding horizon manner, i.e., the first step or move is implemented and the entire sequence recalculated at the next sampling instant. We show that this approach is equivalent to designing a T-optimal (trace optimal) input for autoregressive exogenous (ARX) input models. The application of the model predictive input design approach is demonstrated through examples on both well and ill-conditioned processes.

This approach has numerous advantages: (a) the input is

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designed by solving a convex optimization problem, (b) the receding horizon nature of the algorithm ensures that the "true" process model is not needed, (c) the input and output constraints are explicitly included in the input design optimization problem, (d) the plant tests are done in closed loop, and (e) the implementation in off-the-shelf MPC technology is rather straightforward.

II. Model Predictive Control

Model Predictive Control (MPC) is a technique commonly used in advanced process control (APC) applications in the industry for the last 30 years. Its success in industry has been due to (1) its ability to capture the fundamental relationships in a process unit in the form of an empirical model, and (2) its ability to handle tradeoffs between process constraints and drive the unit to its most profitable constraint.

The dynamic layer of any industrial MPC application relies on minimization of a multi-step objective function to calculate a sequence of moves for the inputs. Only the first of these moves is implemented and the rest are discarded. The calculations are repeated at each sampling instant to give it a receding horizon nature. Based on a linear dynamic model, the following quadratic objective is minimized at every instant to calculate the sequence of future moves,

$$J(k) = (\mathbf{r}(k) - \mathbf{y}(k))^T \boldsymbol{\Gamma}(\mathbf{r}(k) - \mathbf{y}(k)) + \Delta\mathbf{u}(k)^T \boldsymbol{\Lambda} \Delta\mathbf{u}(k) \quad (1)$$

where

$$\begin{aligned}
\mathbf{r}(k) &= \left[\mathbf{r}_1(k)^T \;\; \mathbf{r}_2(k)^T \;\; \cdots \;\; \mathbf{r}_{n_y}(k)^T\right]^T \in \mathbb{R}^{Pn_y \times 1} \\
\mathbf{r}_i(k) &= \left[r_i(k+1) \;\; r_i(k+2) \;\; \cdots \;\; r_i(k+P)\right]^T \in \mathbb{R}^{P \times 1} \\
\mathbf{y}(k) &= \left[\mathbf{y}_1(k)^T \;\; \mathbf{y}_2(k)^T \;\; \cdots \;\; \mathbf{y}_{n_y}(k)^T\right]^T \in \mathbb{R}^{Pn_y \times 1} \\
\mathbf{y}_i(k) &= \left[y_i(k+1) \;\; y_i(k+2) \;\; \cdots \;\; y_i(k+P)\right]^T \in \mathbb{R}^{P \times 1} \\
\Delta\mathbf{u}(k) &= \left[\Delta\mathbf{u}_1(k)^T \;\; \Delta\mathbf{u}_2(k)^T \;\; \cdots \;\; \Delta\mathbf{u}_{n_u}(k)^T\right]^T \in \mathbb{R}^{Mn_u \times 1} \\
\Delta\mathbf{u}_i &= \left[\Delta u_i(k+1) \;\; \Delta u_i(k+2) \;\; \cdots \;\; \Delta u_i(k+M)\right]^T \in \mathbb{R}^{M \times 1}.
\end{aligned}$$

In the above formulation, P and M are the prediction and control horizons, respectively, and $\boldsymbol{\Gamma}$ and $\boldsymbol{\Lambda}$ are the output and input weighting matrices. $r(k)$, $y(k)$, and $u(k)$ denote the set points, the outputs and the inputs at at the sampling instant k. The number of inputs and outputs are denoted by n_u and n_y, respectively. Δ is the difference operator. The estimated model is used to predict the future outputs over the prediction horizon. This objective function is minimized subject to process operating constraints. The following quadratic program is solved at every sampling instant,

$$\begin{aligned}
\underset{\Delta\mathbf{u}(k)}{\text{minimize}} \quad & J(k) \\
\text{subject to} \quad & \mathbf{y}_L \le \mathbf{y}(k) \le \mathbf{y}_H \\
& \mathbf{u}_L \le \mathbf{u}(k) \le \mathbf{u}_H \\
& \Delta\mathbf{u}_L \le \Delta\mathbf{u}_k \le \Delta\mathbf{u}_H
\end{aligned}$$

where $()_L$ and $()_H$ are lower and upper bounds on the corresponding variables.

One of the main advantages of the MPC technology is its ability to formulate the input and output constraints in a consistent way, and ensure that they are satisfied to the extent possible in any process situation.

III. Optimal Input Design

A. The Approach

The problem of optimal test input design is one where the information content in a given data set has to be maximized in the context of the process model being identified and the controller for which the model is being developed. For example, if one is designing a PID controller with the model being identified, the frequency ranges in the input signal could be different than the frequency ranges if the intended application is a model predictive controller. The input design for a multivariable process is often based on different principles than that for a univariate process, especially when the process is ill-conditioned (Koung and MacGregor, 1994).

When designing a test sequence for a MPC application, the main tradeoffs are between exciting the process to the maximum extent possible and maintaining the process within the constraints. The level of process excitation or information content is often expressed through the condition of persistent excitation, which ensures that the "information matrix" is well conditioned. Shouche *et al.* (1998) had taken the approach of imposing the condition number of the information matrix as an explicit constraint in the MPC objective function. This approach ensures that the designed input is capable of exciting the process to the extent permitted by the constraints.

In the present approach, we reformulate the MPC objective function to calculate a sequence of input moves that are capable of maximizing the variability in the outputs while attempting to satisfy input/output constraints. The duality of the control/identification problems is exploited in formulating the objectives of the input design problem. A receding horizon approach is taken to meet the process constraints in the presence of time-varying disturbances.

We propose to solve the following optimization problem to design the optimal input for exciting a process with the model predictive controller,

$$\begin{aligned}
\underset{\Delta\mathbf{u}(k)}{\text{maximize}} \quad & J(k) \\
\text{subject to} \quad & \mathbf{y}_L \le \mathbf{y}(k) \le \mathbf{y}_H \\
& \mathbf{u}_L \le \mathbf{u}(k) \le \mathbf{u}_H \\
& \Delta\mathbf{u}_L \le \Delta\mathbf{u}(k) \le \Delta\mathbf{u}_H.
\end{aligned}$$

Note that similar optimization problems are solved in the input design and control formulations. The only difference being whether the objective function is maximized (input design) or minimized (control). The proposed formulation of input design will result in calculation of a set of moves that maximize the input/output variability to the extent allowed by the constraints. The first move in the test sequence is implemented and the rest are discarded, similar to the MPC control sequence.

B. T-Optimality

Optimal inputs are often designed by minimizing some function of the model parameter covariance matrix. In this section, we show that the MPC objective $J(k)$ is in fact proportional to the trace of inverse of parameter covariance matrix (in other words trace of the information matrix) for ARX models and therefore the proposed input design method is T-optimal.

The information matrix depends on the type of model structure used in the identification exercise. Typical model structures used in MPC identification are: (1) finite impulse response (FIR) models, and (2) ARX/Box Jenkins model structures. If a FIR model structure is used, the model identification typically involves solving a least squares problem. The information matrix consists of past inputs all the way up to the steady-state time of the process. If an ARX structure is used, the information matrix at the identification step comprises of past inputs and outputs up to the model order chosen. Many modern identification methods (Zhu, 2001) rely on using high order ARX models to initially identify the process. Subsequently, these models are reduced to lower order models to present the models in a form compatible with the intended MPC or PID application. Therefore in the following paragraphs we show that maximizing MPC objective is equivalent to maximizing the information matrix of an ARX model.

Let us consider a multivariable ARX model of the following form (for the ith output)

$$\begin{aligned} A_i(q)y_i(t) &= B_{i1}(q)\Delta u_1(t) + B_{i2}(q)\Delta u_2(t) + \cdots \\ &\quad + B_{im}(q)u_m(t) + e_1(t) \end{aligned}$$

where $A_i(q)$ is a polynomial of order n_i for $i = 1$ to n and B_{ij} is a polynomial of order m_{ij} for $j = 1$ to m. Assume that the noise sequences $e_i(k)$ are independent. The $A_i(q)$ and $B_{ij}(q)$ polynomials are of the form

$$\begin{aligned} A_i(q) &= a_i^{(0)} + a_i^{(1)} q^{-1} + \cdots + a_i^{(n_i)} q^{-n_i} \\ B_{ij}(q) &= b_{ij}^{(0)} + b_{ij}^{(1)} q^{-1} + \cdots + b_{ij}^{(m_{ij})} q^{-m_{ij}} \end{aligned}$$

where $a_i^{(.)}$ and $b_{ij}^{(.)}$ are coefficients of the respective polynomials. These coefficients can be easily estimated from data by solving a simple least squares problem (Ljung, 1999). Note that the ARX model uses differenced input. Let us consider a time period from $t = k$ to $k+N$, where N denotes the number of samples considered. By stacking the outputs during this period and expanding the corresponding ARX models, we can write

$$\mathbf{y}_i = \mathbf{Z}_i \theta_i + \mathbf{e}_i \tag{2}$$

where $\mathbf{y}_i = [y_i(k) \cdots y_i(k+N)]^T$, $\mathbf{Z_i}$ is a corresponding data matrix obtained using the ith ARX equation and θ_i is a vector of corresponding parameters in $A_i(q)$ and $B_{ij}(q)$. Similarly $e_i = [e_i(k) \cdots e_i(k+N)]^T$. Now stacking together similar linear equations for each output, we can create the following set of equations

$$\mathbf{y} = \mathbf{Z}\theta + \mathbf{e}$$

where $\mathbf{y} = [\mathbf{y_1}^T \ \mathbf{y_2}^T \ \cdots \ \mathbf{y_n}^T]^T$, $\mathbf{Z} = diag(\mathbf{Z_1}, \mathbf{Z_2}, \cdots, \mathbf{Z_n})^1$, $\theta = [\theta_1{}^T \ \theta_2{}^T \ \cdots \ \theta_n{}^T]^T$ and $\mathbf{e} = [\mathbf{e_1}^T \ \mathbf{e_2}^T \ \cdots \ \mathbf{e_n}^T]^T$. The least squares solution to the parameter vector, θ is given by

$$\hat{\theta} = (\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{y} \tag{3}$$

and the corresponding variance of the estimated parameters, $\hat{\theta}$ is proportional to (Ljung, 1999)

$$cov(\hat{\theta}) \propto (\mathbf{Z}^T\mathbf{Z})^{-1}. \tag{4}$$

The matrix $\mathbf{F} := (\mathbf{Z}^T\mathbf{Z})$ is also called Fisher information matrix and is inversely proportional to the parameter covariance matrix. Inputs for system identification are often designed by minimizing some function of this parameter covariance matrix. For instance, we can minimize the trace (A - optimal design), eigenvalue (E - optimal), or determinant (D - optimal) of the covariance matrix. The *minimization* of the inverse of the covariance matrix often is a nonlinear and complex function of the inputs and therefore not amenable to convex optimization techniques. Instead, *maximization* of a function of the information matrix tends to be convex problem. The following proposition shows that the maximization of the trace of information matrix (also called T - optimal design) is equivalent to maximization of the MPC objective function under some mild technical constraints.

Proposition 1: The MPC objective $J(k)$ as defined in (1) is proportional to $trace(\mathbf{F})$ for small order ARX models with appropriate choice of input and output weights, and prediction horizons.

An outline of the proof of this proposition is presented in the appendix. This result essentially states that the input designed by maximizing $J(k)$ subject to process constraints will also maximize the trace of the information matrix. Therefore the proposed approach is T-optimal.

The use of the MPC objective function ensures that outputs are given adequate importance during the test sequence design process. Additionally due to the process conditioning, if it is necessary to move one or more inputs in a correlated fashion, this will automatically result from the above optimization step. The input and output prediction horizons are assigned the same value to ensure consistency with the identification step.

The use of the prediction horizon idea ensures that the designed test sequence will be controller relevant, i.e., the frequency ranges that are of interest to the MPC are given more attention indirectly. The presence of the input/output constraints in the test design formulation ensures that the information content as defined by the objective function is maximized to the extent allowed by the process constraints at every sampling instant. The receding horizon approach, like the MPC calculation, allows for the presence of unmeasured disturbances. This leads to increased probability of constraint satisfaction during the plant test.

A desirable consequence of retaining the MPC objective function structure in the test sequence calculation is that

[1] $diag(.)$ is used to denote a matrix obtained by stacking its arguments along the diagonal

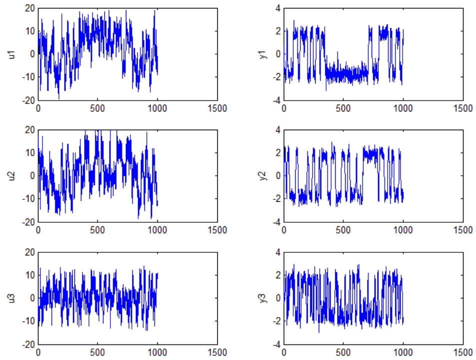

Fig. 1. Closed loop data from the MPC operation.

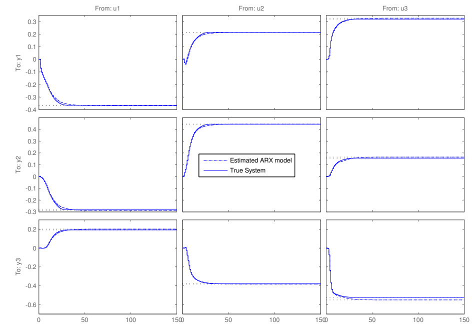

Fig. 2. Comparison of the estimated model (dash-dotted) and actual (solid) step responses from the experiment data.

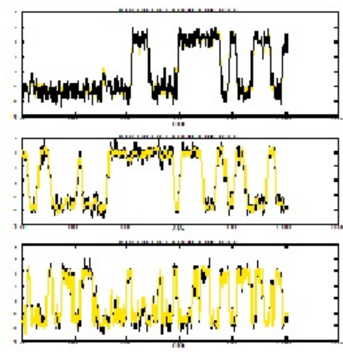

Fig. 3. Model predictions from estimated model.

in theory any MPC application is inherently capable of switching into a testing mode without the need to use additional software. Simply changing the objective of the optimization problem, from minimization to maximization, will cause the application to go into a testing mode. Once sufficient data has been collected, the application can return to control mode by switching back the optimization mode to minimize. In the following section we present results of using this formulation on two representative processes - one which is well conditioned and the second an ill-conditioned one

IV. Simulation Examples

A. Example 1 - Well Conditioned Process

A 3×3 process was used for this example. The model is based on a crude distillation column with the outputs being purities and inputs being flows and temperatures. A MPC was designed and implemented on this simulated process with the following tuning parameters and constraints. No model plant mismatch was considered for this case. The following parameters were used in simulations: $P = M = 5$, $\mathbf{u}_L = -20$, $\mathbf{u}_H = 20$, $\mathbf{y}_L = -2$, $\mathbf{y}_H = 2$, $\Gamma = \Lambda = -I$. White noise of variance 0.1 was added to each output.

The controller generated data looked very similar to a standard closed loop experiment. Figure 1 shows the data for the three inputs and outputs generated by the controller alone without the addition of any external persistent excitation.

A higher order ARX model was used to identify models from the above data. The above data was divided into equal estimation and validation data sets. Figure 2 shows a comparison of the identified models with the true system.

Model validation was done through monitoring of the residuals, quality of the predictions on the validation data set and the confidence intervals on the identified parameters. Figures 3 shows the fit between data and the model for the three outputs.

B. Example 2 - Ill Conditioned Process

The proposed approach is demonstrated here on a high purity distillation column simulation. By its very nature, high purity distillation columns tend to be ill-conditioned from a systems point of view. As such, conventional perturbation methods do not yield expected results for model identification purposes. Moving the process inputs independently leads to inaccurate identification of the weak gain directions - (Cooley and Lee, 2001; Koung and MacGregor, 1994). To design optimal perturbation for these types of systems one has to adopt one of the following approaches: (1) Use a priori knowledge to move inputs in a correlated fashion, the degree of correlation being dependent on the process model, which is often unknown at the identification stage, and (2) conduct the experiment under closed loop conditions and rely on the controller to provide the necessary correlation to identify the strong and weak gain directions accurately. The process model along with its singular values is shown in Figure 4. This is a 2×2 process with the following transfer function,

$$G(s) = \begin{bmatrix} \frac{0.878}{\tau s+1} & -\frac{0.864}{\tau s+1} \\ \frac{1.0819}{\tau s+1} & -\frac{1.0958}{\tau s+1} \end{bmatrix} \tag{5}$$

where $\tau = 194$, and

$$W = \begin{bmatrix} -0.6246 & -0.7809 \\ -0.7809 & 0.6246 \end{bmatrix} \quad V = \begin{bmatrix} -0.7066 & 0.7077 \\ -0.7077 & -0.7066 \end{bmatrix}$$

$$\Sigma = \frac{1}{\sqrt{\tau^2\omega^2+1}} \begin{bmatrix} 1.9721 & 0 \\ 0 & 0.0139 \end{bmatrix}$$

$$\text{cond}(G) = \frac{\sigma_1(\omega)}{\sigma_2(\omega)} = 141.732$$

where W and V are the unitary matrices of singular value decomposition, Σ is the matrix of singular values and ω

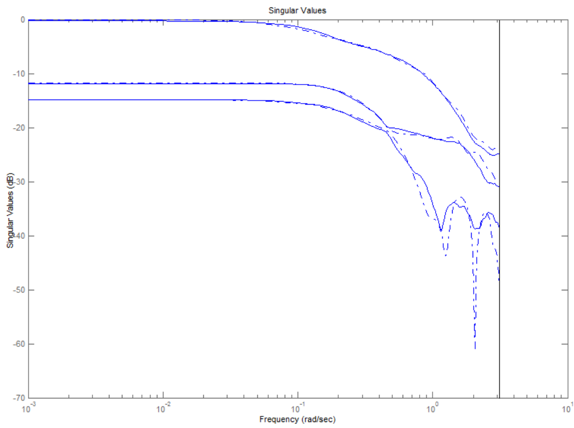

Fig. 4. Comparison of the singular values - estimated model (dash-dotted) with the true system (solid).

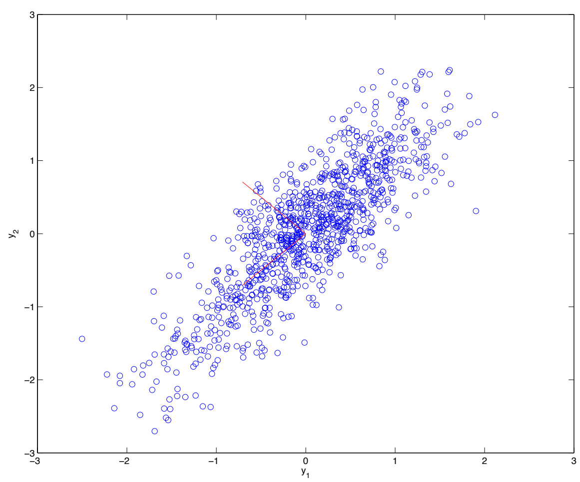

Fig. 5. Open loop excitation shown in the output space (red straight lines denote the directions of the two column vectors in V).

denotes the frequency. As can be seen from the singular values, the system is poorly conditioned. Conventional open loop step testing approaches can often lead to models which estimate only the strong gain direction. The input space of an open loop PRBS type test is shown in Figure 5. The receding horizon formulation was next used to carry out a closed loop experiment for the system. The following tuning parameters and constraints were used during the experiment: $P = M = 10$, $\mathbf{u}_L = -200$, $\mathbf{u}_H = 200$, $\mathbf{y}_L = -2$, $\mathbf{y}_H = 2$, $\Gamma = -0.01I$, $\Lambda = -I$. White noise of variance 0.1 was added to each output with power of 0.1. The data from the closed loop step test is shown in Figure 6. The data shows significant excitation along the weak gain direction as opposed to the open loop experiment in the Figure 5 where the strong direction was dominant. The models estimated from the closed loop experiment are compared in Figure 7.

It is even more instructive to look at the gains estimated

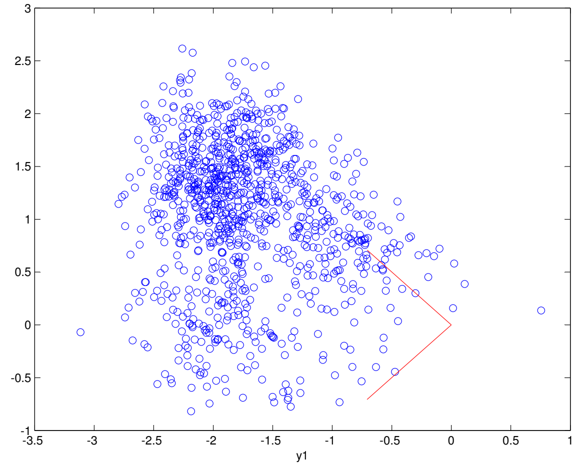

Fig. 6. Output space with the receding horizon approach to experiment design (red straight lines denote the directions of the two column vectors in V).

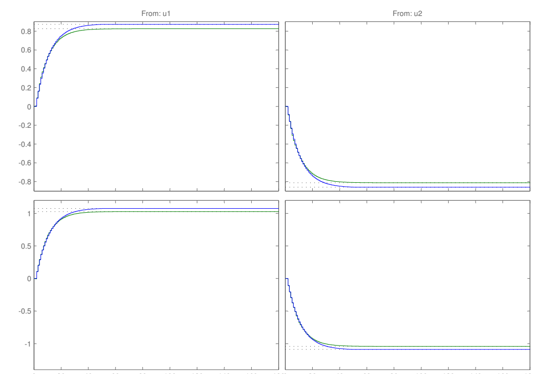

Fig. 7. Comparison of the estimated model with the true system (green - model, blue - true system).

from the two different approaches and their inverse,

$$K = \begin{bmatrix} 0.8724 & -0.8585 \\ 1.0751 & -1.0890 \end{bmatrix} \quad K^{-1} = \begin{bmatrix} 40.20 & -31.69 \\ 39.68 & -32.20 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 0.8253 & -0.8114 \\ 1.0279 & -1.0417 \end{bmatrix} \quad K_1^{-1} = \begin{bmatrix} 40.59 & -31.62 \\ 40.05 & -32.16 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 0.6968 & -0.7424 \\ 0.8940 & -0.9269 \end{bmatrix} \quad K_2^{-1} = \begin{bmatrix} -38.10 & -30.51 \\ -36.75 & 28.35 \end{bmatrix}$$

where K is the gain of the true system, K_1 is the gain estimated from the closed loop receding horizon experiment data and K_2 is the gain estimated from the open loop data. Note how different the inverse is for the open loop based model from the true inverse. A controller based on the second model can end up making moves in the wrong direction. This is a direct result of not estimating both the gain directions accurately. Inaccurate estimation of gain directions can have significant impact on the controller performance, especially for the optimization or linear programming layer of the controller.

V. CONCLUSIONS

The proposed approach has many advantages: (1) ability to handle constraints in a predictive way during the step test,

(2) ability to deal with ill-conditioned processes, (3) ability to account for unmeasured disturbances and mitigating their impact on constraint violations during the step test and (4) ability to switch between control and step testing merely by switching the objectives of the MPC.

On the other hand compared to traditional experiment design approaches, it is not clear how the proposed method will address excitation over different frequency ranges. It is expected that the choice of the prediction horizon will influence the frequency content of the implemented signal. This is a topic that needs further research. One of the advantages of a conventional step testing approach is the transparency of the move plan and complete control over the implemented move sequence. In the case of the proposed approach, an automated move plan is generated, the first move is implemented and the rest discarded. The generated move plan is a function of the: (1) input/output weightings, (2) input/output constraints, (3) prediction horizon, (4) current model plant mismatch, and (5) unmeasured disturbances. More work is needed to establish the relationships between these parameters and the calculated move sequence.

APPENDIX

Only a brief outline of the proof of proposition 1 is presented in this section. We can easily show the following,

$$trace(\mathbf{F}) = trace(\mathbf{Z}_1^T\mathbf{Z}_1) + \cdots + trace(\mathbf{Z}_n^T\mathbf{Z}_n) \quad (6)$$

and

$$\begin{aligned} trace(\mathbf{Z}_i^T\mathbf{Z}_i) &= \sum_{t=k}^{k+N}\sum_{s=1}^{n_i} y_i^2(t-s) + \sum_{j=1}^{m}\sum_{t=k}^{k+N}\sum_{s=1}^{m_{ij}} \Delta u_j^2(t-s) \\ &= \sum_{l=k-n_i}^{k+N-1} \alpha_i(l) y_i^2(l) + \sum_{j=1}^{m}\sum_{l=k-m_{ij}}^{k+N-1} \beta_{ij}(l)\Delta u_j^2(l) \end{aligned} \quad (7)$$

where

$$\alpha_i(l) = \begin{cases} l+1-k+n_i & (k-n_i) \le l \le (k-1) \\ n_i & (k-1) \le l \le (k+N-n_i) \\ k+N-l & (k+N-n_i) \le l \le k+N-1 \\ 0 & \text{Otherwise} \end{cases}$$

and

$$\beta_{ij}(l) = \begin{cases} l+1-k+m_{ij} & (k-m_{ij}) \le l \le (k-1) \\ m_{ij} & (k-1) \le l \le (k+N-m_{ij}) \\ k+N-l & (k+N-m_{ij}) \le l \le k+N-1 \\ 0 & \text{Otherwise} \end{cases}$$

Now defining $\boldsymbol{\alpha}(l) = diag(\alpha_1(l), \alpha_2(l), \cdots, \alpha_n(l)$, and $\boldsymbol{\beta}(l)$ a matrix with elements $\beta_{ij}(l)$. Therefore

$$\begin{aligned} trace(F) = &\sum_{l=k-\max_i(n_i)}^{k+N-1} \mathbf{y}(l)^T\boldsymbol{\alpha}(l)\mathbf{y}(l) + \\ &\sum_{l=k-\max_{i,j}(m_{ij})}^{k+N-1} \Delta\mathbf{u}(l)^T\boldsymbol{\beta}(l)\Delta\mathbf{u}(l) \end{aligned} \quad (8)$$

Now for $N >> \max_i(n_i)$ and $\max_{i,j} m_{ij}$,

$$\begin{aligned} trace(F) \approx &\sum_{l=k-1}^{k+N-\max_i(n_i)} \mathbf{y}(l)^T\boldsymbol{\alpha}(l)\mathbf{y}(l) + \\ &\sum_{l=k-1}^{k+N-\max_{i,j}(m_{ij})} \Delta\mathbf{u}(l)^T\boldsymbol{\beta}(l)\Delta\mathbf{u}(l) \end{aligned} \quad (9)$$

Noting that $\boldsymbol{\alpha}(l)$ and $\boldsymbol{\beta}(l)$ are constant in the above equation, we have an MPC objective function with appropriate input and output weights, and appropriate prediction horizons.

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