Nonlinear Process Control: Novel Controller Designs for Chemical Processes with Uncertainty and Constraints

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Abstract

This paper highlights some of our recent results on control of single-input single-output constrained uncertain nonlinear processes. The main issues and challenges that arise in control of such processes are discussed and a novel Lyapunov-based framework for nonlinear controller synthesis is presented to address these issues. The proposed framework leads to the explicit synthesis of nonlinear robust optimal state feedback controllers, with well-characterized stability and performance properties, that cope effectively with the simultaneous problems of severe process nonlinearities, significant model uncertainty, and hard constraints on the manipulated input. The impact of the proposed framework is analyzed and its implications for nonlinear process control are identified.

Keywords

Uncertainty, Lyapunov's direct method, Input constraints, Bounded control, Inverse optimality

Introduction

In designing an effective process control system, there are several fundamental issues that transcend the boundaries of specific process applications. Although they may vary from one application to another and have different levels of significance, these issues remain generic in their relationship to the control design objectives. Central to these issues is the requirement that the control system provide satisfactory performance in the face of severe process nonlinearities, modeling errors, process variations, and actuator constraints. Nonlinear behavior, model uncertainty and input constraints represent some of the more salient features whose frequently-encountered co-presence in a multitude of chemical processes renders the task of controlling such processes a challenging one. For example, many important industrial processes including highly exothermic chemical reactions, high purity distillation columns, and batch systems exhibit strong nonlinear behavior and cannot be effectively controlled with controllers designed on the basis of approximate linear or linearized process models. The limitations of traditional linear control methods in dealing with nonlinear chemical processes have become increasingly apparent as chemical processes may be required to operate over a wide range of conditions due to large process upsets or set-point changes. This realization has consequently motivated intense research activity in the area of nonlinear process control over the last fifteen years. The literature on nonlinear process control is really extensive (see, e.g., (Allgöwer and Doyle III, 1997) for references).

In addition to nonlinear behavior, many industrial processes are characterized by the presence of model uncertainty such as unknown process parameters and external disturbances which, if not accounted for in the controller design, may cause performance deterioration and even closed-loop instability. Significant research work has consequently focused on this problem including the use of integral action in conjunction with feedback linearizing controllers to compensate for constant model uncertainty and the design of robust controllers through Lyapunov's direct method for processes with time-varying uncertain variables (e.g., (Christofides et al., 1996)).

The above approaches, however, do not lead in general to controllers that are optimal with respect to a meaningful cost and therefore do not guarantee achievement of the control objectives with the smallest possible control action. This is an important limitation of these methods, especially in view of the fact that the capacity of control actuators used to regulate chemical processes is almost always constrained. The problems caused by input constraints have consequently motivated many recent studies on the dynamics and control of chemical processes subject to input constraints. Notable contributions in this direction include controller design and stability analysis within the model predictive control framework (e.g., (Rawlings, 1999; Kurtz et al., 2000)), constrained quadratic-optimal control (Chmielewski and Manousiouthakis, 1996), the design of "anti-windup" schemes in order to prevent excessive performance deterioration of an already designed controller when the input saturates (Kothare et al., 1994; Valluri and Soroush, 1998; Kapoor and Daoutidis, 1999). However, these control methods do not explicitly account for robust uncertainty attenuation.

Summarizing, a close look at the available literature reveals the fact that, at this stage, existing nonlinear process control methods lead to the synthesis of controllers that can deal with either model uncertainty or input constraints, but not simultaneously or effectively with both. This clearly limits the achievable control quality and closed-loop performance, especially in view

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of the commonly-encountered co-presence of uncertainty and constraints in chemical processes. Therefore, the development of a unified framework for control of nonlinear processes that explicitly accounts for the presence of model uncertainty and input constraints is expected to have a significant impact on chemical process control. Motivated by this, we outline in this paper some of our recent results on the development of this framework and discuss, in particular, its implications for enriching our existing arsenal of nonlinear control tools.

Nonlinear Processes with Uncertainty and Constraints

We consider single-input single-output nonlinear processes with uncertain variables and input constraints modeled by the class of continuous-time nonlinear ordinary differential equation systems with the following state-space description:

$$\begin{aligned} \dot{x} &= f(x) + g(x)sat(u) + \sum_{k=1}^{q} w_k(x)\theta_k(t) \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the vector of process state variables, $u \in \mathbb{R}$ denotes the manipulated input, $\theta_k(t) \in \mathcal{W} \subset \mathbb{R}$ denotes the k-th uncertain (possibly time-varying) but bounded variable taking values in a nonempty compact convex subset $\mathcal{W}$ of $\mathbb{R}$, $y \in \mathbb{R}$ denotes the output to be controlled, and *sat* refers to the standard saturation nonlinearity. The presence of the *sat* operator in Equation 1 signifies the presence of hard constraints on the manipulated input. The uncertain variable $\theta_k(t)$ may describe time-varying parametric uncertainty and/or exogenous disturbances. It is assumed that the origin is the equilibrium point of the nominal (i.e., $u(t) = \theta_k(t) \equiv 0$) system of Equation 1.

Main Issues on Control of Constrained Uncertain Nonlinear Processes

Towards our end goal of developing an effective control strategy that enforces the desired stability and performance properties in nonlinear processes subject to model uncertainty and actuator constraints, we begin in this section by identifying some of the outstanding issues that arise in this problem and must be addressed properly. While the individual presence of either model uncertainty or input constraints poses its own unique set of problems that must be dealt with at the stage of controller design, the combined presence of both uncertainty and constrains is far more problematic for process stability and performance. The difficulty here emanates not only from the cumulative effect of the co-presence of the two phenomena but is, more importantly, due to the additional issues that arise from the interaction of the two.

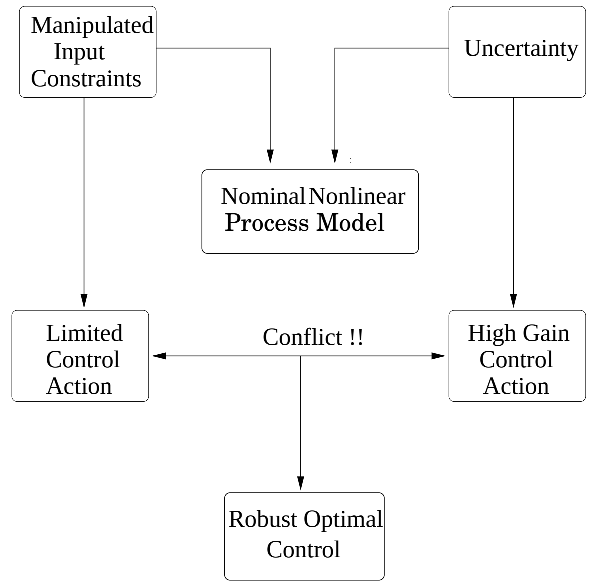

Figure 1: Paradigm for control of constrained uncertain nonlinear processes.

At the core of these issues are the following two problems:

1. The co-presence of model uncertainty and input constraints creates an inherent conflict in the controller design objectives and control policy to be implemented. On one hand, suppressing the influence of significant external disturbances requires typically large (high-gain) control action. On the other hand, the availability of such action is often limited by the presence of input constraints. Failure to resolve this conflict will render any potential control strategy essentially ineffective. A schematic representation of this conflict is depicted in Figure 1.
2. The set of feasible process operating conditions under which the process can be operated safely and reliably is significantly restricted by the co-presence of uncertainty and constraints. While, on their own, input constraints place fundamental limitations on the size of this set (and consequently on our ability to achieve certain control objectives), these limitations become even stronger when uncertainty is present. At an intuitive level, many of the feasible operating conditions permitted by constraints under nominal conditions (i.e. predicted using a nominal model of the process) cannot be expected to continue to be feasible in the presence of significant plant-model mismatch. The success of a control strategy in effectively addressing the problems of uncertainty and constraints hinges, therefore, not only on the design of effective controllers, but also on the ability to predict a priori the feasible conditions under which the designed controllers are guar-

anteed to work in the presence of both uncertainty and constraints.

Having discussed some of the main issues involved, we are now motivated to proceed in the next two sections with the presentation of some of our recent results on this problem, which directly address the issues outlined above and culminate in the development of a general Lyapunov-based framework for control of nonlinear processes with model uncertainty and input constraints. More specifically, the developed framework entails:

1. The synthesis of nonlinear robust optimal controllers that account explicitly for the problems of significant model uncertainty and input constraints and enforce the desired stability, robustness, optimality, and explicit constraint-handling properties in the closed-loop system.

2. The explicit and quantitative characterization of the limitations imposed by the co-presence of uncertainty and constraints on our ability to steer the nonlinear process in a desired direction.

To lay the appropriate foundation for the development of the proposed framework, we begin in the next section by presenting first some of the key tools necessary to address the first issue. Then, in the following section, we show how these tools can be built upon to address the second issue, leading finally to the desired framework.

Robust Optimal Control

The control paradigm presented in Figure 1 suggests a natural approach to resolve the inherent conflict between the need to compensate for the effect of model uncertainty and the limitations imposed by input constraints on the available control action. This is the robust optimal control approach which involves the synthesis of robust controllers that use minimal or reasonably small control action to compensate for the effect of significant model uncertainty and achieve robust stabilization. In (El-Farra and Christofides, 2001a), using a novel combination of Lyapunov's direct method and the inverse optimal control approach, we synthesized robust optimal controllers that enforce, in the presence of significant model uncertainty and absence of constraints: a) global stability, b) robust asymptotic output tracking with arbitrary degree of attenuation of the effect of uncertainty on the output, and c) optimal performance, in the closed-loop system. The controllers take the general form

$$u = -\frac{1}{2}R^{-1}(x, \theta_b, \phi)L_{\bar{g}}V \tag{2}$$

where

$$\begin{aligned}\frac{1}{2}R^{-1}(\cdot) = c_0 + \frac{L_{\bar{f}}V + \sqrt{(L_{\bar{f}}V)^2 + (L_{\bar{g}}V)^4}}{(L_{\bar{g}}V)^2} \\ + \frac{\rho + \chi\sum_{k=1}^{q}\theta_{bk}|L_{\bar{w}k}V||2b^TPe|}{(L_{\bar{g}}V)^2(|2b^TPe| + \phi)}\end{aligned} \tag{3}$$

is a strictly positive definite function, θ_{bk}'s represent the available bounds that capture the magnitude of uncertainty for all time, c_0, ρ, χ, and ϕ are tuning parameters that can be adjusted to achieve the desired degree of uncertainty attenuation, $V = e^TPe$ is a scalar quadratic Lyapunov function of the tracking error whose time-derivative is rendered negative definite by the feedback law of Equation 2 along the trajectories of the closed-loop system, and $L_{\bar{g}}V$ is the standard Lie derivative notation. Further details on the controller synthesis procedure and notation used can be found in (El-Farra and Christofides, 2001a). In what follows, we outline some of the key desirable features of the robust optimal controllers proposed and their implications for the problem of controlling constrained uncertain nonlinear processes.

1. The robust optimal controllers of Equation 2 possess two desirable properties not present in other nonlinear controllers designed on the basis of feedback linearization concepts. The first property is their ability to recognize the presence of beneficial (stabilizing) nonlinearities in the process and prevent their unnecessary cancellation. As a result, these controllers do not waste unnecessary control effort to accomplish the desired closed-loop objectives. An important implication of this property is the fact that these controllers (though not designed to handle constraints explicitly yet) are better equipped to cope with the limitations imposed by input constraints on the available control action than other controllers which may request unnecessarily large control effort. The second property is the use of domination, rather than cancellation, to eliminate the effects of harmful (destabilizing) nonlinearities. This property guards against the non-robustness of nonlinear cancellation designs which increases the risk of instability due to the presence of other uncertainty not taken into account in the controller design.
2. The use of Lyapunov's direct method in the controller design allows the synthesis of robust controllers that can effectively attenuate the effect of time-varying persistent uncertainty on the closed-loop output which cannot be achieved using classical uncertainty compensation techniques, including the incorporation of integral action and parameter adaptation in the controller. For constant disturbances, the Lyapunov approach offers an alternative method for disturbance rejection that avoids

the use of integral action which contributes to the problem of windup in the presence of constraints. Furthermore, robust stabilization is accomplished regardless of how large the disturbances are so long as bounds are available that capture their magnitude. For nonlinear controller design, in general, Lyapunov methods provide useful and systematic tools (see, e.g.,(Kazantzis and Kravaris, 1999)).

3. Using the inverse optimal control approach (Freeman and Kokotovic, 1996; El-Farra and Christofides, 2001a), one can rigorously prove that the robust Lyapunov-based controller of Equation 2 is optimal with respect to a meaningful performance index of the form:

$$J = \int_0^\infty (l(e) + uR(x)u)dt \tag{4}$$

which imposes physically sensible penalties on both the tracking error ($l(e)$ is a smooth positive definite nonlinear function bounded below by a quadratic function of the norm of the tracking error) and the control action. The inverse optimal approach provides a sound theoretical basis for explaining the origin of the controllers' optimality properties outlined in the first remark. Another major advantage of using the inverse optimal approach in controller design is the fact that it provides a convenient route for the synthesis of robust controllers that are optimal with respect to meaningful cost functionals without recourse to the unwieldy task of solving the Hamilton-Jacobi-Isaacs partial differential equation which is the optimality condition for the robust stabilization problem for nonlinear systems.

Integrating Robustness, Optimality, and Constraints

While optimality is certainly a desirable feature that every well designed controller must possess to cope with the limitations imposed by input constraints on the control action, it might not always be sufficient to address the problem in its entirety. For example, one can envision situations where the control objectives cannot be achieved in the presence of constraints, irrespective of the particular choice of the control law. Therefore, for an optimal control policy, such as the one presented in the previous section, to effectively address the problem of constraints, it is imperative that it also identifies, explicitly, the feasible operating conditions under which stability of the process can be guaranteed in the presence of constraints. In this regard, we note that although the robust optimal controllers of Equation 2 are equipped with the appropriate tools to resolve the conflict between uncertainty and constraints, they are not designed to address the second issue of explicitly characterizing the limitations imposed by input constraints on the feasible operating conditions and cannot therefore be expected to enforce the same closed-loop properties in the presence of arbitrary input constraints.

To address this issue, we developed in (El-Farra and Christofides, 2001a) a novel scaling procedure that bounds the robust optimal controllers in Equation 2:

$$|u| \leq u_{max} \tag{5}$$

where $|\cdot|$ is the Euclidean norm and u_{max} represents the available information on the actuator constraints. The result of this bounding procedure was the direct synthesis of nonlinear bounded robust optimal controllers that are conceptually aligned with the robust optimal controllers of Equation 2, but possess the additional capabilities of: a) incorporating both uncertainty and actuator constraints explicitly in the controller synthesis formula, and b) characterizing explicitly the set of feasible initial conditions starting from where the desired stability and performance properties are guaranteed in the closed-loop system under uncertainty and constraints. The resulting bounded robust optimal controllers have the general form:

$$u = -\frac{1}{2}\bar{R}^{-1}(x, u_{max}, \theta_b, \phi)L_{\bar{g}}V \tag{6}$$

where

$$\frac{1}{2}\bar{R}^{-1}(\cdot) = \frac{L_{\bar{f}}^* V + \sqrt{(L_{\bar{f}}^* V)^2 + (u_{max}L_{\bar{g}}V)^4}}{(L_{\bar{g}}V)^2[1 + \sqrt{1 + (u_{max}L_{\bar{g}}V)^2}]} \tag{7}$$

is a strictly positive definite function of its arguments and $L_{\bar{f}}^* V = L_{\bar{f}}V + \sum_{k=1}^{q}\theta_{bk}|L_{\bar{w}k}V| + \rho|e|$. For details on the notation used, see (El-Farra and Christofides, 2001a). We have shown in (El-Farra and Christofides, 2001a) that whenever the following inequality is satisfied:

$$L_{\bar{f}}V + \rho|e| + \chi\sum_{k=1}^{q}\theta_{bk}|L_{\bar{w}k}V| \leq u_{max}|L_{\bar{g}}V| \tag{8}$$

the bounded robust optimal controllers of Equation 6 enforce the following properties in the constrained closed-loop system, including: a) stability, b) robust asymptotic output tracking with arbitrary degree of uncertainty attenuation, and c) optimal performance with respect to a meaningful performance index of the general form of Equation 4 that imposes meaningful penalties on the tracking error and control action. In what follows, we outline some of the key features of the proposed bounded robust optimal controller design method and discuss its implications for the development of the desired unified framework for control of constrained uncertain nonlinear processes.

1. The inequality of Equation 8 describes explicitly the largest region in state space where the time-derivative of the Lyapunov function is guaranteed to be negative definite along the trajectories of the

closed-loop system, in the presence of uncertainty and constraints. Therefore, guided by this inequality, one can explicitly identify the set of admissible initial states, starting from where the aforementioned closed-loop properties are guaranteed, and ascertain a priori (before implementing the controller) whether stability and robust set-point tracking can be guaranteed in the constrained closed-loop system for a given initial condition. This aspect of the proposed method has important practical implications for efficient process operation since it provides the plant operators with a systematic guide to select feasible operating conditions. This is particularly significant in the case of unstable plants (e.g., exothermic chemical reactors) where lack of such a priori knowledge can lead to undesirable outcomes.

2. In addition to providing the desired characterization of the region of guaranteed closed-loop stability, the inequality of Equation 8 also depicts the region where the control action satisfies the input constraints. Within this region, no mismatch exists between the controller output and the actual process input.

3. The inequality of Equation 8 captures, in an intuitive way, the limitations imposed by uncertainty and constraints on the size of the closed-loop stability region. To this end, note that Equation 8 predicts that the tighter the input constraints (i.e., smaller u_{max}) and/or the larger the plant-model mismatch (i.e., larger θ_{bk}), the fewer the initial conditions that can be used for stabilization.

4. In light of the above remarks, it's important to compare the above control method with other methods of nonlinear control for nonlinear processes with input constraints. For example, in contrast to the traditional two-step approaches employed in analytical process control, which involves first the design of a controller for the unconstrained process and then accounts for input constraints through a suitable anti-windup modification, the bounded robust optimal control method offers a direct approach to the problem whereby the controllers of Equation 6 use directly the available information on actuator constraints (u_{max}) and uncertainty (θ_b) to compute the necessary control action; thus integrating robustness, optimality, and explicit-constraint handling capabilities in a single design. Other direct approaches for dealing with input constraints include optimization-based methods such as model predictive control. In these methods, however, the feedback law is not given explicitly, but implicitly through the optimization problem, which must be solved at each time step. Furthermore, issues of computational effort, robustness, and the a priori characterization of the region of closed-loop stability have yet to be addressed satisfactorily within these approaches.

The proposed Lyapunov-based control approaches reviewed here were applied successfully in (El-Farra and Christofides, 2001a) to benchmark examples including nonisothermal chemical reactors with unstable dynamics. Finally, we note that the problem of output feedback was recently addressed in (El-Farra and Christofides, 2001b, 2000) through combination of the robust optimal state feedback controllers with high gain observers. This approach was shown to practically preserve both the optimality properties as well as the region of guaranteed closed-loop stability obtained under state feedback.

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