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Multi–Controller Adaptive Control (MCAC) for a Tracking Problem using an Unfalsification approach

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Abstract— **In this paper, we apply a multiple controller based adaptive method to solve a tracking problem, where the plant output is required to track a reference input. The proposed methodology is based on an unfalsification approach. The method relies on a finite set of candidate–controllers; depending on the evolving plant data, it learns and selects an optimal controller from the candidate controller set. Although prior plant knowledge is helpful in selecting the candidate controller set, the method makes no use of, nor tries to identify, the plant structure or its parameters while deciding the optimal switching sequence. Probable performance of candidate controllers is evaluated directly from the plant data.**

Index Terms— **Multi–controller based adaptive control (MCAC), adaptive pole placement, unfalsification, unfalsified control.**

I. INTRODUCTION

In the past few decades multiple controller based adaptive schemes has gained popularity among the adaptive control researchers. The early work in this field was done by Martensson [1] in mid 80s. He proposed a pre–routed search among the candidate controllers, *i.e.* , the controllers were switched in the feedback loop in a pre–determined sequence until one controller was found which could achieve the control objective. However, such schemes might take a long time to switch to a stabilizing controller. Specifically when the number of candidate controllers is large, the method gives poor transient response. Other pre–routed based switching schemes can be found in [2], [3].

Most of the recent switching based algorithms select the candidate controllers more intelligently, based upon on–line evaluation of the plant input/output data. These switching schemes are more efficient and result in better performance than the blind pre–routed search. These algorithms can broadly be divided in two categories: those based on indirect method of process estimation [4]–[8]; and those based on direct evaluation of the potential performance of candidate controllers [9]–[14].

The indirect estimator based switching [4]–[8] relies on system identification. Here, the actual plant behavior is constantly compared with that of several candidate plant models and the model that best approximates the actual plant is identified at each time. The basic assumption here is that there is at least one plant model in the candidate model set which is *sufficiently* close to the actual process to be controlled.

An alternative to the above method is the more direct controller performance based switching, popularly referred to as the unfalsification approach in some recent publications [9]–[14]. Here, the potential performance of every candidate controller is evaluated at each time directly from the measured data using some suitably defined performance index, without trying to estimate or identify the actual process. This index is a measure of how closely the output of the closed loop system would have followed some reference input, had the candidate controller been in the feedback loop. Note that the performance index of all the candidate controllers can be evaluated from the open loop plant input–output data (u_p and y_p), without actually inserting all the controllers in the feedback loop. This method is completely plant–model free; the only assumption here is the presence of at least one stabilizing controller in the candidate controller set, provided one judiciously chooses a *cost detectible* performance index.

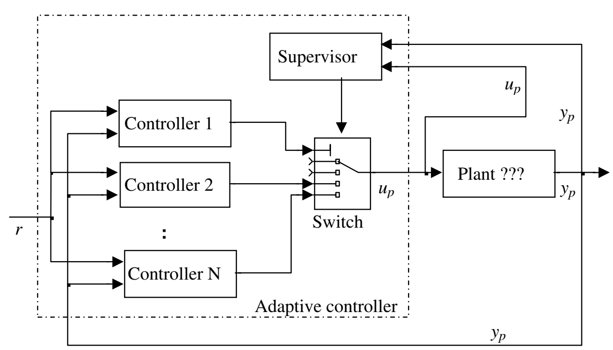

Fig. 1. A general architecture for switching based control

A popular class of control scheme that involves stabilizing the plant by changing the closed loop pole was studied by Ioannou and Sun in [15] and several references therein. In [15], the pole changing method was coupled with the internal model principal of the reference signal, which resulted

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in perfect target tracking for a class of reference signals [15]. In this paper, we take an alternate approach; we apply the direct controller performance based multiple controller based switching to solve the pole placement problem.

The paper is organized as follows. In the next section, we present the problem statement and the structure of the controller used. In Section III, we discuss the basic concepts of a MCAC system and the unfalsification theory. In Section IV, we apply these theories to adapt the controller for the problem discussed earlier. In the last two sections, we give simulation results and our conclusion.

II. Problem Statement

Consider a linear, time invariant plant, with input u_p and output y_p, related by

$$y_p = G_p(s)u_p,\ G_p(s) = \frac{Z_p(s)}{R_p(s)}, \tag{1}$$

where $G_p(s)$ is proper and $R_p(s)$ is a monic polynomial. The tracking error is given by

$$e = r - y_p, \tag{2}$$

where r is the reference signal. The control objective is to choose the plant input u_p such that all the closed-loop poles are in the left half plane so that the system is stabilized; moreover y_p is required to track the reference signals r, which we achieve using the internal model principle.

The reference signal is assumed to satisfy

$$Q_m(s)r \ = \ 0, \tag{3}$$

where the polynomial $Q_m(s)$ is the internal model of r. It is a monic polynomial of degree, say, q. Using prior knowledge about the reference signal, $Q_m(s)$ can be calculated easily [15]. For example, for a step input, $Q_m(s) = s$.

Assumption 1: The only assumption needed for unfalsified control theory, as stated later in Section III, is the existence of a robustly stabilizing controller in the candidate controller set and an appropriately chosen cost detectible performance index; the switching scheme stated in the Algorithm 1 in Section III ensures convergence to a stabilizing controller. *However, to form the algebraic structure of the controller, we for now make the following assumptions (which will be later relaxed) [15]:*

1. $R_p(s)$ is a monic polynomial whose degree n is known.

***Remark** 1:* We can relax the above by assuming knowledge of an upper bound on the degree of $R_p(s)$ and including candidate controller for all possible orders of the plant.

2. $Z_p(s)$ and $R_p(s)$ are coprime and degree of $Z_p < n$. Note that unlike Model Reference Adaptive Control (MRAC) problem, we do not require the plant to be minimum phase [4], [10].

3. $Q_m(s)$ and $Z_p(s)$ are coprime. ◇

The input to the plant is given by [15]

$$u_p = \frac{\Lambda(s) - L(s)Q_m(s)}{\Lambda(s)}u_p - \frac{P(s)}{\Lambda(s)}(y_p - r), \tag{4}$$

where the polynomials $P(s)$ and $L(s)$ forms the controller parameters with degree
$P(s) : q + n - 1$
$L(s) : n - 1$ (L(s) monic)
and $\Lambda(s)$ is any monic Hurwitz polynomial of degree $n + q - 1$. The controller structure is as shown in Fig. 2. The closed loop plant characteristic polynomial is given by

$$LQ_m(s)R_p(s) + P(s)Z_p(s). \tag{5}$$

Using Bezout identity, it can be shown that for any given $R_p(s), Z_p(s), Q_m(s)$, we can choose polynomials $P(s), L(s)$ such that

$$LQ_m(s)R_p(s) + P(s)Z_p(s) = A^*(s)$$

is satisfied for any monic Hurwitz polynomial $A^*(s)$ of degree $2n + q - 1$.

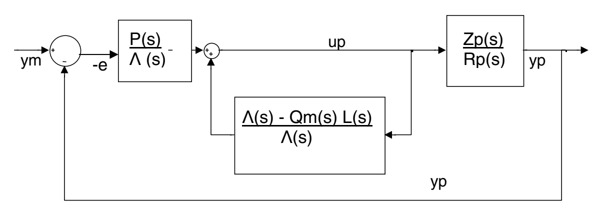

Fig. 2. Controller structure for pole placement problem [15]

The error signal can be calculated as

$$e = -\frac{L(s)R_p(s)}{LQ_m(s)R_p(s) + P(s)Z_p(s)}Q_m(s)r(s), \tag{6}$$

where the denominator polynomial is the characteristic polynomial of the closed loop plant. If the closed loop plant poles can be placed in the left half plane, then e converges to zero exponentially as $Q_m r = 0$ (where Q_m is the internal model polynomial of r). This objective (of placing the closed loop poles in the left plane) can be met by monitoring a suitable norm of the error signal (performance index) for each candidate controller (corresponding to a different reference signal, the *fictitious reference signal*, discussed in Section III) and placing the candidate controller in the loop with the best performance index.

***Remark** 2:* Note that the internal model of the reference signal, $Q_m(s)$ is assumed to be known (*i.e.*, the reference signal is known). In this case, the problem can be made much simpler by segregating the pole placement problem and problem of tracking the reference signal by using the internal model principal. In that case, $\frac{1}{Q_m(s)}$ can be cascaded in series with the plant and we can define control input to the plant as

$$u_p = Q_m(s)u_p',$$

where u_p' is the output of the controller of Fig. 2, without the term $Q_m(s)$. There would be corresponding change in the order of the controller parameters.

If there is no a priori information about the reference (i.e., $Q_m(s)$ is unknown), then we can employ multiple candidate $Q_m(s)$ (along with multiple $P(s)$ and $L(s)$) and select the optimal one using the switching logic.

III. Unfalsified Control Theory

In this section, a brief description of unfalsification approach is presented, and then the proposed methodology is discussed for the problem stated in the previous section. Consider an unknown plant P. Let $\mathbb{Z}$ be the universum of possible signals, and $z = [u_p, y_p] \in \mathbb{Z}$ be the plant Input–Output (I/O) signal measured from time 0 to current time t. We need to determine a control law K^* from a class of candidate control law $K_i, i = 1, \ldots, N, \;\; K_i \in \mathbb{K}$, so that the closed loop system response, say T, satisfies a specification requiring that for all command inputs $r \in \Re$, the triple (r, y_p, u_p) be in the given specification set $\mathbb{T}_{spec}$.
Given a set of past plant I/O data (u_p, y_p), and a candidate controller $K_i \in \mathbb{K}$, we now define fictitious reference input $\widetilde{r}(K_i, y_p, u_p)$ for the candidate controller.

***Definition** 1: (Fictitious Reference Signal)* Given a set of past measured plant I/O data (u_p, y_p), and a candidate controller $K_i \in \mathbb{K}$, a fictitious reference signal for this candidate controller is a hypothetical reference signal that would have produced exactly the measured data (u_p, y_p) had the candidate controller K_i been in the feedback loop with the unknown plant during the entire time period over which the measured data (u_p, y_p) were collected. $\diamond$

If the i^{th} controller is actually in the loop during the entire period over which plant I/O data (u_p, y_p) were collected, then the i^{th} fictitious reference signal would be same as the actual reference signal; else it would be different from the actual reference signal (hence the name *fictitious*). Throughout the paper, we will denote all the fictitious signals by placing a $\widetilde{}$ above the signal.

***Remark** 3:* To determine a unique fictitious reference signal, the controller has to be *Stably Causally-Left-Invertible* **(SCLI)**, that is, from the past and present output of the controller, one should be able to uniquely determine its present input and it should also be stable in $\mathcal{L}_{2e}$ space. The fictitious reference signal for the candidate controller K_i is represented by $\widetilde{r}(K_i, u_p, y_p)$ and for notational convenience, would also be denoted by $\widetilde{r}_i$. $\diamond$

As an example, consider the controller structure in Fig. 3;

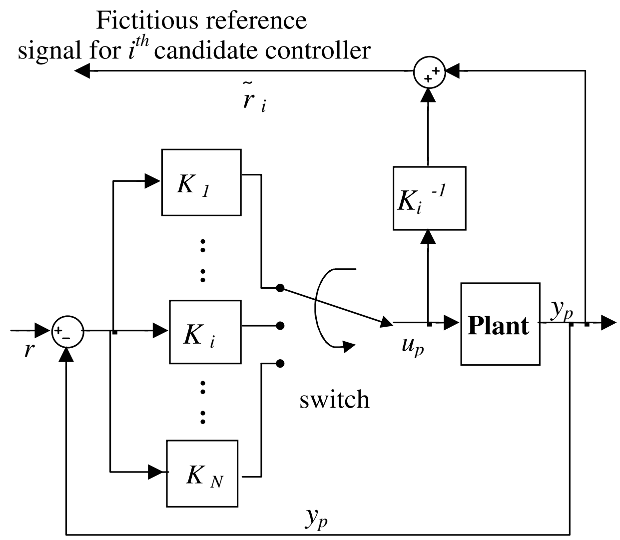

Fig. 3. Fictitious reference signal generator for the i^{th}controller for an example controller structure.

the i^{th}controller is SCLI if the inverse of the K_i exists and is stable, *i.e.* , the candidate controller is minimum phase and bi–proper. For the above example, the fictitious reference signal for the i^{th}controller is given by

$$\widetilde{r}(K_i, u_p, y_p) \triangleq K_i^{-1}u_p + y_p. \tag{7}$$

Controllers for several popular problems, like Model Reference Adaptive Control (MRAC) [10], PID controller with positive proportional, integral and approximate derivative gains [14], the controller structure used in this paper satisfy the SCLI property.

***Definition** 2: (Fictitious error signal)* For each candidate controller, a fictitious error signal can be defined as the error between its fictitious reference signal and the actual plant output. Hence, this would have been the error signal, had that candidate controller been in the feedback loop with (u_p, y_p) as the measured plant data and $\widetilde{r}_i$ as the reference signal. $\diamond$

The fictitious error signal for the candidate controller K_i is represented by $\widetilde{e}(K_i, u_p, y_p)$ and for notational convenience, would henceforth be denoted by $\widetilde{e}_i$ and is given by

$$\widetilde{e}_i(K_i, u_p, y_p) \triangleq \widetilde{r}_i(K_i, u_p, y_p) - y_p \tag{8}$$

Performance index: Now, y_p is the output of the actual process with K_i controller in the loop and with the i^{th} fictitious reference signal $\widetilde{r}_i$ as the command signal (follows from the definition of fictitious reference signal). Hence, the error $\widetilde{e}_i$ would have been the control error, had the i^{th} candidate controller K_i been in the feedback loop during the entire time period over which the measured data (u_p, y_p) were collected, with plant I/O data as (u_p, y_p) and fictitious reference signal $\widetilde{r}_i(t)$ as the reference signal. Thus, the error $\widetilde{e}_i$ is a measure of effectiveness of the i^{th} candidate controller if it is placed in the loop, i.e., how closely will the feedback system with the i^{th} controller in the loop follow the given reference signal. So, ordering the candidate controllers based on their potential performance and the switching among the controllers should be based on some norm of this error. We propose the following performance index for the i^{th} candidate controller

$$\widetilde{J}(\widetilde{r}_i, u_p, y_p) \triangleq \begin{cases} \max\limits_{l\in(0,t)} \dfrac{\|\widetilde{e}_i\|_l^2 + \alpha\|u_p\|_l^2}{\|\widetilde{r}_i\|_l^2}, & if\|\widetilde{r}_i\|_l \neq 0 \\ 0 & \|\widetilde{r}\|_l = 0 \end{cases}, \tag{9}$$

where $\widetilde{e}_i$ and $\widetilde{r}_i$ are the fictitious error and fictitious reference signals corresponding to the i^{th}controller,

$$\|x\|_l^2 = \int_0^l exp^{-\lambda(l-\zeta)} x^2(\zeta) d\zeta,$$

and α is a non–negative weighting constant. The norm of the signals also includes the non–negative exponential forgetting factor λ, that de–emphasizes the importance of distant past signals and places more emphasis on the current value of the signal. The error term $\|\widetilde{e}_i\|$ is required for stabilization and the term $\|u_p\|$ ensures bounded and small control signal. The max operator in (9) ensures monotonically increasing cost. Detailed discussion on the choice of the performance index and the rationale behind can be found in [16].

***Definition** 3:* (*Unfalsified stability* [16]) Given a candidate controller K_i and corresponding fictitious reference signal $\widetilde{r}(K_i, u_p, y_p)$, we say the stability of the system with the i^{th}candidate controller is *falsified* if there exists some $\widetilde{r}(K_i, u_p, y_p)$ such that $\lim\limits_{t\to\infty} \dfrac{\|y_p\|}{\|\widetilde{r}(K_i, u_p, y_p)\|} = \infty$. $\diamond$

***Definition** 4: (Cost detectability of the performance index)* A system is said to be cost detectable, if, whenever the stability of the system, with controller K_i in the loop is falsified by data (u_p, y_p), then $\widetilde{J}(K_i, u_p, y_p) = \infty$. $\diamond$

***Remark** 4:* Cost detectability means the destabilizing behavior of a candidate controller is reflected in its performance index. It can be shown that the proposed performance index of (9) is *cost detectable*, however a formal proof is omitted because this is an application based paper and due to space limitation.

Assumption 1a: The candidate controller set $K_i, i = 1, \ldots, N$ contains at least one stabilizing controller for the unknown plant. $\diamond$

***Remark** 5:* In Section II, we had certain assumption on the plant (Assumption 1) which we needed to show that the controller structure has sufficient degree of freedom to satisfy the control requirement. However, here we modify that assumption, because for the algorithm to work, this is the minimum assumption needed. We agree that to satisfy Assumption 1a in practice, some prior knowledge about the plant would be needed. However, we feel that Assumption 1a presented above is less restrictive than Assumption 1 in Section II.

Given plant I/O data (u_p, y_p), a finite set of candidate controllers given by $K_i, i = 1, \ldots, N$, the following algorithm is used to switch among the candidate controller.

***Algorithm** 1:* **Initialization:**

- A finite set of N number of candidate controllers, given by $K_i,\ i \in I \triangleq 1, \ldots, N$.
- Set initial performance index $\widetilde{J}_i(0) = 0,\ \forall\, i \in I$.
- Let the controller actually in the loop be given by K^*. At time $t = 0$, select any candidate controller as the initial controller.

Procedure:

(1) Calculate the following signals: $\widetilde{r}_i, \widetilde{e}_i, \widetilde{J}_i,\ \forall\ i \in I$, using (7)–(9).

(2) If $\widetilde{J}(K^*, u_p, y_p) > \min\limits_{K_i, i\in I} \widetilde{J}(K_i, u_p, y_p) + \epsilon$, then $K^* = \arg \min\limits_{K_i, i\in I} \widetilde{J}(K_i, u_p, y_p)$.

(3) Go to step 1. $\diamond$

The above algorithm says that the switching occurs when the performance index of current controller in the loop exceeds the minimum of the performance index of other candidate controllers by at least ϵ. The non–negative hysteresis factor ϵ prevents arbitrary fast switching among candidate controllers and ensures a non-zero dwell time between switches.

***Proposition** 1:* [16] If there exists at least one stabilizing controller among the set of candidate controllers (Assumption 1a), then Algorithm 1, with the performance index in (9), will always converge to a stabilizing controller with finitely many switches.

Proof: See [16]. $\diamond$

Proposition 1 guarantees that a stabilizing controller is chosen in finitely many switches under Assumption 1. In that case, the internal model principal (polynomial Q_m) will ensure perfect tracking, as stated in Section II.

IV. Unfalsified control theory applied to the pole placement problem

In this section, we apply the unfalsified switching based theory to the problem in hand. From the control law in (4) and from Definition 1, the fictitious reference signal for the i^{th} controller, with parameters as $P_i(s)$, $L_i(s)$ and $Q_{m\,i}(s)$,

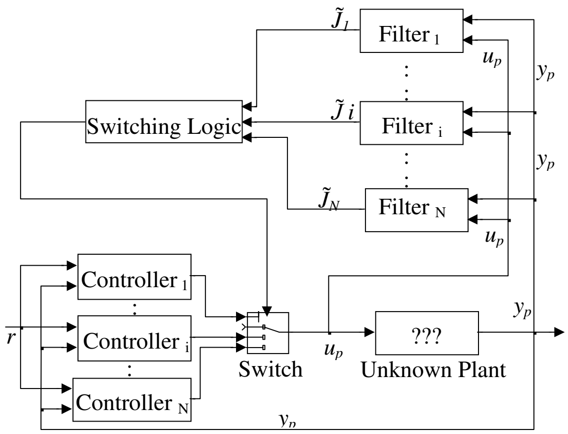

Fig. 4. General architecture for MCAC

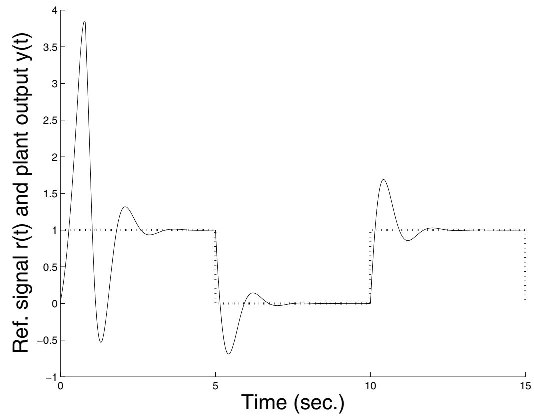

Fig. 6. Simulation: $\cdots$ reference signal r(t), – plant output $y_p(t)$

can calculated as

$$\widetilde{r}_i = \frac{\lambda - P_i}{\lambda}\widetilde{r}_i + \frac{Q_{m\ i}L_i}{\lambda}u_p + \frac{1}{\lambda}y_p. \qquad (10)$$

The fictitious error for the i^{th} candidate controller $\widetilde{e}_i$ can be calculated using (8). As stated in the previous section, this would have been the control error, had the i^{th}candidate controller been in the loop, with $\widetilde{r}_i$ as the reference signal and (u_p, y_p) as the measured plant I/O data. Hence this signal is an indication of the ability of the controller to track the reference input, if placed in the feedback loop with the unknown plant. Thus as per justification given in the previous section, the switching and ordering of controllers is based on a performance index, which is a quadratic function of this error, given by (9). The fictitious reference signal generator for the i^{th}controller is as shown in Fig. 5.

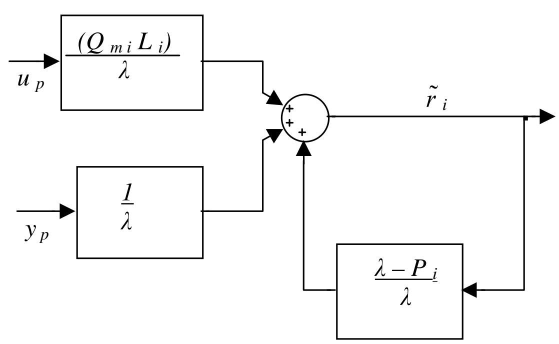

Fig. 5. Fictitious reference signal generator for the i^{th}candidate controller (from (10)).

V. SIMULATION RESULTS

In this section, we present some simple simulation results to illustrate the efficacy of the proposed method. A first order plant is considered, with transfer function as $G_p(s) = \frac{2}{s-4}$. To keep simulation results simple, it is also assumed that the degree of the plant is known. But let us also note that for the theory to work, the only assumption needed is the existence of a stabilizing controller in the candidate controller set (Assumption 1a).

The reference signal is a rectangular wave generator and is assumed to be known. The reference signal can be viewed as step change and hence, the internal model is given by $Q_m(s) = s$. The monic, stable polynomial $\lambda(s)$ is taken as $(s+1)$. For the first order plant used, the polynomial $L(s) = 1$. Six numbers of candidate controllers are used, with different candidate polynomial $P(s)$, given by $P_i, i = 1, \ldots, 6$. The candidate $P(s)$ are as follows: $(s+3), (s+10), (4s+3), (4s+10), (2.8s+3), (2.8+10)$ and would be denoted as $P_1, \ldots, P_6$ in the order presented. It can be shown that the closed loop plant, with P_1 or P_2 in the loop, is unstable. With P_3 or P_4 in the loop, the real part of the poles of the closed loop plant is at –2. With P_5 or P_6, the real part of the poles of the closed loop plant is at –0.8. Hence, the closed loop system with 3^{rd} and the 4^{th} candidate controller has roots with real part in the left–most of the complex plane. The plot in Fig. 7 shows the switching signal and it is seen that the switching settles down within the first few seconds to the 4^{th} candidate controller, which is a stabilizing one. Also, from Fig. 6, the system is seen to track the reference signal asymptotically.

Based on extensive experiments with various simulation scenarios, plants models and candidate controller sets, we have confirmed that the switching always settles down to a stabilizing controller, provided there exists one. Moreover, we have observed, as in this simulation, that in presence of more than one stabilizing controller, the switching stabilizes to the controller have faster response (closed loop poles having greater negative real part). This can be explained by the fact that for the stabilizing controllers, irrespective of the reference signal (*i.e.* , even with the *fictitious* reference signal), the response is faster, thereby decreasing the (*fictitious*) error at a much faster rate.

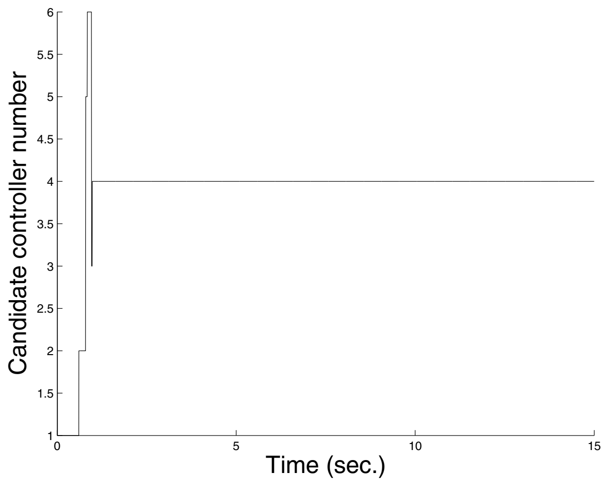

Fig. 7. Simulation: Switching sequence

VI. Conclusion and Discussion

In this paper, we have studied a stabilization problem coupled with the internal model principal of the reference signal for perfect tracking. Although this kind of controller structure has been studied before [15] using standard assumptions (Assumption 1), here we have solved the problem using multiple controller and switching using unfalsification concept and less restrictive assumption (Assumption 1a). The only assumption necessary is the presence of a stabilizing controller in the candidate controller set, provided one judiciously chooses a performance index. The novelty of our method is that it does not try to identify or estimate the plant; rather it tries to determine the probable performance of the candidate controllers, all at the same time, without placing all of them in the loop. Future work will focus on comparing this methodology with other available adaptive methods, study in more details the minimum set of assumptions needed by those other methods and propose a systematic method of choosing a candidate controller set in which at least one stabilizing controller exists.

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