Feedback Control of Stable, Non-minimum-phase, Nonlinear Processes

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Abstract

A nonlinear control law is presented for stable, multiple-input, multiple-output processes, whether their delay-free part is minimum- or non-minimum-phase. It is derived by exploiting the connections between continuous-time model-predictive control and input-output linearization. The differential-geometric, control law induces a linear closed-loop response approximately. It has a few tunable parameters (one for each controlled output), and thus, is easily tuned.

Keywords

Nonlinear control, State feedback design, Input-output linearization, Non-minimum-phase systems, Model-based control

During the past decade, the problem of analytical (non-model-predictive) control of non-minimum-phase, nonlinear processes without deadtime has received considerable attention, leading to several solutions (Doyle III et al., 1996; Kravaris et al., 1998; Morari and Zafiriou, 1989; Isidori and Byrnes, 1990; Devasia et al., 1996; Isidori and Astolfi, 1992; van der Schaft, 1992; Isidori, 1995). These methods are applicable to a very small class of nonlinear processes, their application requires solving partial differential equations, or are not applicable to the class of general, nonlinear, stable, multivariable processes with time delays. More on advantages and disadvantages of these methods can be found in Kanter et al. (2000).

In the framework of model-predictive control, it is well known that large prediction horizons are needed for non-minimum-phase processes. For example, Hernández and Arkun (1992) developed a p-inverse (long prediction horizon) control law for nonlinear, single-input single-output (SISO), non-minimum-phase, discrete-time processes with arbitrary order and relative order.

An objective of this work is to derive a nonlinear control law for MIMO, stable, nonlinear, continuous-time processes, whether their delay-free part is non-minimum-phase or minimum-phase. This builds upon the single-input single-output controllers presented in Kanter et al. (2001), leading to the derivation of a continuous-time, differential-geometric control law that is approximately input-output linearizing (Allgöwer and Doyle III, 1998).

The paper is organized as follows. The scope of the study and some mathematical preliminaries are given in Section 2. Section 3 presents a method of nonlinear feedforward/state-feedback design. A nonlinear feedback control law with integral action is given in Section 4.

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Scope and Mathematical Preliminaries

Consider the class of MIMO, nonlinear processes of the form:

$$\left.\begin{array}{rcl}\dfrac{d\bar{x}(t)}{dt} &=& f[\bar{x}(t),u(t)], \quad \bar{x}(0)=\bar{x}_0 \\ \bar{y}_i(t) &=& h_i[\bar{x}(t-\theta_i)]+d_i, \quad i=1,\cdots,m\end{array}\right\} \quad (1)$$

where $\bar{x}=[\bar{x}_1\cdots\bar{x}_n]^T \in X$ is the vector of the *process* state variables, $u=[u_1\cdots u_m]^T \in U$ is the vector of manipulated inputs, $\bar{y}=[\bar{y}_1\cdots\bar{y}_m]^T$ is the vector of *process* outputs, $\theta_1,\cdots,\theta_m$ are the measurement delays, $d=[d_1\cdots d_m]^T \in D$ is the vector of constant unmeasured disturbances, $f(.,.)$ is a smooth vector field on $X\times U$, and $h_1(.),\cdots,h_m(.)$ are smooth functions on X. Here $X\subset\Re^n$ is a connected open set that includes $\bar{x}_{ss}$ and $\bar{x}_0$, $U\subset\Re^m$ is a connected open set that includes u_{ss}, and $D\subset\Re^m$ is a connected set, where $(\bar{x}_{ss},u_{ss})$ denotes the nominal steady-state (equilibrium) pair of the process; that is, $f[\bar{x}_{ss},u_{ss}]=0$.

The system:

$$\left.\begin{array}{rcl}\dfrac{d\bar{x}(t)}{dt} &=& f[\bar{x}(t),u(t)], \quad \bar{x}(0)=\bar{x}_0 \\ \bar{y}_i^*(t) &=& h_i[\bar{x}(t)]+d_i, \quad i=1,\cdots,m\end{array}\right\} \quad (2)$$

is referred to as the delay-free part of the process. The relative orders (degrees) of the controlled outputs $y_1,\cdots,y_m$ with respect to u are denoted by $r_1,\cdots,r_m$, respectively, where r_i is the smallest integer for which $\frac{d^{r_i}\bar{y}_i^*}{dt^{r_i}}$ explicitly depends on u for every $x\in X$ and every $u\in U$. The relative order (degree) of a controlled output y_i with respect to a manipulated input u_j is denoted by r_{ij} $(i=1,\cdots,m,\ j=1,\cdots,m)$, where r_{ij} is the smallest integer for which $\frac{d^{r_{ij}}\bar{y}_i^*}{dt^{r_{ij}}}$ explicitly depends on u_j for every $x\in X$ and every $u\in U$. The set-point and the set of acceptable set-point values are denoted by y_{sp} and Y, respectively, where $Y\subset\Re^m$ is a connected set.

The following assumptions are made:

(A1) For every $y_{sp}\in Y$ and every $d\in D$, there exists an equilibrium pair $(\bar{x}_{ss},u_{ss})\in X\times U$ that satisfies $y_{sp}-d=h(\bar{x}_{ss})$ and $f[\bar{x}_{ss},u_{ss}]=0$.

(A2) The nominal steady-state (equilibrium) pair of the process, $(\bar{x}_{ss}, u_{ss})$, is hyperbolically stable; that is, all eigenvalues of the open-loop process evaluated at $(\bar{x}_{ss}, u_{ss})$ have negative real parts.

(A3) For a process in the form of (1), a model in the following form is available:

$$\left.\begin{array}{rcll} \dfrac{dx(t)}{dt} & = & f[x(t), u(t)], & x(0) = x_0 \\ y_i(t) & = & h_i[x(t-\theta_i)], & i = 1, \cdots, m \end{array}\right\} \quad (3)$$

where $x = [x_1 \cdots x_n]^T \in X$ is the vector of *model* state variables, and $y = [y_1, \cdots, y_m]^T$ is the vector of *model* outputs.

(A4) The relative orders, $r_1, \cdots, r_m$, are finite.

The following notation is used:

$$\begin{array}{rcl} h_i^1(x) & \doteq & \dfrac{dy_i^*}{dt} \\ & \vdots & \\ h_i^{r_i-1}(x) & \doteq & \dfrac{d^{r_i-1}y_i^*}{dt^{r_i-1}} \\ h_i^{r_i}(x,u) & \doteq & \dfrac{d^{r_i}y_i^*}{dt^{r_i}} \\ h_i^{r_i+1}(x, u^{(0)}, u^{(1)}) & \doteq & \dfrac{d^{r_i+1}y_i^*}{dt^{r_i+1}} \\ & \vdots & \\ h_i^{p_i}(x, u^{(0)}, u^{(1)}, \ldots, u^{(p_i-r_i)}) & \doteq & \dfrac{d^{p_i}y_i^*}{dt^{p_i}} \end{array} \quad (4)$$

where $p_i \geq r_i$ and $u^{(\ell)} = d^\ell u/dt^\ell$.

Input-Output Linearization

For a process in the form of Equation 1, responses of the closed-loop process outputs are requested, having the linear form:

$$\begin{bmatrix} (\epsilon_1 D + 1)^{r_1} \bar{y}_1(t+\theta_1) \\ \vdots \\ (\epsilon_m D + 1)^{r_m} \bar{y}_m(t+\theta_m) \end{bmatrix} = y_{sp}, \quad (5)$$

where D is the differential operator (i.e., $D \doteq \frac{d}{dt}$), and $\epsilon_1, \cdots, \epsilon_m$ are positive, constant, adjustable parameters that set the speed of the response of the closed-loop process outputs $\bar{y}_1, \cdots, \bar{y}_m$ respectively. Substituting for the process output derivatives from the model in Equation 5, one obtains:

$$\begin{bmatrix} h_1(\bar{x}) + \binom{r_1}{1}\epsilon_1 h_1^1(\bar{x}) + \cdots + \binom{r_1}{r_1}\epsilon_1^{r_1} h_1^{r_1}(\bar{x}, u) \\ \vdots \\ h_m(\bar{x}) + \binom{r_m}{1}\epsilon_m h_m^1(\bar{x}) + \cdots + \binom{r_m}{r_m}\epsilon_m^{r_m} h_m^{r_m}(\bar{x}, u) \end{bmatrix} \quad (6)$$

$$= y_{sp} - d$$

where

$$\binom{a}{b} \doteq \frac{a!}{b!(a-b)!}.$$

Under the assumption of the nonsingularity of the characteristic (decoupling) matrix:

$$\frac{\partial}{\partial u}\begin{bmatrix} h_1^{r_1}(\bar{x}, u) \\ \vdots \\ h_m^{r_m}(\bar{x}, u) \end{bmatrix}$$

on $X \times U$, Equation 6 represents a feedforward/state feedback. When the process delay-free part exhibits nonminimum-phase behavior, the input-output behavior of the closed-loop system under the feedforward/state feedback of Equation 6 is governed by the linear response of Equation 5, but the internal dynamics (unobservable modes) of the closed-loop system are unstable.

The dynamic feedforward/state feedback

$$\Phi_p(\bar{x}, u, \mathcal{U}) = y_{sp} - d \quad (7)$$

where the ith component of $\Phi_p(\bar{x}, u, \mathcal{U})$:

$$\begin{array}{l} [\Phi_p(\bar{x}, u, \mathcal{U})]_i \doteq h_i(\bar{x}) + \binom{p_i}{1}\epsilon_i h_i^1(\bar{x}) + \cdots + \\ \quad \binom{p_i}{r_i-1}\epsilon_i^{r_i-1} h_i^{r_i-1}(\bar{x}) \\ + \binom{p_i}{r_i}\epsilon_i^{r_i} h_i^{r_i}(\bar{x}, u) + \binom{p_i}{r_i+1}\epsilon_i^{r_i+1} h_i^{r_i+1}(\bar{x}, u, u^{(1)}) \\ \quad + \cdots + \binom{p_i}{p_i}\epsilon_i^{p_i} h_i^{p_i}(\bar{x}, u, u^{(1)}, \cdots, u^{(p_i-r_i)}) \end{array} \quad (8)$$

$$\mathcal{U} = \left[u^{(1)} \cdots u^{(\max[p_1-r_1, \cdots, p_m-r_m])}\right]^T, \quad p = [p_1 \cdots p_m]$$

with

$$\partial\Phi_p/\partial\left[u_1^{\max(p_1-r_{11}, \cdots, p_m-r_{m1})}, \cdots, u_m^{\max(p_1-r_{1m}, \cdots, p_m-r_{mm})}\right]^T$$

nonsingular, $\forall x \in X$, also induces a linear, closed-loop, output response of the form:

$$\begin{bmatrix} (\epsilon_1 D + 1)^{p_1} \bar{y}_1(t+\theta_1) \\ \vdots \\ (\epsilon_m D + 1)^{p_m} \bar{y}_m(t+\theta_m) \end{bmatrix} = y_{sp}, \quad (9)$$

Similarly, the dynamic feedforward/state feedback of Equation 7 cannot ensure asymptotic stability of the closed-loop system when the delay-free part of the process exhibits non-minimum-phase behavior. Consequently, an objective of this study is to design a feedback control law that ensures asymptotic stability of the closed-loop system, whether the delay-free part of the process is minimum- or non-minimum-phase.

Nonlinear Feedforward/State Feedback Design

Assume that for every $x \in X$, every $d \in D$, and every $y_{sp} \in Y$, the algebraic equation:

$$\phi_p(\bar{x}, u) = y_{sp} - d \tag{10}$$

where

$$\phi_p(\bar{x}, u) \doteq \Phi_p(\bar{x}, u, 0) \tag{11}$$

has a real root inside U for u, and that for every $\bar{x} \in X$ and every $u \in U$, $\frac{\partial \phi_p(\bar{x},u)}{\partial u}$ is nonsingular. The corresponding feedforward/state feedback that satisfies Equation 10 is denoted by

$$u = \Psi_p(\bar{x}, y_{sp} - d) \tag{12}$$

Note that the preceding feedforward/state feedback was obtained by setting all the time derivatives of u in Equation 7 to zero.

Theorem 1 *For a process in the form of Equation 1, the closed-loop system under the feedforward/state feedback of Equation 12 is asymptotically stable, if the following conditions hold:*

(a) The nominal equilibrium pair of the process, $(\bar{x}_{ss}, u_{ss})$, corresponding to y_{sp} and d, is hyperbolically stable.

(b) The tunable parameters $p_1, \cdots, p_m$ are chosen to be sufficiently large.

(c) The tunable parameters $\epsilon_1, \cdots, \epsilon_m$ are chosen such that for every $\ell = 1, \cdots, m$, all eigenvalues of $\epsilon_\ell J_{ol} \doteq \epsilon_\ell [\frac{\partial}{\partial x} f(x,u)]_{(\bar{x}_{ss}, u_{ss})}$ lie inside the unit circle centered at $(-1, 0j)$.

Furthermore, as $p_1, \cdots, p_m \longrightarrow \infty$, the state feedback places the n eigenvalues of the Jacobian of the closed-loop system evaluated at the nominal equilibrium pair at the n eigenvalues of the Jacobian of the open-loop process evaluated at the nominal equilibrium pair.

The proof can be found elsewhere (Kanter et al., 2000).

Condition (c) states that $\epsilon_1, \cdots, \epsilon_m$ should be chosen such that for every ϵ_ℓ ($\ell = 1, \cdots, m$) and for every λ_i ($i = 1, \cdots, n$), $|\epsilon_\ell \lambda_i + 1| < 1$, where $\lambda_1, \cdots, \lambda_n$ are the eigenvalues of J_{ol}. For an overdamped, stable process, $\epsilon_1, \cdots, \epsilon_m$ should be chosen such that for every ϵ_ℓ, $\ell = 1, \cdots, m$, and for every τ_i, $i = 1, \cdots, n$, $0 < \dfrac{\epsilon_\ell}{\tau_i} < 2$, where $\tau_1, \cdots, \tau_n$ are the open-loop time constants of the process. In other words, $\epsilon_1, \cdots, \epsilon_m$ should be chosen to be less than $2\tau_{min}$, where τ_{min} is the smallest time constant of the process [i.e., $\tau_{min} = min(\tau_1, \cdots, \tau_n)$].

Note that the feedforward/state feedback of Equation 12 does not induce the linear, closed-loop response of Equation 9, since in the derivation of the feedforward/state feedback the time derivatives of u were set to zero. The nonlinearity of the resulting delay-free output response is the price of ensuring closed-loop stability for processes with a non-minimum-phase delay-free part.

P	$\lambda_1(J_{cl})$	$\lambda_2(J_{cl})$	$\lambda_3(J_{cl})$
1	7.28	-20.00	-10.00
2	64.75	-13.19	-7.14
3	-4.34	-15.69+2.13i	-15.69-2.13i
4	-3.16	-11.18+3.25i	-11.18-3.25i
5	-2.60	-9.63+2.25i	-9.63-2.25i
6	-2.28	-8.76+0.96i	-8.76-0.96i
7	-2.10	-9.63	-6.74
10	-1.86	-9.97	-4.45
20	-1.78	-10.00	-2.38
30	-1.55	-10.00	-2.02
40	-1.31	-10.00	-2.00
50	-1.18	-9.70	-2.00

Table 1: Closed-loop eigenvalues of Example 1 for several values of $p_1 = p_2 = P$.

Example 1 *Consider a linear process without deadtime in state space form with*

$$A = \begin{bmatrix} -2 & 5 & -3 \\ 0 & -1 & 3 \\ 0 & 0 & -10 \end{bmatrix}, \ B = \begin{bmatrix} 2 & 5 \\ 5 & 0 \\ 2 & 22 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

It is non-minimum-phase (has a right-half plane [RHP] transmission zero at $s = 7.28$) and hyperbolically stable (has three left-half plane [LHP] eigenvalues at $s = -1$, $s = -2$ and $s = -10$). Its relative orders $r_1 = 1$ and $r_2 = 1$. With $\epsilon_1 = 0.1$ and $\epsilon_2 = 0.05$, the eigenvalues of $\epsilon_1 A$ are -0.2, -0.1 and -1, while the eigenvalues of $\epsilon_2 A$ are -0.5, -0.1 and -0.05. These eigenvalues are inside the unit circle centered at (-1,0i), so $\rho(\epsilon_1 A + I) = 0.9 < 1$ and $\rho(\epsilon_2 A + I) = 0.95 < 1$. The location of the closed-loop eigenvalues, $\lambda_1(J_{cl_p})$, $\lambda_2(J_{cl_p})$ and $\lambda_3(J_{cl_p})$, for several values of $p_1 = p_2 = P$ are given in Table 1; the closed-loop eigenvalues converge to the open-loop eigenvalues ($s = -1$, $s = -2$, $s = -10$), as $P \to \infty$.

Nonlinear Feedback Controller Design

The feedforward/state feedback of Equation 12 lacks integral action, and thus, it cannot induce offset-free response of the process output when process-model mismatch exists. To resolve this problem, the feedforward/state feedback of Equation 12 is implemented with a disturbance estimator, leading to the derivation of the feedback control law with integral action, given in Theorem 2.

Theorem 2 *For a process in the form of Equation 1 with incomplete state measurements, the closed-loop sys-*

tem under the error-feedback control law

$$\left.\begin{aligned} \frac{dx}{dt} &= f[x,u] \\ u &= \Psi_p\left[x, e + \begin{bmatrix} h_1[x(t-\theta_1)] \\ \vdots \\ h_m[x(t-\theta_m)] \end{bmatrix}\right] \end{aligned}\right\} \quad (13)$$

where $e = y_{sp} - \bar{y}$*, is asymptotically stable, if the following conditions hold:*

(a) The nominal equilibrium pair of the process, $(\bar{x}_{ss}, u_{ss})$*, corresponding to* y_{sp} *and* d*, is hyperbolically stable.*

(b) The tunable parameters $p_1, \cdots, p_m$ *are chosen to be sufficiently large.*

(c) The tunable parameters $\epsilon_1, \cdots, \epsilon_m$ *are chosen such that for every* $\ell = 1, \cdots, m$*, all eigenvalues of* $\epsilon_\ell J_{ol} \doteq \epsilon_\ell[\frac{\partial}{\partial x} f(x,u)]_{(\bar{x}_{ss}, u_{ss})}$ *lie inside the unit circle centered at* $(-1, 0j)$*.*

Furthermore, the error-feedback control law of Equation 13 has integral action.

The proof can be found elsewhere (Kanter et al., 2000).

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