

# A TWO-STAGE STOCHASTIC PROGRAMMING MODEL FOR PLANNING OF OPTIMAL HYDROGEN SUPPLY NETWORKS UNDER UNCERTAINTY IN SELLING PRICE AND SUPPLY COST

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## *Abstract*

Many papers have studied the cost minimizing design of the hydrogen supply network that consists of the various activities such as production, storage and transportation. Selling price and supply cost of hydrogen are difficult to estimate precisely in the future. In this study, a two-stage stochastic programming model is developed for planning of hydrogen supply networks under uncertainty in price and supply cost. The proposed model can help to determine where and how much hydrogen to be produced, stored and transported for the purpose of maximizing the expected total profit while handling uncertain hydrogen selling price and supply cost. The capability of the proposed model to provide correct decisions despite the uncertainty is tested by applying into a real case study based on Korea in 2030. The results include not only the investment strategy for the optimal hydrogen supply network configuration but also the effect of the uncertain parameters.

## *Keywords*

Hydrogen supply network, uncertainty, hydrogen selling price, supply cost, stochastic programming.

## **1. Introduction**

The CO<sub>2</sub> is known as a primary factor of climate change. Global attention on the need to reduce CO<sub>2</sub> emissions has been given to developing a sustainable energy and transport model. Recently, hydrogen has gradually drawn increasing attention as a promising alternative energy carrier, since it is environmental-friendly and has a wide range of applications. In order to accelerate hydrogen economy, constructing a hydrogen supply network model to address the economic supply of hydrogen is thus an eminent issue. Past research efforts more directed toward modeling hydrogen supply networks to evaluate the possibility of the hydrogen economy. For example, there have been proposed some deterministic mathematical models to design hydrogen supply networks (Almansoori and Shah 2006), (Han, Ryu et al. 2011). These studies have addressed stochastic models for the hydrogen supply

networks under uncertainties such as hydrogen demand and supply cost (Sabio, Gadalla et al. 2010), (Kim, Lee et al. 2008). However, these studies did not address the variation and uncertainty of hydrogen selling price and supply cost. Hence, to obtain more realistic results, hydrogen supply networks model considers more various impact uncertainties such as selling price and supply cost. Therefore, this study aims to address the modeling of the hydrogen supply networks under uncertainty in selling prices and supply costs. The proposed model is formulated as a two-stage stochastic programming based on an uncertain scenario approach. This approach compares a stochastic model with a deterministic model to assess the variation of the selling price and supply cost for hydrogen supply network. This study then uses the proposed models to examine a real case study based on Korea in 2030.

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## 2. Problem statement

The design problem addressed in this work has the objective of determining the configuration of the hydrogen supply networks with the goal of maximizing the expected total profit of hydrogen supply network. The hydrogen supply network model includes three main components such as production, storage and transportation. The decision-making problem of the model is to determine where and how to produce, store and transport hydrogen under given conditions, which include hydrogen local demand, capacity limitations of hydrogen supply technologies and uncertain parameters (i.e. selling price and supply cost) in order to maximize the expected total profit of the hydrogen supply network.

## 3. Mathematical model

The model presented in this work is inspired by previous formulations (Han, Ryu et al. 2011). Specially, the model considers the uncertainty of the coefficients (e.g. selling prices, production operating costs, storage operating costs and transportation operating cost) of the objective function via a multi-scenario stochastic programming approach.

### 3.1 Demand constraints

In the hydrogen supply network, each region has its own deterministic demand. This demand must be fulfilled by production plants established within a particular region:

$$\sum_g D_g^T Y_{p,g',g} \leq \sum_i PCap_{p,i}^{\max} NOP_{p,i,g'} \quad \forall p, g' \quad (1)$$

Assuming a steady-state operation, a total mass balance on a region is written as follows:

$$\sum_i \sum_p P_{p,i,g} = \sum_i \sum_l \sum_{g'} (Q_{i,l,g,g'} - Q_{i,l,g',g}) + D_g^T \quad \forall g \quad (2)$$

### 3.2 Production constraints

The production rate of plant type  $p$  established in region  $g$  is constrained by the minimum and maximum production limits and the number of production facilities associated with plant type  $p$ :

$$PCap_{p,i}^{\min} NOP_{p,i,g} \leq P_{p,i,g} \leq PCap_{p,i}^{\max} NOP_{p,i,g} \quad \forall p, i, g \quad (3)$$

### 3.3 Transportation constraints

The flow of a hydrogen form  $i$  from a region  $g$  to a different region  $g'$  will exist if the transportation mode  $l$  has been established. And there is a minimum and maximum flow rate of hydrogen:

$$Q_{i,l}^{\min} X_{i,l,g,g'} \leq Q_{i,l,g,g'} \leq Q_{i,l}^{\max} X_{i,l,g,g'} \quad \forall i, l, g, g': g \neq g' \quad (4)$$

### 3.4 Storage constraints

The average amount of hydrogen stored in a region  $g$  is constrained by the number of storage facilities. And the total average amount of hydrogen will be bound between the minimum and maximum capacities of each facility of type  $s$ :

$$SCca_{s,i}^{\min} NOS_{s,i,g} \leq S_{s,i,g} \leq SCap_{s,i}^{\max} NOS_{s,i,g} \quad \forall i, s, g \quad (5)$$

### 3.5 Objective function

The model presented in this work aims to maximize the expected total net profits (eTNP).

Max eTNP

The objective function consists of total daily cost and total selling profit for demand considering uncertainty of supply cost and selling price for hydrogen.

$$eTNP = \sum_w prob_w \times (TSP_w - TDC_w) \quad (6)$$

Total selling profit ( $TSP_w$ ) in scenario  $w$  is calculated by multiplying the total demand of hydrogen from each region  $g$  and the price for hydrogen.

$$TSP_w = \sum_e \sum_g pTD_{e,g} \times pPTD_{e,g,w} \quad \forall w \quad (7)$$

Total daily cost ( $TDC_w$ ) in scenario  $w$  is calculated by summing the capital costs ( $FCC_e$ ,  $TCC_e$ ) as well as operating costs ( $FOC_{e,w}$ ,  $TOC_{e,w}$ ).

$$TDC_w = \sum_e (FCC_e + TCC_e) + \sum_e (FOC_{e,w} + TOC_{e,w}) \quad \forall w \quad (8)$$

The detailed explanations for the objective and its constraints were described by (Han, Ryu et al. 2011).

## 4. Case study

The case study in previous paper (Han, Ryu et al. 2011) is optimizing the ideal configuration to supply variable inventory of hydrogen within Korea. That model was applied to infrastructure of hydrogen of Korea in 2030. Hydrogen is assumed to be produced from four different plants, namely steam methane reforming (SMR), coal gasification (CG), biomass gasification (BMG) and electrolysis (ELE). In this study, the inventory period was fixed and two case studies are examined according to supply cost and selling price considering uncertainty. Using the previous model in the paper (Han, Ryu et al. 2011), we generated 50 uncertain parameters using normal distribution of the operating cost of hydrogen supply network configuration and selling price of hydrogen. In the

results, we obtained the optimal supply network of hydrogen under uncertainty in selling price and supply cost. In the study, two case studies are examined according to model types such as deterministic and stochastic model.

### 5. Results and Discussion

The stochastic problems were calculated using the CPLEX 9.0 solver of GAMS.

Figure 1 can compare total net profits of the deterministic model with the stochastic model for hydrogen supply network. Most of this decreased profit of case 2 is derived from the total daily cost, as depicted in Figure 2 and 3. It is explained by the fact that the uncertain impact of cost is larger than one of price.

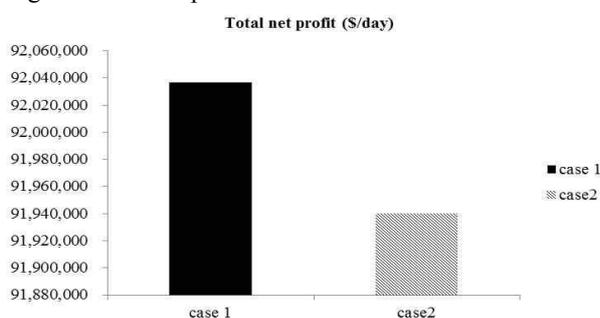


Figure 1. Comparison of the total net profit of the deterministic model (case 1) and stochastic model (case 2)

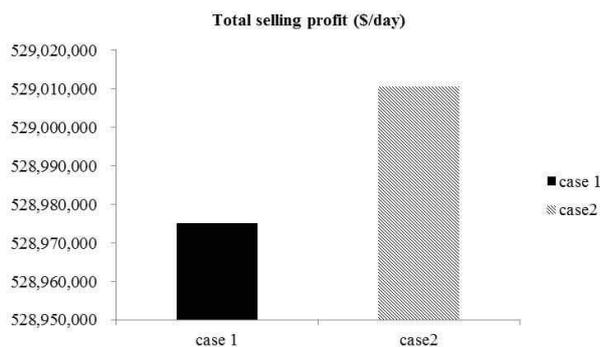


Figure 2. Comparison of the total selling profit of the deterministic model (case 1) and stochastic model (case 2)

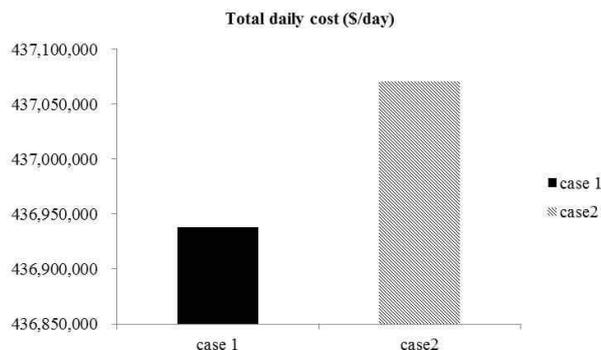


Figure 3. Comparison of the total daily cost of the deterministic model (case 1) and stochastic model (case 2)

Additionally, the design configurations of the deterministic model and stochastic model for hydrogen supply network are also compared (Table 1). The number of facilities, the number of transportation modes varied among cases, according to the variation of selling price and operating cost. The facility operating cost is insensitive to the uncertainty, while the transportation operating cost is very sensitive to it.

Table 1. The optimal solution of hydrogen supply network design

Case	Case 1 (D)	Case 2 (S)
Number of production plants	165 (SMR.LH2)	165 (SMR.LH2)
Number of storage facilities	1 (SMR.CH2)	1 (SMR.CH2)
Number of transport modes	2204 (LH2s.LH2)	2206 (LH2s.LH2)
	3 (CH2s.CH2)	3 (CH2s.CH2)
	1007 (tanker truck.LH2)	1023 (tanker truck.LH2)
	1 (pipeline.CH2)	1 (pipeline.CH2)

\*Model : D (Deterministic), S (Stochastic)

\*\*Product type : LH2(liquid hydrogen), CH2(compressed-gas hydrogen)

\*\*\*Technology type : SMR (steam methane reforming), LH2s (liquid hydrogen storage), CH2s (compressed-gas hydrogen storage)

### 6. Conclusions

This work has introduced a mathematical model for hydrogen supply network under uncertainty in selling prices and supply costs. The problem is formulated as a two- stage stochastic programming model. We also studied

the effect of selling price and supply cost uncertainty by comparing the solutions of the deterministic and the stochastic model. Simulation results have shown that the comparison of the absolute values of each case has provided information on elasticity trends for uncertain price and supply cost.

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## APPENDIX

### Nomenclature

#### Indices

- $g$  regions  
 $g'$  regions such that  $g' \neq g$   
 $p$  plant type  
 $i$  product physical form  
 $l$  type of transportation modes  
 $e$  product form of electricity  
 $w$  Scenarios

#### Parameters

- $D_g^T$  total demand for product H<sub>2</sub> in region  $g$ , kg d<sup>-1</sup>  
 $PCap_{p,i}^{max}$  maximum H<sub>2</sub> production capacity of facility type  $p$  for product physical form  $i$ , kg d<sup>-1</sup>  
 $PCap_{p,i}^{min}$  minimum H<sub>2</sub> production capacity of facility type  $p$  for product physical form  $i$ , kg d<sup>-1</sup>  
 $NOP_{p,i,g}$  number of production facilities of type  $p$  for product physical form  $i$ , in region  $g$ , day  
 $NOS_{s,i,g}$  number of storage facilities of type  $s$  for product physical form  $i$ , in region  $g$ , day

- $Q_{i,l}^{min}$  minimum flowrate of product physical form  $i$  by transportation mode  $l$ , kg d<sup>-1</sup>  
 $Q_{i,l}^{max}$  maximum flowrate of product physical form  $i$  by transportation mode  $l$ , kg d<sup>-1</sup>  
 $SCap_{s,i}^{min}$  minimum H<sub>2</sub> storage capacity of facility type  $s$  for product physical form  $i$ , kg d<sup>-1</sup>  
 $SCap_{s,i}^{max}$  maximum H<sub>2</sub> storage capacity of facility type  $s$  for product physical form  $i$ , kg d<sup>-1</sup>  
 $pTD_{e,g}$  total demand of hydrogen in region  $g$ , kg d<sup>-1</sup>  
 $pPTD_{e,g,w}$  total price of hydrogen in scenario  $w$ , \$ d<sup>-1</sup>  
 $prob_w$  Probability of occurrence of scenario  $w$

#### Binary variables

- $X_{i,l,g,g'}$  has value of 1 if product form  $i$  of product  $e$  is to be transported from region  $g$  to  $g'$  at onshore by transportation mode  $l$  otherwise 0

#### Positive variables

- $Y_{p,g,g'}$  fraction of demand in region  $g$  served from plant type  $p$  in region  $g'$   
 $P_{p,i,g}$  Amount of physical product form  $i$  produced by plant type  $p$  in region  $g$ , kg d<sup>-1</sup>  
 $S_{s,i,g}$  Amount of physical product form  $i$  stored by facility type  $s$  in region  $g$ , kg d<sup>-1</sup>  
 $Q_{i,l,g,g'}$  flow amount of product form  $i$  of product  $e$  by transportation mode  $l$  between regions  $g$  and  $g'$ , kg d<sup>-1</sup>

#### variables

- $eTNP$  the expected total net profit, \$ d<sup>-1</sup>  
 $TSP_w$  total selling profit in scenario  $w$ , \$ d<sup>-1</sup>  
 $TDC_w$  total daily cost in scenario  $w$ , \$ d<sup>-1</sup>  
 $FCC_e$  facility fixed cost in product form  $e$ , \$ d<sup>-1</sup>  
 $TCC_e$  transportation fixed cost in product form  $e$ , \$ d<sup>-1</sup>  
 $FOC_{e,w}$  facility variable cost in scenario  $w$ , \$ d<sup>-1</sup>  
 $TOC_{e,w}$  transportation variable cost in scenario  $w$ , \$ d<sup>-1</sup>