Efficient Nonlinear Model Predictive Control

Rolf Findeisen* and Frank Allgöwer†
Institute for Systems Theory in Engineering
University of Stuttgart, Germany

Moritz Diehl‡, H. Georg Bock§, and Johannes P. Schlöder¶
Interdisciplinary Center for Scientific Computing (IWR)
University of Heidelberg, Germany

Zoltan Nagy‖
Faculty of Chemistry and Chemical Engineering
"Babes-Bolyai" University of Cluj, Romania

Abstract

The growing interest in model predictive control for nonlinear systems, also called NMPC, is motivated by the fact that today's processes need to be operated under tighter performance specifications to guarantee profitable and environmentally safe production. One of the remaining essential problems for NMPC is the high on-line computational load. At each sampling instant, a nonlinear optimal control problem must be solved. In this paper, we summarize recent results showing the practical applicability of NMPC for process control. We show how recent advances in NMPC theory and dynamic optimization can be used to make the real-time application of NMPC feasible even for high dimensional problems. As an application example the real-time control of a high purity distillation column is considered.

Keywords

Nonlinear model predictive control, Real-time optimization, Quasi-infinite horizon, Multiple shooting, Computational effort, Large scale, Distillation control

Introduction

Over the last two decades model predictive control (MPC), also referred to as moving horizon control or receding horizon control, has become an attractive feedback strategy. Linear MPC approaches have found successful applications, especially in the process industry (Qin and Badgwell, 1996). Nowadays, tighter product quality specifications, increasing productivity demands and environmental regulations require systems to be operated closer to the boundary of the admissible operating region. To allow operation near the boundary, a linear model is often not adequate to describe the process dynamics. This motivates the use of nonlinear system models, non-quadratic cost functions and nonlinear constraints in the predictive framework, thus leading to nonlinear model predictive control (NMPC).

Recently NMPC schemes with favorable properties including guaranteed closed-loop stability or reduced computational demand have been developed, see for example De Nicolao et al. (2000); Allgöwer et al. (1999) for a review. Despite these advances concern has been raised that due to the high on-line computational load none of the available NMPC schemes can be used for real-time control in practice. This concern is based on the fact that at every sampling instant a high-dimensional nonlinear, finite horizon optimal control problem has to be solved.

In this paper we summarize results of an ongoing study (Nagy et al., 2000; Bock et al., 2000b; Allgöwer et al., 2000) showing the practical applicability of NMPC to medium/high dimensional processes. We consider the control of a high purity distillation column using NMPC. In contrast to (Nagy et al., 2000; Bock et al., 2000b) we consider the output feedback case in this paper.

Our goal is to outline the key components for real-time application of NMPC. The conclusion is that a successful application of NMPC is possible even nowadays, if a combination of special dynamic optimization strategies (Bock et al., 2000b; Biegler, 2000) and NMPC schemes with reduced online computational load (Chen and Allgöwer, 1998; De Nicolao et al., 1996) is used.

The paper is organized as follows: In the first section, we review NMPC strategies that require reduced computational load. In the second section, one specially tailored dynamic optimization strategy for the solution of the occurring optimal control problems is described. Finally, the control of a high-purity distillation column is considered.

Nonlinear Model Predictive Control

In Figure 1 the general principle of model predictive control is shown. For simplicity of exposition, we assume that the control and prediction horizon have the same length. Based on measurements obtained at time t, the controller predicts the future dynamic behavior of the system over a control horizon T_c and determines the manipulated input such that a predetermined open-loop performance objective functional is optimized. In order to incorporate some feedback mechanism, the open-loop manipulated input function obtained is implemented only until the next measurement becomes available. We assume that this is the case every δ seconds (sampling time). Using the new measurement, at time $t+\delta$, the whole procedure—prediction and optimization—is repeated to find a new input function.

*findeise@ist.uni-stuttgart.de
†allgower@ist.uni-stuttgart.de
‡moritz.diehl@iwr.uni-heidelberg.de
§bock@iwr.uni-heidelberg.de
¶schloeder@iwr.uni-heidelberg.de
‖znagy@chem.ubbcluj.ro

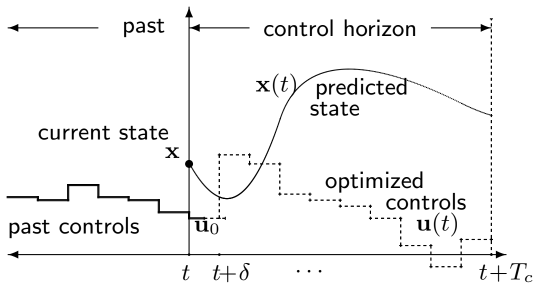

Figure 1: Principle of model predictive control.

Mathematical Formulation of NMPC

We assume that the system is given by nonlinear index-one differential algebraic equations (DAE) of the form

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{z}(t), \mathbf{u}(t)), \qquad \mathbf{x}(0) = \mathbf{x}_0 \tag{1a}$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}(t), \mathbf{z}(t), \mathbf{u}(t)), \tag{1b}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ denotes the differential variables, $\mathbf{z}(t) \in \mathbb{R}^p$ the algebraic variables and $\mathbf{u}(t) \in \mathbb{R}^m$ the inputs. The control objective is to stabilize this system around a given setpoint, denoted by $(\mathbf{x}_s, \mathbf{z}_s, \mathbf{u}_s)$, while satisfying constraints on the input and states of the form:

$$\mathbf{h}(\mathbf{x}, \mathbf{z}, \mathbf{u}) \geq \mathbf{0}. \tag{2}$$

In the simplest case the constraints are box constraints, i.e. $\mathbf{u}_{min} \leq \mathbf{u} \leq \mathbf{u}_{max}$, $\mathbf{x}_{min} \leq \mathbf{x} \leq \mathbf{x}_{max}$. The control is given by the following open-loop optimization problem that is solved at every sampling instant:

$$\min_{\bar{\mathbf{u}}(\cdot)} J(\bar{\mathbf{u}}(\cdot); \mathbf{x}(t)) \tag{3a}$$

with:

$$J(\bar{\mathbf{u}}(\cdot); \mathbf{x}(t)) = \int_t^{t+T_c} F(\bar{\mathbf{x}}(\tau), \bar{\mathbf{u}}(\tau)) d\tau \tag{3b}$$

subject to:

$$\dot{\bar{\mathbf{x}}}(\tau) = \mathbf{f}(\bar{\mathbf{x}}(\tau), \bar{\mathbf{z}}(\tau), \bar{\mathbf{u}}(\tau)), \quad \bar{\mathbf{x}}(t) = \mathbf{x}(t) \tag{3c}$$

$$\mathbf{0} = \mathbf{g}(\bar{\mathbf{x}}(\tau), \bar{\mathbf{z}}(\tau), \bar{\mathbf{u}}(\tau)) \tag{3d}$$

$$\mathbf{h}(\bar{\mathbf{x}}(\tau), \bar{\mathbf{z}}(\tau), \bar{\mathbf{u}}(\tau)) \geq \mathbf{0} \quad \tau \in [t, t+T_c]. \tag{3e}$$

Internal controller variables are denoted by a bar. The function F, often called stage cost function, specifies the desired control performance. Often, F is chosen as quadratic in x and u: $F(\mathbf{x}, \mathbf{u}) = (\mathbf{x} - \mathbf{x}_s)^T Q (\mathbf{x} - \mathbf{x}_s) + (\mathbf{u} - \mathbf{u}_s)^T R (\mathbf{u} - \mathbf{u}_s)$. The system input during the sampling time δ is given by the optimal input resulting from the solution of the open-loop control problem at time t: $\mathbf{u}(\tau) = \bar{\mathbf{u}}^\star(\tau)$, $\tau \in [t, t+\delta)$.

Efficient Formulation of the NMPC Problem

While the NMPC formulation described above can be applied straightforwardly in practice, in general nothing can be said about stability of the closed loop and performance. One way to achieve good closed-loop performance and stability is the use of an *infinite* horizon length (Keerthi and Gilbert, 1988), i.e. T_c is set to ∞. However, for an infinite horizon, the resulting nonlinear program (NLP) is in practice not solvable in finite time. If finite prediction and control horizons are used, the closed-loop system trajectories differ from the predicted open-loop ones. As a consequence it is not clear how the resulting performance is related to the "optimal" infinite horizon cost and whether the closed-loop is stable.

To allow an efficient solution of the resulting open-loop optimal control problem while guaranteeing stability and good performance several NMPC schemes have been proposed (Chen and Allgöwer, 1998; De Nicolao et al., 1996). These methods lead to a similar open-loop optimization problem as Equation 3a, however the cost function, Equation 3b, is augmented by a terminal penalty term $E_s(\cdot)$

$$J(\bar{\mathbf{u}}(\cdot); \mathbf{x}(t)) = \int_t^{t+T_c} F(\bar{\mathbf{x}}(\tau), \bar{\mathbf{u}}(\tau)) d\tau + E_s(\bar{\mathbf{x}}(t+T_c)) \tag{4}$$

and the following final region constraint is added

$$r(\bar{\mathbf{x}}(t+T_c)) \geq 0. \tag{5}$$

Roughly speaking the terminal state penalty term E_s gives an (upper bound) estimate of the infinite horizon cost and thus approximately recovers the infinite horizon. For this approximation however, the final predicted state has to be restricted to a predetermined region given by the terminal region constraint r. Detailed descriptions of these approaches and how to obtain E_s and r can be found in Allgöwer et al. (2000); Chen and Allgöwer (1998); Findeisen and Allgöwer (2000); De Nicolao et al. (2000). The computational advantage of the described schemes lies in the fact, that shorter horizons can be used, while not jeopardizing performance and stability. We propose to use this kind of NMPC schemes in combination with specially tailored dynamic optimization strategies as outlined in the next section.

Efficient Solution of NMPC Problems

An numerically efficient solution of the NMPC optimization problem should: 1) take advantage of the special problem structure of the open loop optimization problem, 2) reuse as much information as possible from the previous sampling interval in the current sampling interval. One dynamic optimization scheme that can be adapted to provide all these properties is the so-called direct multiple shooting approach (Bock and Plitt, 1984).

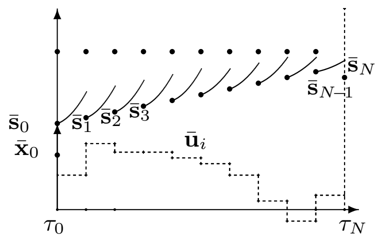

Figure 2: Principle of direct multiple shooting.

Direct Multiple Shooting for NMPC

In the following, the basic idea of direct multiple shooting is outlined. More can be found in Bock et al. (2000b). We assume, that the controls are parametrized as piecewise constant on each of the $N = \frac{T_c}{\delta}$ so-called *multiple shooting intervals*, i.e. $\bar{\mathbf{u}}(\tau) = \bar{\mathbf{u}}_i$ for $\tau \in [\tau_i, \tau_{i+1})$, $\tau_i = t + i\delta$. The DAE solution is decoupled on these intervals by considering the initial values $\bar{\mathbf{s}}_i^x$ and $\bar{\mathbf{s}}_i^z$ of differential and algebraic states at the times τ_i as additional optimization variables (compare Figure 2). The solution of the "decoupled" initial value problems obeys the following *relaxed* DAE formulation on the intervals $[\tau_i, \tau_{i+1})$:

$$\dot{\bar{\mathbf{x}}}_i(\tau) = \mathbf{f}(\bar{\mathbf{x}}_i(\tau), \bar{\mathbf{z}}_i(\tau), \bar{\mathbf{u}}_i) \tag{6a}$$

$$\mathbf{0} = \mathbf{g}(\bar{\mathbf{x}}_i(\tau), \bar{\mathbf{z}}_i(\tau), \bar{\mathbf{u}}_i) - \mathbf{g}(\bar{\mathbf{s}}_i^x, \bar{\mathbf{s}}_i^z, \bar{\mathbf{u}}_i) \tag{6b}$$

$$\bar{\mathbf{x}}_i(\tau_i) = \bar{\mathbf{s}}_i^x, \qquad \bar{\mathbf{z}}_i(\tau_i) = \bar{\mathbf{s}}_i^z. \tag{6c}$$

The subtrahend in Equation 6b is introduced to allow an efficient DAE solution for initial values and controls $\bar{\mathbf{s}}_i^x, \bar{\mathbf{s}}_i^z, \bar{\mathbf{u}}_i$ that violate temporarily the consistency conditions $\mathbf{0} = \mathbf{g}(\bar{\mathbf{s}}_i^x, \bar{\mathbf{s}}_i^z, \bar{\mathbf{u}}_i)$ (Bock et al., 2000a). The contribution of the integral cost term on $[\tau_i, \tau_{i+1})$ is determined by:

$$J_i(\bar{\mathbf{s}}_i^x, \bar{\mathbf{s}}_i^z, \bar{\mathbf{u}}_i) = \int_{\tau_i}^{\tau_{i+1}} F(\bar{\mathbf{x}}(\tau), \bar{\mathbf{u}}_i) d\tau.$$

The resulting large but structured NLP takes the form:

$$\min_{\bar{\mathbf{u}}_i, \bar{\mathbf{s}}_i} \sum_{i=0}^{N-1} J_i(\bar{\mathbf{s}}_i^x, \bar{\mathbf{s}}_i^z, \bar{\mathbf{u}}_i) + E_s(\bar{\mathbf{s}}_N^x) \tag{7}$$

subject to:

$$\bar{\mathbf{s}}_0^x = x(t), \tag{8a}$$

$$\bar{\mathbf{s}}_{i+1}^x = \bar{\mathbf{x}}_i(\tau_{i+1}), \qquad i = 0, 1, \ldots, N-1, \tag{8b}$$

$$\mathbf{0} = \mathbf{g}(\bar{\mathbf{s}}_i^x, \bar{\mathbf{s}}_i^z, \bar{\mathbf{u}}_i), \qquad i = 0, 1, \ldots, N-1, \tag{8c}$$

$$\mathbf{h}(\bar{\mathbf{s}}_i^x, \bar{\mathbf{s}}_i^z, \bar{\mathbf{u}}_i) \geq \mathbf{0}, \qquad i = 0, 1, \ldots, N-1, \tag{8d}$$

$$r_s(\bar{\mathbf{s}}_N^x) \geq 0. \tag{8e}$$

This large structured NLP problem is solved by a specially tailored *partially reduced* SQP algorithm (see Bock et al. (2000a) for a detailed description). To further improve the solution time, one should taken into account that the optimization problems at consecutive sampling instants are quite similar. We propose to consider the following strategy to decrease the computation time:

Initial Value Embedding Strategy: Optimization problems at subsequent sampling instants differ only by different initial values $\mathbf{x}(t)$, that are imposed via the initial value constraint, Equation 8a: $\bar{\mathbf{s}}_0^x = \mathbf{x}(t)$. Accepting an initial violation of this constraint, the complete solution trajectory of the previous optimization problem can be used as an initial guess for the current problem. Furthermore, all problem functions, derivatives as well as an approximation of the Hessian matrix have already been found for this trajectory and can be used in the new problem, so that the first QP solution can be performed *without any additional DAE solution.*

The application of this strategy does improve the robustness and speed of the optimization algorithm significantly. More details can be found in Diehl et al. (2001a).

Example Process

We outlined two key components for a computationally feasible application of NMPC: 1) the use of efficient, tailored dynamic optimization algorithms and 2) the use of efficient NMPC formulations. In this section we utilize these components to show that a successful application of NMPC to a nontrivial process control example is feasible already nowadays.

High Purity Distillation

As an application example the control of a high purity binary distillation column for the separation of Methanol and n-Propanol is considered (see Figure 3). The binary mixture is fed into the column with flow rate F and molar feed composition x_F. Products are removed at the top and bottom of the column with concentrations x_B and x_D. The liquid flow rate L and the vapor flow rate V are the control inputs (L/V configuration). The control problem is to maintain the specifications on the product concentrations x_B and x_D despite disturbances in the feed flow F and the feed concentration x_F. It is assumed that only the temperatures on the 14^{th} tray and 28^{th} tray can be measured and that the disturbance quantities x_F and F are not measured.

System Models and State Estimation

For comparison, two models of different complexity for the prediction are used in the controller. Modeling of the distillation column under the assumptions of constant relative volatility, constant molar overflow, no pressure losses, no energy balances and hydrodynamics leads to a 42^{nd} order ODE model. The second model considered is a 164^{th} order model with 122 algebraic states and 42 differential states. A more detailed description is given in Nagy et al. (2000). The controller needs estimates of all differential states, as well as of the disturbances F

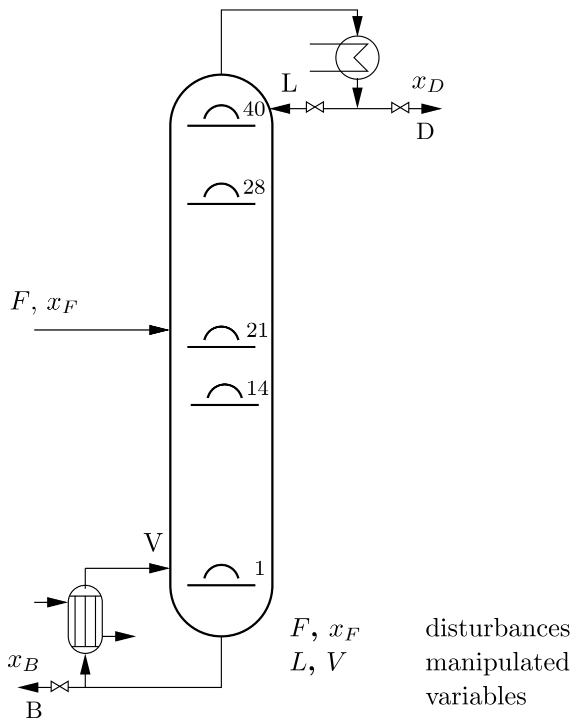

Figure 3: Considered distillation column.

model size	max	avrg
42	1.86s	0.89s
164	6.21s	2.48s

Table 1: Necessary CPU time for one sampling time.

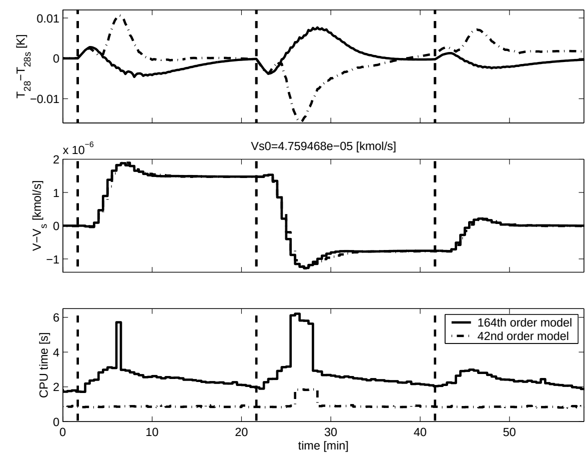

Figure 4: Behavior of the closed-loop.

and x_F. They are reconstructed from measurements of the temperatures T_{14}, T_{28} using an Extended Kalman Filter (EKF).

Controller Setup

As common in distillation control, the product concentrations x_B and x_D are not controlled directly, i.e. they do not appear directly in the cost function. Instead an inferential control scheme, which controls the deviation of the concentrations on tray 14 and 28 from the setpoints, is used. Based on the estimates from the EKF in a first step, the system state at the next sampling instant is predicted. Using this state, the open-loop optimal control problem is solved. The resulting first input is implemented at the next control instant and the procedure is repeated. Note that this leads to a delay in the control scheme. This is necessary since the solution of the dynamic optimization problem cannot be obtained instantaneously. As NMPC scheme, the quasi-infinite horizon NMPC scheme for index–one DAE systems is applied (Findeisen and Allgöwer, 2000). A quadratic stage cost and a quadratic terminal penalty term are used. The choice of the weighting matrices and the derivation of the terminal region is described in (Nagy et al., 2000). For all simulations, the real plant is given by the 164^{th} order model. The 42^{nd} and 164^{th} order models are used for the controller predictions. The control input parameterization (controller sampling time) δ is $30s$, while the EKF is updated with the plant measurements every 10s. The control horizon T_c is fixed to 10 minutes ($N = 20$).

Performance and Computational Complexity

In Table 1, the necessary solution times for the input disturbance scenario as shown in Figure 4 are given[1]. One can see that the proposed strategy of combining NMPC schemes that require reduced computational time with a direct multiple shooting approach does lead to a rather low computational load. In our example the solution is easily feasible in the sampling time of 30s, even for the 164^{th} order model and a horizon length of $N = 20$. In Figure 4 the performance of the closed-loop for the different model sizes is compared. The temperatures T_{14} and T_{28} are kept in a narrow band, which is certainly more than satisfying. As shown, real-time application of NMPC is possible even for rather large models, if NMPC schemes with a low computational load and specialized optimization schemes are employed. Currently, the presented algorithms are experimentally applied to control a medium scale distillation column. Results will be presented in a forthcoming paper (Diehl et al., 2001b).

Conclusions

From an industrial/application point of view there is a strong demand to use NMPC schemes, since these methods allow to directly use (nonlinear) first principle models that are able to describe a wider range of operation than linear models can do. However, concern has been raised that NMPC cannot be applied in practice, since at every sampling instant a nonlinear optimization prob-

[1] All computations are carried out on a Compaq Alpha XP1000.

lem has to be solved. In this paper, we outlined the key components for a computationally feasible application of NMPC: The use of efficient NMPC schemes like quasi-infinite horizon NMPC in combination with specially tailored, efficient dynamic optimization techniques. Using these techniques, a successful application of NMPC even for high dimensional systems is feasible. This has been demonstrated considering the real-time control of a high purity distillation column.

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