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Designing an observer-based controller for a network control system

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***Abstract*— We propose a numerical procedure to design a linear output-feedback controller for a remote linear plant in which the loop is closed through a network. The controller stabilizes the plant in the presence of delay, sampling, and dropout effects in the measurement and actuation channels. We consider two types of control units: anticipative and non-anticipative. In both cases the closed-loop system with delay, sampling and packet dropout effects can be modeled as a delay differential equation. Our method of designing the controller is based on the Lyapunov-Krasovskii theorem and a linear cone complementarity algorithm. Numerical examples show that our method is significantly better than the existing ones.**

I. INTRODUCTION

Network Control Systems (NCSs) are spatially distributed systems in which the communication between plants, sensors, actuators and controllers occurs through a shared band-limited digital network. Using networks as a medium to connect elements of the system reduces wiring cost and maintenance, since there is no need for point to point wiring. Consequently, NCSs have been finding application in a broad range of areas such as mobile sensor networks, remote surgery, haptics collaboration over the Internet and unmanned aerial vehicles [1].

Data is sent through the network as atomic units called *packets*. Therefore any continuous-time signal must be appropriately sampled to be carried over a network. Hence there are some similarities between NCSs and sampled-data systems due to the *sampling effect*. However NCSs are significantly different from standard sampled-data systems since the delay in the control system loop can be highly variable due to both *access delay* (i.e., the time it takes for a shared network to accept data) and *transmission delay* (i.e., the time during which data are in transit inside the network) depend on highly variable network conditions such as congestion and channel quality. Since access and transmission delays have the same effect with respect to the stability of NCSs, throughout the paper we use the term *(NCS) delay* referring to access/transmission delay.

Data packets may be discarded at any point between the source and the destination. *Packet dropout* occurs along the network due to uncertainty and noise in communication channels. It may also occur at the destination when out of order delivery takes place. In reliable transmission protocols that guarantee the eventual delivery of packets, data is resent again. However NCSs should operate with non-reliable transport protocols since transmission of old data is not suitable as new data is available.

We want to design an observer-type output feedback control unit that remotely stabilizes the plant even in the presence of network effects, i.e., delays, sampling, and packet dropouts in the (sensor) measurement and actuation channels. We consider two types of control units: non-anticipative and anticipative. Let's assume that the sampling interval in actuation channel is constant and eqaul to h^a. A non-anticipative control unit sends control updates at times ℓh^a, $\ell \in \mathbb{N}$, equal to $\{u_\ell : \ell \in \mathbb{N}\}$, where u_ℓ is a single valued control command applied to the plant until the next command arrives. An anticipative control unit sends control updates at times ℓh^a, $\ell \in \mathbb{N}$, equal to $\{u_\ell(\cdot) : \ell \in \mathbb{N}\}$, where $u_\ell(\cdot)$ is a signal with a proper duration sent at times ℓh^a to be used until the next control command arrives. If we assume the actuation channel delay is constant, each $u_\ell(\cdot)$ is a control signal of duration equal to the sampling interval h^a. Through an example we compare the anticipative control unit and the non-anticipative one.

In this paper we follow the same steps as in [2] to model network effects as a delay differential equation (DDE). An NCS with LTI plant model, anticipative or non-anticipative controller and network effect can be modeled as a DDE of the form

$$\dot{\bar{x}}(t) = A_0\bar{x}(t) + \sum_{i=1}^{2} A_i\bar{x}\big(t-\tau_i(t)\big), \tag{1a}$$

$$\tau_i(t) \in \big[\tau_{i\min}, \tau_{i\max}\big), \quad \forall t \geq 0,\ \dot{\tau}_i(t) = 1\ a.e. \tag{1b}$$

The delay's bounds $\tau_{i\min}$ and $\tau_{i\max}$ for $i = 1,2$, are positive and functions of sampling intervals, maximum number of dropouts[1] and upper and lower bounds on the delay in the measurement and actuation channels. We find sufficient conditions for asymptotic stability of (1), based on a Lyapunov-Krasovskii functional, formulated in the form of matrix inequalities. Our stability result is closely related to [3], [4], [5] where the stability and the state feedback stabilization of (1a) are studied for either $\tau_i(t) \in \big[\tau_{i\min}, \tau_{i\max}\big)$, $\dot{\tau}_i(t) < d < 1$, or for $\tau_i \in \big[0, \tau\big)$, $\dot{\tau}_i(t) = 1$ almost everywhere. In [2] the Razumikhin theorem is used to design an output-feedback controller, which generally leads to conservative designs [6]. However our analysis is based on a new descriptor system approach and the Lyapunov-Krasovskii functional, proposed by Fridman and Shaked [3]. We expect less conservative results and we will illustrate the improvement with respect to the previous results by applying our method to the example in [2].

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[1] Number of dropouts means number of *consecutive* packet dropouts.

For a given controller, the matrix inequalities that guarantee asymptotic stability of (1a) are linear matrix inequalities (LMIs). However for designing a controller the matrix inequalities are bilinear matrix inequality (BMI) and non-convex, with no tractable method to solve them. We propose a numerical method based on a linear cone complementarity introduced in [7] to solve the problem. This method converts the feasibility of the original non-convex matrix inequalities to convex optimization of a linear function subject to a set of LMIs, which can be effectively solved by numerical packages such as MATLAB.

Stability and stabilization of NCSs have received significant attention in the literature. Montestruque and Antsaklis [8], [9] study the stability of model-based NCSs. They use an explicit model of the plant to produce an estimate of the plant's state between transmission times which allows reduced the network usage. In [10] stability of NCSs with uncertain time delays and packet dropouts in the framework of switched systems is investigated. Branicky et al. [11] analyze the influence of the sampling and delay on the system stability by using hybrid system stability analysis techniques. Yu et al. [2] design an observer-type output feedback controller to stabilize a plant through a network with admissible bounds on dropouts and delays, based on the Razumikhin theorem. In survey paper [1] a collection of works in the area of NCSs can be found.

This paper is organized as follows: In section II we introduce anticipative and non-anticipative control units. We show that the system equations of both types can be written as (1), since sampling and packet dropout effects can be captured as fictitious delay with derivative one almost everywhere. In section III we find a sufficient condition for asymptotic stability of system (1) in the form of LMIs. In section IV a numerical procedure is proposed to design a controller to stabilize the plant for admissible bounds on delays, dropouts and sampling intervals. Then through examples we illustrate the use of our method.

II. Network control system modeling

Figure 1 shows an NCS consisting of a plant, actuator, sensor and control unit where the plant, actuator and sensor are compound. The plant is LTI with state space model of the form

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad y(t) = Cx(t), \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^l$, $y(t) \in \mathbb{R}^m$ are the state, the input and the output of the plant, respectively. The measurements are sampled with periodic sampling interval equal to h^s[2] and sent at times kh^s, $k \in \mathbb{N}$. Assuming for now, that there are no dropouts, the measurements $\{y(kh^s) : k \in \mathbb{N}\}$ are received by the control unit at times $kh^s + \tau_k^s$ where τ_k^s is the delay that measurement sent at kh^s experiences. These are used

[2]Superscripts s and a are used to label the network effects in (sensor) measurement and actuation channels respectively.

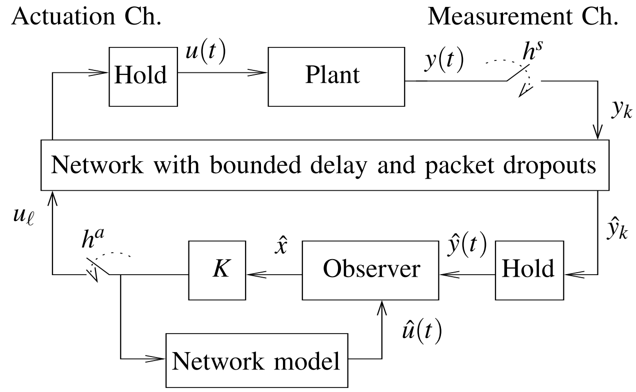

Fig. 1. Two channel feedback NCS with an observer-based controller.

to construct an estimate of the plant state using

$$\begin{aligned}\dot{\hat{x}}(t) =& A\hat{x}(t) + B\hat{u}(t) + L\big(\hat{y}(kh^s) - C\hat{x}(kh^s)\big), \\ &\forall t \in \big[kh^s + \tau_k^s, (k+1)h^s + \tau_{k+1}^s\big),\end{aligned} \tag{3}$$

where $\hat{u}(t)$ is an estimate of the plant's input at time t and $\hat{y}(kh^s)$ is equal to the last successfully received measurement data. Since u is constructed from data sent by the controller, in general we have $\hat{u} = u$. We consider two types of control units: Non-anticipative and anticipative.

A. Non-anticipative control unit

Control signal The control unit sends control updates at times ℓh^a, equal to $\{-K\hat{x}(\ell h^a), \ell \in \mathbb{N}\}$, where K is static gain. In the absence of dropouts, these arrive at the plant at times $\ell h^a + \tau_\ell^a$, $\ell \in \mathbb{N}$, leading to

$$u(t) = -K\hat{x}(\ell h^a), \quad \forall t \in \big[\ell h^a + \tau_\ell^a, (\ell+1)h^a + \tau_{\ell+1}^a\big). \tag{4}$$

Delay differential equation formulation Defining

$$\bar{\tau}_k^s := t - kh^s, \quad \forall t \in \big[kh^s + \tau_k^s, (k+1)h^s + \tau_{k+1}^s\big), \tag{5}$$

$$\bar{\tau}_\ell^a := t - \ell h^a, \quad \forall t \in \big[\ell h^a + \tau_\ell^a, (\ell+1)h^a + \tau_{\ell+1}^a\big), \tag{6}$$

we can re-write (3) and (4) as

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + B\hat{u}(t) + L\big(y(t-\bar{\tau}^s) - C\hat{x}(t-\bar{\tau}^s)\big), \\ u(t) &= -K\hat{x}(t-\bar{\tau}^a),\end{aligned} \tag{7}$$

in which $\bar{\tau}^s(t) := \bar{\tau}_k^s$ and $\bar{\tau}^a(t) := \bar{\tau}_\ell^a$, $\bar{\tau}_k^s$ and $\bar{\tau}_\ell^a$ are defined according to (5), (6) and

$$\bar{\tau}^s \in \big[\min_k\{\tau_k^s\}, h^s + \max_k\{\tau_{k+1}^s\}\big),\ \forall k \in \mathbb{N} \quad \dot{\bar{\tau}}^s = 1 \quad \text{a.e.},$$

$$\bar{\tau}^a \in \big[\min_\ell\{\tau_\ell^a\}, h^a + \max_\ell\{\tau_{\ell+1}^a\}\big),\ \forall \ell \in \mathbb{N} \quad \dot{\bar{\tau}}^a = 1 \quad \text{a.e.}$$

Fig. 2.a shows $\bar{\tau}^s$ with respect to time where $\tau_k^s = \tau^s, \forall k$ and constant sampling interval h^s. The derivative of $\bar{\tau}^s$ is almost always one, except at the sampling times, where $\bar{\tau}^s$ drops to τ^s.

Packet dropouts Packet dropouts can be viewed as a delay which grow beyond the defined bounds. If m^s and

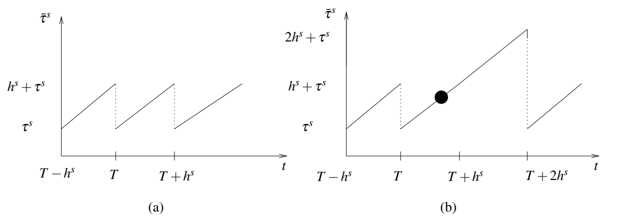

Fig. 2. Evolution of $\bar{\tau}^s$ with respect to time when (a)There is no packet dropout, (b) The packet sent at kh^s is dropped. In the picture $T := kh^s + \tau^s$.

m^a dropouts happen in the measurement and actuation channels, then

$$\bar{\tau}^s \in \left[\min_k\{\tau_k^s\}, (m^s+1)h^s + \max_k\{\tau_{k+m^s+1}^s\}\right), \quad \forall k \in \mathbb{N},$$
$$\bar{\tau}^a \in \left[\min_\ell\{\tau_\ell^a\}, (m^a+1)h^a + \max_\ell\{\tau_{\ell+m^a+1}^a\}\right), \quad \ell \in \mathbb{N}.$$

Fig. 2.b shows the situation that the measurement packet sent at kh^s is dropped and $\bar{\tau}^s$ grows up to $2h^s + \tau^s$.

Closed-loop Defining $e := x - \hat{x}$, with regard to (2) and (7), the closed-loop can be written as

$$\begin{bmatrix}\dot{\hat{x}}(t)\\ \dot{e}(t)\end{bmatrix} = \begin{bmatrix}A & 0\\ 0 & A\end{bmatrix}\begin{bmatrix}\hat{x}(t)\\ e(t)\end{bmatrix} + \begin{bmatrix}0 & LC\\ 0 & -LC\end{bmatrix}\begin{bmatrix}\hat{x}(t-\bar{\tau}^s)\\ e(t-\bar{\tau}^s)\end{bmatrix} + \begin{bmatrix}-BK & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\hat{x}(t-\bar{\tau}^a)\\ e(t-\bar{\tau}^a)\end{bmatrix},$$

or alternatively we can choose $\begin{bmatrix}x(t)' & e(t)'\end{bmatrix}'$ as the state of the augmented system.

B. Anticipative control unit

Control signal For simplicity, we assume that the actuation channel delay is constant and equal to τ^a and there is no dropout in the measurement and actuation channels. At each sampling time ℓh^a, $\ell \in \mathbb{N}$, the controller sends a time-varying control signal $u_\ell(\cdot)$ that should be used from the time $\ell h^a + \tau^a$ at which it arrives until the time $(\ell+1)h^a + \tau^a$ at which the next control update will arrive. This leads to

$$u(t) = u_\ell(t), \quad \forall t \in [\ell h^a + \tau^a, (\ell+1)h^a + \tau^a),\ \ell \in \mathbb{N}. \quad (8)$$

To stabilize (2), $u_\ell(t)$ should be equal to $-K\hat{x}(t)$. However, the estimates $\hat{x}(\cdot)$ needed in the interval $[\ell h^a + \tau^a, (\ell+1)h^a + \tau^a)$ must be available at the transmission time ℓh^a, which requires the control unit to estimate the plant's state up to $h^a + \tau^a$ time units into the future.

Remark 1: Anticipative controllers send actuation signals to be used during time intervals of duration h^a, therefore the sample and hold blocks in Fig. 1 should be understood in a broad sense. In practice, the sample block would send over the network some parametric form of the control signal $u_\ell(\cdot)$ (e.g., the coefficients of a polynomial approximation to this signal).

State predictor An estimate $z(t)$ of $x(t + h^a + \tau^a)$ is constructed as follows:

$$\dot{z}(t) = Az(t) + B\hat{u}(t + h^a + \tau^a) + L\big(\hat{y}(kh^s) - Cz(kh_s - h^a - \tau^a)\big), \quad (9)$$

for $\forall t \in [t_k^s + \tau_k^s, t_{k+1}^s + \tau_{k+1}^s)$, $\forall k \in \mathbb{N}$. To compensate for the time varying delays and dropouts in the actuation channel, z would have to estimate x further into the future. Hence the assumptions of constant delay and loss-less actuation channel can be relaxed by predicting x more into the future.

Control signal construction With such estimate available, the signal $u_\ell(t)$ sent at time ℓh^a, to be used in $\left[\ell h^a + \tau^a, (\ell+1)h^a + \tau^a\right)$, is then given by

$$u_\ell(t) = -Kz(t - h^a - \tau^a), \quad (10)$$
$$\forall t \in [\ell h^a + \tau^a, (\ell+1)h^a + \tau^a),\ \forall \ell \in \mathbb{N},$$

which only requires knowledge of $z(.)$ in the interval $t \in \left[(\ell-1)h^a, \ell h^a\right)$, and therefore is available at transmission time ℓh^a.

Delay differential equation formulation Defining

$$\bar{\tau}_k^s := t - t_k^s, \qquad \forall t \in \left[kh^s + \tau_k^s, (k+1)h^s + \tau_{k+1}^s\right),$$

assuming that $\hat{u} = u$, we conclude from (8),(9) and (10) that

$$\dot{z}(t) = (A - BK)z(t) + L\big(y(t - \bar{\tau}^s) - Cz(t - h^a - \tau^a - \bar{\tau}^s)\big), \quad (11)$$
$$\bar{\tau}^s \in \left[\min_k\{\tau_k^s\}, h^s + \max_k\{\tau_{k+1}^s\}\right), \quad k \in \mathbb{N}, \quad \dot{\bar{\tau}}^s = 1 \quad \text{a.e.}$$

Closed-loop Defining $e(t) = x(t + h^a + \tau^a) - z(t)$ with regard to (2) and (11) the closed-loop can be written as

$$\begin{bmatrix}\dot{z}(t)\\ \dot{e}(t)\end{bmatrix} = \begin{bmatrix}A - BK & 0\\ 0 & A\end{bmatrix}\begin{bmatrix}z(t)\\ e(t)\end{bmatrix} + \begin{bmatrix}0 & LC\\ 0 & -LC\end{bmatrix}\begin{bmatrix}z(t - h^a - \tau^a - \bar{\tau}^s)\\ e(t - h^a - \tau^a - \bar{\tau}^s)\end{bmatrix}.$$

III. Stability of Delay Differential Equations

In section II we show that both types of control units with any choice of states have the closed-loop form

$$\dot{\bar{x}}(t) = A_0\bar{x}(t) + \sum_{i=1}^{2} A_i\bar{x}(t - \tau_i), \quad (12a)$$
$$\tau_i \in \left[\tau_{i\min}, \tau_{i\max}\right), \qquad \dot{\tau}_i = 1 \quad a.e. \quad (12b)$$

Until recently the only available tool to study the stability of delay equations of the form (1) was the Razumikin theorem. Fridman and Shaked [4] were able to use the Lyapunov-Krasovskii theorem to study the stability of system (12). In [5] they study the stability of sampled-data systems with input delays as DDEs of the form (12), where $\tau_1 \in [0, h^s)$. In sampled-data systems, at each sampling time the delay drops to zero. However in NCSs as new information arrives the delay drops to NCS delay bounded bellow by $\tau_{i\min} > 0$. If we assume that the maximum delay in both cases is the same, due to the smaller variation of the delay in the latter case, we expect less conservative results than in [4]. The next theorem gives a sufficient condition for the asymptotic stability of the system (12) where A_i is $2n \times 2n$ for $i = 0, 1, 2$.

Theorem 1: The system (12) is asymptotically stable, if there exist $2n \times 2n$ matrices $P_1 > 0$, P_2, P_3, S_i, R_i and

$4n\times 4n$ matrices Z_{1i}, Z_{2i} and $2n\times 4n$ matrices T_i for $i=1,2$, that satisfy the following set of LMIs: [3]

$$\begin{bmatrix} \Psi & P'\begin{bmatrix}0\\A_1\end{bmatrix}-T_1' & P'\begin{bmatrix}0\\A_2\end{bmatrix}-T_2' \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0, \tag{13a}$$

$$\begin{bmatrix} R_i & \begin{bmatrix}0 & A_i'\end{bmatrix}P \\ * & Z_{2i} \end{bmatrix} > 0, \qquad i=1,2, \tag{13b}$$

$$\begin{bmatrix} R_i & T_i \\ * & Z_{1i} \end{bmatrix} > 0, \qquad i=1,2, \tag{13c}$$

where

$$\Psi = P'\begin{bmatrix}0 & I\\ A_0 & -I\end{bmatrix} + \begin{bmatrix}0 & I\\ A_0 & -I\end{bmatrix}' P + \Phi, \tag{14}$$

$$\Phi = \sum_{i=1}^{2}\Big(\begin{bmatrix}S_i & 0\\ 0 & \tau_{i\max}R_i\end{bmatrix} + (\tau_{i\max}-\tau_{i\min})Z_{2i} + \tau_{i\min}Z_{1i} + \begin{bmatrix}T_i\\0\end{bmatrix} + \begin{bmatrix}T_i\\0\end{bmatrix}'\Big), \quad P = \begin{bmatrix}P_1 & 0\\ P_2 & P_3\end{bmatrix},$$

Proof of Theorem 1: See Appendix.

Remark 2: For sampled-data systems $\tau_{i\min}=0$ and by choosing $S_i=0$, $T_i=\begin{bmatrix}0 & A_i'\end{bmatrix}P$ and $Z_{1i}=\varepsilon_i I$, such that (13c) holds, we recover Corollary 1 of [4]. For constant delay and no sampling $\tau_{i\max}-\tau_{i\min}=0$ and the LMIs (13a)-(13c) change to the ones in Lemma 1 of [4] when $Z_{2i}=\varepsilon_2' I$ such that (13b) holds for $i=1,2$. Consequently, Theorem 1 is the generalized form of the relevant results in [4].

IV. Observer-based controller design for NCSs

When the controller parameters L and K of an anticipative or non-anticipative controller are known, the system matrices A_i, $i=0,1,2$, are constant and known. Hence (13a)-(13c) are in the form of LMIs. However when L and K are unknown, the matrices A_i become variables and consequently the matrix inequalities in Theorem 1 are BMIs, and there is no efficient numerical method to solve them. In this section we develop an efficient numerical method to solve the matrix inequalities in Theorem 1. The Next lemma, taken from [12], plays a central role.

Lemma 2: Assume that $Q(M)$ is a symmetric matrix and matrix variables M and N are independent of each other. There exists a symmetric matrix $N>0$ such that

$$J(M)'NU + U'NJ(M) + Q(M) < 0, \tag{15}$$

if and only if there exist symmetric matrices X and Y, and a scalar $\alpha>0$ such that $X = \alpha^2 Y^{-1}$ and

$$\begin{bmatrix} U'XU - Q(M) & J(M)' + \alpha U' \\ * & Y \end{bmatrix} > 0. \tag{16}$$

The proof is obtained by Schur's lemma and $X=\alpha N$. Assume $Q(M)$ and $J(M)$ are linear functions of the matrix variable M, and U is a known matrix. Lemma 2 changes the BMI (15) to the LMI (16) with the non-convex constraint that $X = \alpha^2 Y^{-1}$. We want to write matrix inequalities in Theorem 1 in the form of (15) and consequently in the form of (16) which is suitable to compute K and L. Suppose $P_2>0$ and $P_3>0$, after some manipulations (13a) can be written as $J_0(K,L)'NU_0 + U_0'NJ_0(K,L) + Q_0 < 0$, where

$$Q_0 = \begin{bmatrix}\Gamma & -T_1' & -T_2'\\ * & -S_1 & 0\\ * & * & -S_2\end{bmatrix}, \quad U_0 = \begin{bmatrix} I & 0 & 0 & 0\\ 0 & I & 0 & 0\end{bmatrix},$$

$$\Gamma = \begin{bmatrix}0 & P_1\\ P_1 & 0\end{bmatrix} + \Phi, \qquad N = \begin{bmatrix}P_2 & 0\\ 0 & P_3\end{bmatrix},$$

$$J_0(K,L) = \begin{bmatrix}A_0 & -I & A_1 & A_2\\ A_0 & -I & A_1 & A_2\end{bmatrix},$$

in which Φ is defined in (14). Similarly (13b) can be written as $J_i(K,L)'NU_i + U_i'NJ_i(K,L) + Q_i < 0$, where

$$Q_i = -\begin{bmatrix}R_i & 0\\ 0 & Z_{2i}\end{bmatrix}, \qquad U_i = \begin{bmatrix}0 & I & 0\\ 0 & 0 & I\end{bmatrix},$$

$$J_i(K,L) = -\begin{bmatrix}A_i & 0 & 0\\ A_i & 0 & 0\end{bmatrix},$$

for $i=1,2$. Theorem 1 can read as the following theorem:

Theorem 3: The anticipative or non-anticipative controller given in section II with parameters K and L asymptotically stabilizes the plant with state space model given by (2) for given $\tau_{i\min}$ and $\tau_{i\max}$, if there exist $2n\times 2n$ matrices $P_1>0, X_1>0, X_2>0, Y_1, Y_2, S_i, R_i$, $4n\times 4n$ Z_{1i}, Z_{2i}, $2n\times 4n$ matrices T_i, $n\times 1$ matrix L, $1\times n$ matrix K and $\alpha>0$ that satisfy the following matrix inequalities:

$$\begin{bmatrix} U_0'XU_0 - Q_0 & J_0' + \alpha U_0' \\ * & Y \end{bmatrix} > 0, \tag{17a}$$

$$\begin{bmatrix} U_i'XU_i - Q_i & J_i' + \alpha U_i' \\ * & Y \end{bmatrix} > 0, \qquad i=1,2, \tag{17b}$$

$$\begin{bmatrix} R_i & T_i \\ * & Z_{1i} \end{bmatrix} > 0 \qquad i=1,2, \tag{17c}$$

where

$$X = \begin{bmatrix}X_1 & 0\\ 0 & X_2\end{bmatrix}, \quad Y = \begin{bmatrix}Y_1 & 0\\ 0 & Y_2\end{bmatrix}, \quad X = \alpha^2 Y^{-1}.$$

Theorem 3 transforms the matrix inequalities (13a), (13b) to (17a), (17b) respectively. Since A_0, A_1, A_2 are linear functions of K and L, (17a)-(17c) are LMIs; however, the fact that $X=\alpha^2Y^{-1}$ is a not convex constraint, makes the whole set of matrix inequalities non-convex. Next we introduce a numerical procedure to solve such a non-convex problem.

A. Numerical procedure and example

The cone complementarity linearization algorithm introduced in [7] changes the non-convex feasibility problem in Theorem 3 to the following linear minimization problem:

1) *Choose* α.

[3]Matrix entries by '*' are implicitly defined by the fact that the matrix is symmetric.

m^s	$\tau_{1\min}$	$\tau_{1\max}$	K	L
0	0.765	0.865	$\begin{bmatrix}-1.7436 & 1.1409\end{bmatrix}$	$\begin{bmatrix}0.0675 & 0.0267\end{bmatrix}'$
2	0.525	0.825	$\begin{bmatrix}-1.2990 & 0.6983\end{bmatrix}$	$\begin{bmatrix}0.0720 & 0.0292\end{bmatrix}'$
4	0.292	0.792	$\begin{bmatrix}-1.3556 & 0.7501\end{bmatrix}$	$\begin{bmatrix}0.0727 & 0.0300\end{bmatrix}'$
6	0.073	0.773	$\begin{bmatrix}-0.5310 & 0.1668\end{bmatrix}$	$\begin{bmatrix}0.0564 & 0.0221\end{bmatrix}'$

TABLE I

CONTROLLER PARAMETERS WITH $h^s = 0.1s$ AND m^s DROPOUTS.

2) *Find a feasible point X_0, Y_0 for the set of LMIs (17a)-(17c) and*

$$\begin{bmatrix} X & I \\ I & \alpha^{-2}Y \end{bmatrix} \geq 0 \tag{18}$$

3) *Set $X_j = X_{j-1}$, $Y_j = Y_{j-1}$, and find X_{j+1}, Y_{j+1} that solves the LMI problem*

$$\Sigma_j : \min \operatorname{trace}(X_j Y + X Y_j) \quad \textit{subject to } (17), (18).$$

4) *If stopping criterion is satisfied, exit. Otherwise set $j = j+1$ and go to step 3 if $j < c$ (a preset number) or increase α with a proper amount and go to step 2.*

If the minimum is equal to $8n \times \alpha^{-2}$, then (17a)-(17c) with $X = \alpha^2 Y^{-1}$ are satisfied and the controller with parameters K and L stabilizes the plant for the given specifications of delays. Since obtaining $\operatorname{trace}(X_j Y + X Y_j) = 8n \times \alpha^{-2}$ is numerically difficult, we choose (13a) and (13b) with $N = \alpha^{-1} X$ as the stopping criterion.

In example 1 we compare our method to the one presented in [2], where the controller is directly connected to the actuator, hence the anticipative and non-anticipative controllers result in the same closed-loop system. Then example 2 focuses on the advantage of the anticipative over the non-anticipative control unit.

Example 1: Yu et al. [2] consider the following state space plant model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1.7 & 3.8 \\ -1 & 1.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 5 \\ 2.01 \end{bmatrix} u,$$
$$y(t) = \begin{bmatrix} 10.1 & 4.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

The set of LMIs in [2] is feasible up to $\tau_{1\max} = 0.3195$. Consequently as long as $(m^s+1)h^s + \tau_k^s \leq 0.3195, \forall k \in \mathbb{N}$, the closed-loop system is stable. Our results in Table I show a significant improvement since our set of LMIs is feasible for larger $\tau_{1\max}$. For instance when $h^s = 0.1$ and the measurement channel is loss-less ($\tau_{1\max} - \tau_{1\min} = h^s$), the LMIs are feasible up to $\tau_{1\max} = 0.865$ which means the closed-loop system with the controller parameters given in Table I is stable for any $\tau_k^s \in [0, 0.765], \forall k \in \mathbb{N}$. If we assume that the number of dropouts is $m^s = 6$ ($\tau_{1\max} - \tau_{1\min} = 7 \times h^s$), the LMIs are feasible up to $\tau_{1\max} = 0.773$ and the closed-loop system is stable for any $\tau_k^s \in [0, 0.073], \forall k \in \mathbb{N}$. Table I also contains the expected results that a smaller number of dropouts leads to a larger $\tau_{1\max}$. It justifies taking into account the distinction between the effect of packet dropouts and delays in NCSs.

Example 2: Consider the state space plant model [11]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u, \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Branicky et al. [11] assume that both states are available and moreover full state feedback gain $K = \begin{bmatrix} 3.75 & 11.5 \end{bmatrix}$ is given. The authors obtain that the system with no delay is stable with constant sampling interval up to 4.5×10^{-4}. However, the maximum constant sampling interval for this controller is $1.7s$. With regard to Remark 2 for the same state feedback gain, the system is asymptotically stable for constant sampling interval smaller than 0.87. Now we assume that only the first state is available and the delays are zero and sensor sampling interval is $h^s = 0.5, \forall k \in \mathbb{N}$. Non-anticipative control unit with parameters

$$K = \begin{bmatrix} 3.3348 & 9.9103 \end{bmatrix}, \quad L = \begin{bmatrix} 0.6772 & 0.1875 \end{bmatrix}',$$

stabilizes the plant for maximum sensor sampling interval $h^a \leq 0.7330$. With the same sensor sampling interval, anticipative control unit with parameters

$$K = \begin{bmatrix} 28.5347 & 83.8626 \end{bmatrix}, \quad L = \begin{bmatrix} 0.3518 & 0.0492 \end{bmatrix}',$$

stabilizes the plant for the actuation sampling intervals such that $h^a \leq 0.976$.

V. CONCLUSION AND FUTURE WORK

We introduced two type of control units: non-anticipative and anticipative. NCSs with LTI plant model, anticipative or non-anticipative controller, with network effect can be modeled as a DDE (1). we found sufficient condition for asymptotic stability of DDE (1) in the form of LMI. We presented a procedure to design output-feedback control unit for NCSs. Our method shows significant improvement in compare to the existing results, since it is based on Lyapunov-Krasovskii functional and distinction between the effect of sampling/packet dropout and NCS delay. In future work we will explore the advantages of the anticipative over the non-anticipative controller more. For NCSs that the plant has high computational capability such as remote surgery systems or haptic systems, sending a control signal must have advantages over a single value control command even in the presence of some disturbances.

We will extend our results to the case that from an input to an output some performance is desired. Performance can be H_∞ or H_2 norms or passivity from an input to an output.

APPENDIX

Equation (12a) can be written in as equivalent form [4]

$$\dot{x}(t) = y(t),\ -y(t)\sum_{i=0}^{2} A_i x(t) - \sum_{i=1}^{2} A_i \int_{t-\tau_i}^{t} y(s)ds = 0, \quad (19)$$

and the following Lyapunov-Krasovskii functional:

$$V(t) = x'Px + \sum_{i=1}^{2} \int_{-\tau_{i\max}}^{0} \int_{t+\theta}^{t} y'(s)R_i y(s)dsd\theta \quad (20)$$
$$+ \sum_{i=1}^{2} \int_{t-\tau_{i\min}}^{t} x'(s)S_i x(s)ds,$$

for $i = 1,2$, where $P_1 > 0$. Differentiating the first, second and third term of (20) with respect to t respectively gives

$$2x'P_1\dot{x}(t) = 2\tilde{x}'(t)P'\begin{bmatrix}\dot{x}\\0\end{bmatrix}, \quad (21a)$$

$$\sum_{i=1}^{2}\Big(\tau_{i\max}y'(t)R_i y(t) - \int_{t-\tau_{i\max}}^{t} y'(\tau)R_i y(\tau)d\tau\Big), \quad (21b)$$

$$\sum_{i=1}^{2}\Big(x'(t)S_i x(t) - x'(t-\tau_{i\min})S_i x(t-\tau_{i\min})\Big). \quad (21c)$$

where $\tilde{x} = \begin{bmatrix}x(t)' & y(t)'\end{bmatrix}'$. Substituting (19) into (21a),

$$\frac{dV(t)}{dt} \le \tilde{x}'(t)\tilde{\Psi}\tilde{x} - \sum_{i=1}^{2} x'(t-\tau_{i\min})S_i x(t-\tau_{i\min})$$
$$- \sum_{i=1}^{2}\int_{t-\tau_{i\max}}^{t} y'(\tau)R_i y(\tau)d\tau + \eta,$$
$$\tilde{\Psi} = P'\begin{bmatrix}0 & I\\ \sum_{i=0}^{2}A_i & -I\end{bmatrix} + \begin{bmatrix}0 & I\\ \sum_{i=0}^{2}A_i & -I\end{bmatrix}'P$$
$$+ \sum_{i=1}^{2}\begin{bmatrix}S_i & 0\\ 0 & \tau_{i\max}R_i\end{bmatrix},\ \eta = \sum_{i=1}^{2} 2\tilde{x}'(t)P'\begin{bmatrix}0\\A_i\end{bmatrix}\int_{t-\tau_i}^{t} y(s)ds.$$

By Moon-park inequality [13] a bound on cross term, η, can be found as follows:

$$\eta \le \sum_{i=1}^{2}\int_{t-\tau_{i\min}}^{t}\begin{bmatrix}y(s)\\\tilde{x}(t)\end{bmatrix}'\begin{bmatrix}R_i & T_i - \begin{bmatrix}0 & A_i'\end{bmatrix}P\\ * & Z_{1i}\end{bmatrix}\begin{bmatrix}y(s)\\\tilde{x}(t)\end{bmatrix}ds$$
$$+ \sum_{i=1}^{2}\int_{t-\tau_i}^{t-\tau_{i\min}}\begin{bmatrix}y(s)\\\tilde{x}(t)\end{bmatrix}'\begin{bmatrix}R_i & \tilde{T}_i - \begin{bmatrix}0 & A_i'\end{bmatrix}P\\ * & Z_{2i}\end{bmatrix}\begin{bmatrix}y(s)\\\tilde{x}(t)\end{bmatrix}ds$$
$$= \sum_{i=1}^{2}\int_{t-\tau_i}^{t} y'(s)R_i y(s)ds$$
$$+ 2\sum_{i=1}^{2}\int_{t-\tau_i}^{t-\tau_{i\min}} y'(s)\big(\tilde{T}_i - \begin{bmatrix}0 & A_i\end{bmatrix}P\big)\tilde{x}(t)ds$$
$$+ 2\sum_{i=1}^{2}\int_{t-\tau_{i\min}}^{t} y'(s)\big(T_i - \begin{bmatrix}0 & A_i\end{bmatrix}P\big)\tilde{x}(t)ds$$
$$+ \sum_{i=1}^{2}\tilde{x}'(t)\big(\tau_{i\min}Z_{1i} + (\tau_i - \tau_{i\min}Z_{2i})\tilde{x}(t),$$

where

$$\begin{bmatrix}R_i & T_i\\ * & Z_{1i}\end{bmatrix} \ge 0, \quad \begin{bmatrix}R_i & \tilde{T}_3\\ * & Z_{2i}\end{bmatrix} \ge 0$$

By choosing $\tilde{T}_i = \begin{bmatrix}0 & A_i'\end{bmatrix}P$,

$$\eta \le \sum_{i=1}^{2}\big(\int_{t-\tau_i}^{t} y(s)'R_i y(s)ds + 2x'(t)\big(Y_i - \begin{bmatrix}0 & A_i'\end{bmatrix}P\big)\tilde{x}(t)\big)$$
$$-2\sum_{i=1}^{2}\big(x'(t-\tau_{i\min})\big(Y_i - \begin{bmatrix}0 & A_i'\end{bmatrix}P\big)\tilde{x}(t)\big)$$
$$+\sum_{i=1}^{2}\big(\tilde{x}'(t)\big(\tau_{i\min}Z_{1i} + (\tau_{i\max} - \tau_{i\min})Z_{2i}\big)\tilde{x}(t).$$

Based on the Lyapunov-Krasovskii theorem, (12) is asymptotically stable if $\frac{dV}{dt} \le -\varepsilon\|x\|^2$ for some $\varepsilon > 0$. Hence the system is asymptotically stable if (13a) holds. However any row and column of (13a) except the first block row and column can be zero. The inequality in (13b) and (13c) are in fact non-strict. However for simplicity and since there is no numerical advantage we state them as strict inequality.

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