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Adaptive Control of Uncertain Dynamics at the Nano-scale

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Abstract— **Control of micro and nano-systems imposes new and stringent challenges to be addressed. The nonlinear time varying and uncertain dynamics operating at different time and length scales requires robust ultra-fast feedback systems capable of addressing these unique challenges. A resetting adaptive control architecture based on MRAC is proposed. Stability analysis demonstrates that with a finite number of resets, the feedback system remains stable. The proposed resetting adaptive control is applied to control of a microcantilever in a nonlinear interaction. Simulation results demonstrate the potential of the resetting adaptive control in improving transients while maintaining stability in the presence of uncertainties.**

I. INTRODUCTION

There has been a vast interest in the areas of micro and nanotechnology both for basic research and for applications/products. Extensive research effort has targeted various aspects of micro and nano-systems including analysis, design, fabrication, dynamics and control. Only a sample of the literature on dynamics and control of nano and micro-systems is briefly mentioned below and the list is by no means comprehensive.

In [5] nonlinear dynamics of a microcantilever was studied and bifurcations were analyzed. A theoretical study on chaos was performed in [3] for resonance atomic force microscopy (AFM). Analysis of nano and micro robots in [4] has highlighted issues in the dynamics and control of these novel robots. Control challenges in micro fluidic systems and nanoscale transport phenomena were presented in [6]. Moreover, in [10] iterative control was developed to control coupled dynamics of a high-speed nano-positioner, while robust control algorithms were used in [1], [2] to control uncertain AFM dynamics. More so, adaptive compensation for uncertain AFM dynamics was presented in [7].

The dynamics at the micro and nano-scales can differ greatly from those at the macro-scale. Interactions at these scales are characterized by strong nonlinear dependance on objects separation, presence of long-range or short-range interaction, and rapid change between attractive or repulsive regime. In addition, these interactions strongly depend on geometry, materials and environmental conditions. In addition, possible presence of vastly different multiple time scale dynamics of interest. As a result, severe constraints are placed on feedback performance to compensate for highly nonlinear rapidly changing uncertain dynamics under limited system information.

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Therefore, there is a need to develop simple models that capture essential dynamics/information and use these models for real time robust feedback compensation of rapidly changing uncertain dynamics at the micro and nano-scales.

In this paper a resetting adaptive control architecture is proposed to address some of the aforementioned control challenges. In Section II, a resetting model reference adaptive control is presented along with stability analysis to analyze the effect of controller resetting on feedback stability. Section III applies the proposed resetting control to a case study on a microcantilever in a nonlinear interaction. Simulation results are presented. Finally, conclusions are given in Section IV.

II. RESETTING ADAPTIVE CONTROL

Adaptive control is well suited for compensating for constant or slowly varying parametric uncertainties. Generally however, adaptive control suffers from slow and poor transients. It has been suggested that a structure of multiple models/controllers could be used in conjunction with adaptive control to improve transient performance. In [8] switching between multiple fixed and adaptive linear-time invariant (LTI) models was used to speed up response. Nonlinear adaptive backstepping using estimator resetting was developed in [9] for improved transients. More so, based on certainty equivalent principal arguments [11], switching between multiple fixed LTI controllers was proposed.

In this paper, controller resetting is used with a model reference adaptive control (MRAC) structure to explore improvements in transient performance while maintaining robustness to parametric uncertainties. Consider a set of candidate plant parameter vectors $\{\theta_i \, , i = 1, ..., k\}$ representing the plant under different operating conditions. The proposed control structure, Figure 1, relies on using k parallel prediction models each with a vector θ_i. The output of each of the prediction model y_{pi} is compared in real-time with the plant output y. Prediction errors $e_{pi} = y - y_{pi}$ are monitored and a prediction error measure, Equation (1), is computed based on the norm of the prediction error with an exponential forgetting factor.

$$\theta_c = \arg \min_{\theta_i} \int_{t_o}^{t} e^{-2\alpha(t-\tau)} \, e_{pi}^2 \, d\tau \qquad (1)$$

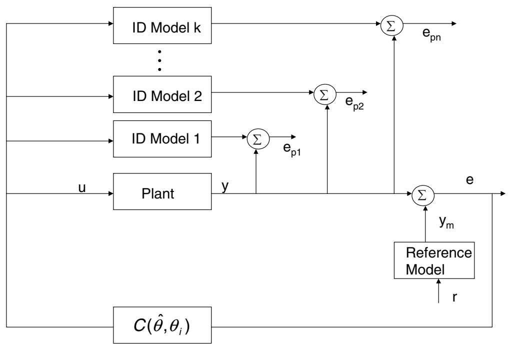

Fig. 1. Resetting adaptive control architecture.

At specific times t_j the parameter estimates $\hat{\theta}$ in the adaptive controller are reset to θ_c which corresponds to the smallest prediction error measure. Now consider a minimum phase plant, Equation (2), with a known relative degree and upper bound on plant order. In the following analysis, focus will be on plants with relative degree of 1 and 2. In addition, the sign of the high frequency gain of the plant k_p is assumed to be known. This assumption however may be relaxed [12].

$$\begin{aligned} \dot{x} &= A\,x + b\,u \qquad (2)\\ y &= C\,x \end{aligned}$$

where $x \in R^n$, $u \in R$, and $y \in R$. The objective is for the system output y to track the output of a reference model y_m given by

$$\begin{aligned} \dot{x}_m &= A_m\,x_m + b_m\,r \qquad (3)\\ y_m &= C_m\,x_m \end{aligned}$$

where r is a reference signal, and $y_m \in R$. Let W_m be the transfer function of the reference model of Equation (3). In addition, consider the filters w_1 and w_2 and the unknown parameter vector θ defined by

$$\begin{aligned} \dot{w}_1 &= \Lambda\,w_1 + l\,u\\ \dot{w}_2 &= \Lambda\,w_2 + l\,y\\ w &= [r, w_1^T, y, w_2^T]^T\\ \theta &= [k, \theta_1^T, \theta_0, \theta_2^T]^T \end{aligned}$$

where

$$\begin{aligned} \Lambda &= \begin{bmatrix} 0_{n-2\times 1} & I_{n-2}\\ -\lambda_1 \ldots & -\lambda_{n-1} \end{bmatrix}\\ l &= [0 \ldots 0\ 1]^T \end{aligned}$$

Λ is a Hurwitz matrix. For the case of relative degree 2 $L(s) = s + a$ is chosen such that $W_m(s)L(s)$ is strictly positive real (SPR). For the case of relative degree 1 $L(s) = 0$. In standard MRAC, the control law u and update law $\dot{\hat{\theta}}$ are defined by:

$$\begin{aligned} u &= \hat{\theta}^T w - sgn(k_p)\,e\,\bar{w}^T\bar{w} \qquad (4)\\ \bar{w} &= \frac{1}{s+a}\,w\\ \dot{\hat{\theta}} &= -sgn(k_p)\,\Gamma\,e\,\bar{w} \qquad (5) \end{aligned}$$

where $\Gamma = \Gamma^T > 0$. Using adaptive resetting we have the following update law:

$$\begin{aligned} \dot{\hat{\theta}}(t) &= -sgn(k_p)\,\Gamma\,e\,\bar{w} \qquad t \neq t_j\\ \hat{\theta}^+(t) &= \theta_c \qquad t = t_j \qquad (6) \end{aligned}$$

As explained earlier, the parameter estimates are updated as usual except at specific times t_j where the parameter estimates $\hat{\theta}$ are reset to θ_c chosen among candidate θ_i values according to Equation (1).

For analyzing stability of the system consider the Lyapunov function

$$V = e^T P\,e + \tilde{\theta}^T \Gamma^{-1}\,\tilde{\theta} \qquad (7)$$

where $e = y - y_m$ and $\tilde{\theta} = \hat{\theta} - \theta$. By using the control and the standard update laws presented above, $\dot{V}$ reduces to

$$\dot{V} = -e^T Q\,e\ ,\ \ Q > 0$$

hence, Lyapunov stability is shown. Then using Barbalat's lemma it can be shown that $e(t) \to 0$ as $t \to \infty$, see [14] for details.

Now consider the system with the resetting update laws of Equation (6). Since the resetting is due to inputs only, the adaptation law can be equivalently represented as, see [13] for details on resetting systems

$$\dot{\hat{\theta}}(t) = -sgn(k_p)\,\Gamma\,e\,\bar{w} + \Delta_\theta$$

where

$$\Delta_\theta = \sum_{j=1}^{N} M(t_j)\delta(t - t_j)$$

where $\delta(t - t_j)$ is the dirac delta function, N is the total number of resets, and $M(t)$ is a uniformly bounded vector $\|M(t)\| \leq m$ representing the jump in $\hat{\theta}$ due to resetting such that $M(t_j)$ equals the jump in $\hat{\theta}$ at $t = t_j$. Now using the Lyapunov function V of Equation (7), and the resetting update law $\dot{V}$ reduces to

$$\dot{V} = -e^T Q\,e + 2\tilde{\theta}^T \Gamma^{-1} \Delta_\theta \qquad (8)$$

Define $\eta = [\eta_1, \eta_2]^T = [e, \tilde{\theta}]^T$ and $\eta = S\,x$, where $S = diag(P^{1/2}, \Gamma^{-1/2})$ is a symmetric positive definite matrix.

Hence, the Lyapunov function can be written as $V = \|\eta\|^2$. Therefore, Equation (8) can be rewritten as

$$\begin{aligned}\dot{V}(\eta) &= 2\|\eta\| \frac{d}{dt}\|\eta\| \\ &= -\eta^T \begin{bmatrix} P^{1/2}QP^{1/2} & 0 \\ 0 & 0 \end{bmatrix} \eta + 2\,\eta_2^T\,\bar{\Delta}_\theta \\ &\leq 2\,\|\eta\|\,\|\bar{\Delta}_\theta\|\end{aligned}$$

where $\bar{\Delta}_\theta = \Gamma^{-1/2}\Delta_\theta$. Hence,

$$\|\eta\|(\frac{d}{dt}\|\eta\| - \|\bar{\Delta}_\theta\|) \leq 0$$

Which implies that

$$\frac{d}{dt}\|\eta\| \leq \|\bar{\Delta}_\theta\|$$

Hence

$$\begin{aligned}\|\eta(t)\| &\leq \|\eta(t_o)\| + N\|\Gamma^{-1/2}\|\|M(t)\| \int_{t_o}^{t} \delta(t-t_j)\,dt \\ &\leq \|\eta\|(t_o)\| + c_o\end{aligned}$$

where $c_o = N\|\Gamma^{-1/2}\|m$ is a constant for a finite number of resets N. To show the bound on x, note that

$$\|x(t)\|\,\underline{\sigma}(S) \leq \|Sx\| \leq \|x\|\,\overline{\sigma}(S)$$

where $\underline{\sigma}(S)$ and $\overline{\sigma}(S)$ are the minimum and maximum singular values of S respectively. Hence

$$\|x(t)\| \leq \frac{\overline{\sigma}(S)}{\underline{\sigma}(S)}\,\|x(t_o)\| + \frac{c_o}{\underline{\sigma}(S)} \tag{9}$$

Therefore, the feedback system will remain stable for a finite number of resets.

III. Case Study: Control of Microcantilever

Microcantilevers are among the most used mechanical structures in applications of micro and nano-actuation and sensing. They are easy to fabricate and simple to model. Microcantilevers are typically actuated using piezoelectric implants or electrostatic actuation. On the other hand, cantilevers with piezoresistive elements or combined with optical or capacitive sensing are commonly used as sensors. A single-mode model of a microcantilever is given by

$$m\,\ddot{x} + c\,\dot{x} + k\,x = b\,u + f \tag{10}$$

where m is cantilever modal mass, c is modal damping coefficient, k is the modal cantilever stiffness, u is the control input, b is a control input gain, and f is an interaction force acting on the cantilever. The nature of f depends on the application and it could be magnetic, electrostatic, intermolecular such as van der Waals, capillary, hydrogen bonding, or double layer. The common characteristics of these forces that they are a strong nonlinear function of the separation distance Δx between the interacting objects. The interaction range can be long-range or short-range and can be attractive, repulsive or both. In addition, f depends on geometry of interacting objects and it is extremely sensitive to the environment medium and conditions including humidity, temperature, and surface chemistry.

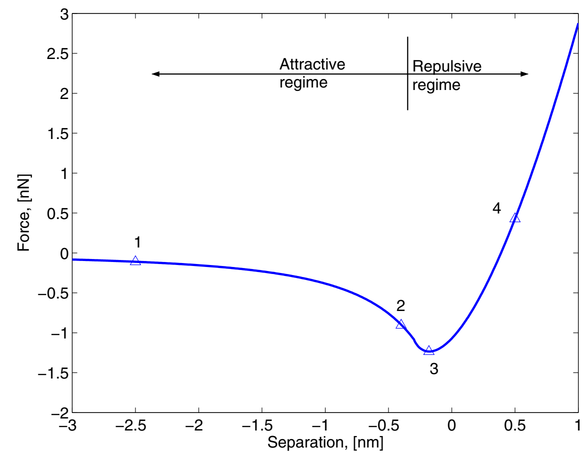

Fig. 2. Interaction force f vs. probe-object separation Δx.

Now consider a scenario where a microcantilever is interacting with an object where the interaction force $f(\Delta x)$ is given in Figure 2. The separation is defined as the difference between the absolute position of the cantilever end x and the probed object position x_{ob}, $\Delta x = x_{ob} - x$. As seen from Figure 2, f has both an attractive $\Delta x < 0$ and repulsive regimes $\Delta x > 0$. The transition between both regimes does not necessarily occur at $\Delta x = 0$. The transition however occurs over a very short separation (several Angstroms). In a dynamic interaction between the cantilever and an object this transition would occur over a very short time scale. As previously mentioned, the interaction force depends on the separation which in turn depends on the cantilever deflection among other things. Now consider the cantilever dynamics at several possible operating points in the force field f namely, points 1, 2, 3 and 4. The interaction force can be linearized at any of these operating points as

$$f(\Delta x_i) \approx -\frac{\partial\,f}{\partial\,x}|_i\;x + \Delta f \tag{11}$$

where $\Delta\,f$ contains a nominal force, higher order terms in x and terms in x_{ob}. Substituting Equation (11) into (10) leads to

$$m\,\ddot{x} + c\,\dot{x} + (k + \frac{\partial\,f}{\partial\,x}|_i)\,x = b\,u + \Delta f \tag{12}$$

As seen from Equation (12), the resonant frequency of the cantilever changes along with the DC gain. In reference to Figure 2, $\frac{\partial f}{\partial x}|_i$ is small at operating point 1 and 3, hence, the parameters of the cantilever dynamics is expected to be slightly changed at these points by the force field. At operating point 2, $\frac{\partial f}{\partial x}|_i < 0$, hence tending to decrease the cantilever resonance frequency while increasing the DC gain. In the case of $-\frac{\partial f}{\partial x}|_i > k$, this operating point becomes unstable and the cantilever would move away from that point. On the other hand, $\frac{\partial f}{\partial x}|_i > 0$ at operating point 4, corresponding to an increase in the resonant frequency and a decrease in the DC gain. The cantilever

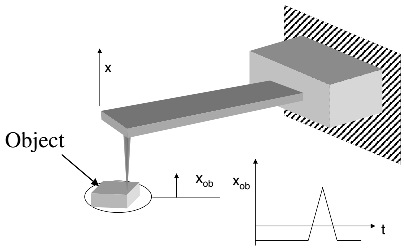

Fig. 3. Schematic of simulation set-up.

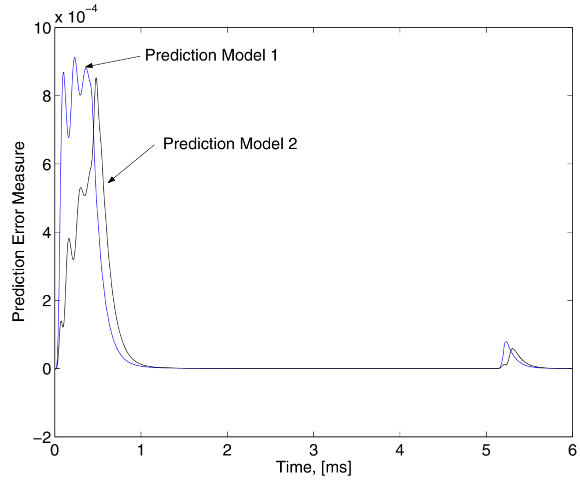

Fig. 5. Prediction error measures.

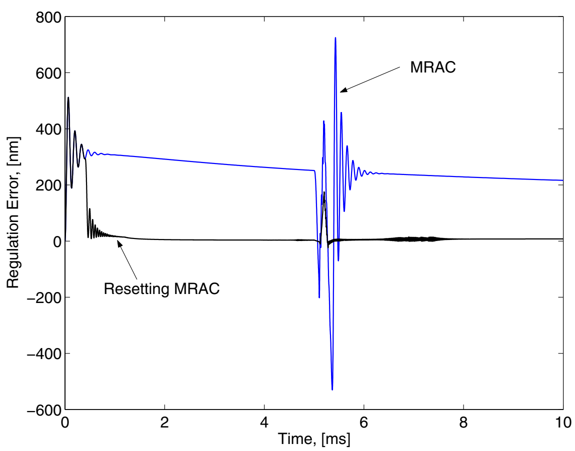

Fig. 4. Regulation error.

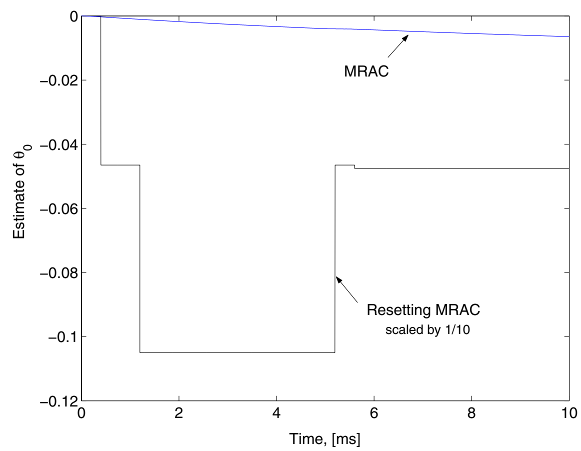

Fig. 6. Parameter estimate $\hat{\theta}_0$.

dynamics can change when interacting with objects that may adhere to the cantilever during interaction. This would increase the effective mass m hence lowering the resonant frequency. Moreover, changes in the dissipation between the cantilever and probed object may result in large changes in c.

Consequently, the cantilever dynamics may be represented by a set of possible models each with a set of parameters θ_i as suggested in Section II. Adaptive control is proposed in order to ensure stable robust operation while resetting is utilized to improve transient and allow fast dynamic response.

A simulation study was performed to investigate the potential of the proposed resetting adaptive control applied to a microcantilever. In this study the parameters of the cantilever are selected for a commercial microcantilever of stiffness $k = 0.025\,N/m$ and resonance frequency of $10\,kHz$. The cantilever is introduced suddenly at $t = 0$ in the force field of Figure 2 at far away from the probed object, causing a small initial attractive force. A schematic of the setup is shown in Figure 3. At $t = 5\,ms$ the object is moved towards the cantilever passing the separation point 4 of Figure 2. Ramping x_{ob} occurs over $200\,\mu s$. Then the object x_{ob} is moved away from the cantilever to the initial separation over $200\,\mu s$. The control objective is to maintain the cantilever deflection x at zero (i.e. maintain a constant force between the cantilever and the probed object). This scenario could represent experiments such as manipulating an object by picking it up by functionalizing the cantilever with an appropriate agent such that the object would adhere to the cantilever.

Two prediction models are used corresponding to operating points 1 and 4 of Figure 2, respectively. The prediction error measure of Equation (1) is computed and resetting is allowed every $2.5\,kHz$. The controller is only allowed to reset if the set-point error is larger than a threshold $10\,nm$. This prevents the potential of continuous resetting in the presence of noise and disturbances. In Figure 4, cantilever set-point error is shown while $\hat{\theta}_0$ and prediction error measures for both models are shown in Figures 6 and 5, respectively. As seen in Figure 4, the slow transient of the MRAC is increased drastically with controller rest to model 1 at time $t = 0.4\,ms$. As the object moves toward the cantilever passing operating point 4 (of Figure 2), the controller automatically resets to model 2 parameters (corresponding to point 4). This can be seen from Figure 5 at times where the prediction error measures for both models change order (smaller/larger).

IV. CONCLUSIONS

Control of micro and nano-systems imposes new and stringent challenges to be addressed. The nonlinear time varying and uncertain dynamics operating at different time and length scales requires robust ultra-fast feedback systems capable of addressing these unique challenges. A resetting adaptive control architecture based on MRAC is proposed. Stability analysis demonstrates that with a finite number of resets, the feedback system remains stable. The proposed resetting adaptive control is applied to control of microcantilever in a nonlinear interaction. Simulation results demonstrate the potential of the resetting adaptive control in improving transients while maintaining stability in the presence of uncertainties. Further work is needed to investigate the full potential of resetting control and address practical implementation issues.

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