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Nussbaum Gain Based Iterative Learning Control for a Class of Multi-input Multi-output Nonlinear Systems

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Abstract—**An adaptive iterative learning control(ILC) approach is proposed for a class of multi-input multi-output (MIMO) uncertain nonlinear systems without prior knowledge about system control gain matrices. The Nussbaum-type gain and the positive definite discrete matrix kernel are proposed for dealing with selection of the unknown control gain and learning of the repeatable uncertainties, respectively. Asymptotic convergence for a trajectory tracking within a finite time interval is achieved through repetitive tracking. Simulations are carried out to show the validity of the proposed control method.**

I. Introduction

Iterative learning control can deal with repeatable uncertainties in a repetitive mode. Typical iterative learning controls are designed based on the discrete Lyapunov method and the control output is updated in an affine fashion such as P type or D type learning [1][2][3]. They require some preconditions of stability on the learning gain. For example, given a linear dynamic system

$$\dot{x}(t) = Ax(t) + Bu(t), x(t) \in R^n, x(0) = x_0 \in R^n,$$
$$y(t) = Cx(t), y(t) \in R^m, u(t) \in R^l, \tag{1}$$

in order to guarantee convergence in terms of the λ-norm, learning gain $L(t)$ should satisfy $\|I - CBL\| < 1$. It implies that *a priori* knowledge about the control gain matrix *CB* has to be available for the learning control design. Reference [4] proposed an adaptive high-gain iterative learning controller for a class of MIMO linear time-invariant systems but *a priori* knowledge about the control gain matrix *CB* is required as well, where it should be positive definite. In some cases, such as uncalibrated visual servoing[5], it is difficult to gain this kind of prior knowledge. In adaptive control, the *Nussbaum gain*[6][7][8] and the *correction vector* method [9] were proposed to deal with this kind of control problem without prior information. For the purpose of robot trajectory imitation with unknown camera-robot model, reference [10] proposed an indirect iterative learning control to avoid control singularity without any prior knowledge about the system model, where the estimated control gain matrix is identified by a least square algorithm and it is modified to avoid control singularity based on the idea of the *correct vector* method. Considering the fact that *Nussbaum gain* has a simpler form and can be more suitable for real-time applications, reference [11] investigated iterative learning control based on a Nussbaum-type learning gain for a class of single-input single-output (SISO) nonlinear systems with an unknown control gain. To date, the Nussbaum-type gain method has mostly been used in SISO systems. How to design an iterative learning controller using a *Nussbaum gain* in MIMO systems is significant for its applications.

This paper proposes an iterative learning control for a class of minimum-phase MIMO nonlinear systems, where the repeatable nonlinear uncertainties are learned via an iterative learning law in a form of the *positive definite discrete matrix kernel* and the unknown control gain matrix, which is either positive definite or negative definite, is dealt with by a *Nussbaum gain*. Under the control of the proposed algorithm, this paper shows that the unknown gain matrix is continuously probed and the control performance is gradually improved from the previous executions. The tracking error sequence is asymptotic to zero when the tracking is repeated.

The paper is organized as follows. In Section 2, the problem formulation and the control objective are introduced. In Section 3, the design and the stability analysis of the iterative learning control are given. In Section 4, simulation works are presented to demonstrate the effectiveness of the proposed scheme. Finally, Section 5 concludes this paper.

II. Problem Formulation

In this paper, we consider a class of nonlinear systems

$$\dot{x}(t,i) = f(t,i) + B[Y(x(t,i)) \cdot a(t,i) + u(t,i)],$$
$$y(t,i) = Cx(t,i), \tag{2}$$

where t denotes the time horizon; i denotes the i-th repetitive tracking; $B \in R^{n\times m}$ and $C \in R^{m\times n}$ are unknown constant input and output matrices, respectively; $Y(\cdot) \in R^{m\times p}$ is a measurable nonlinear matrix; $f(t,i) \in R^n$ and $a(t,i) \in R^p$ are unknown but repeatable vectors, i.e. $f(t,i) = f(t)$ and $a(t,i) = a(t)$; $x(t,i) \in R^n$, $u(t,i) \in R^m$ and $y(t,i) \in R^m$ are the state, the control input and the control output at instant t in the i-th tracking, respectively.

Given a desired trajectories $r(t) \in R^m$ over a finite time interval $[0, T_f]$, the control objective is to design an iterative learning control $u(t,i)$ with the ability of reducing tracking error for the whole trajectory in the time interval $[0, T_f]$ based on the past tracking experience, such that, as $i \to \infty$,

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the system tracking error $e(t,i)=y(t,i)-r(t)\rightarrow 0,\quad t\in[0,T_f]$. The following are the assumptions of the systems:

Assumption 1. The control gain matrix CB is symmetric and has spectrum $\sigma(CB)$ lying in either the open left (C^-) or the open right (C^+) half of the complex plane.

Assumption 2. For every trial, the initial states can be reset to the desired states, i.e. $x_i(0)=x_d(0)=x^0,\forall i$.

The assumption 1 supposed that CB is either positive definite or negative definite but the designers have no prior knowledge about it. The assumption 2 is the initial resetting condition required by the iterative learning control.

III. Design of Control Law

A. Proposed control law

For the system (2), we know that

$$\dot{y}(t,i)=Cf(t,i)+CB[Y(x(t,i))\cdot a(t,i)+u(t,i)]$$

The error equation can then be written as:

$$\dot{e}(t,i)=Cf(t,i)+CB[Y(x(t,i))\cdot a(t,i)+u(t,i)]-\dot{r}(t)$$
$$=(CB)\left[(CB)^{-1}(Cf(t,i)-\dot{r}(t))+Y(x(t,i))\cdot a(t,i)+u(t,i)\right]$$

Notice that $(CB)^{-1}(Cf(t,i)-\dot{r}(t))$ is repeatable, i.e. it is invariant over repetitive index i, because of $f(t,i)=f(t)$. In addition, $a(t)=a(t,i)$ is repeatable. Let $b(t)=(CB)^{-1}(Cf(t)-\dot{r}(t))$, the error equation can be rewritten with regard to two unknown but repeatable uncertainties, $a(t)$ and $b(t)$:

$$\dot{e}(t,i)=(CB)\left[b(t)+Y(x(t,i))\cdot a(t)+u(t,i)\right] \tag{3}$$

If CB is positive definite, an adaptive ILC can be proposed following the way of [5]:

$$u(t,i)=g_l(t,i)+g_f(t,i), \tag{4}$$

where $g_l(t,i)=\hat{b}(t,i)+Y(x(t,i))\hat{a}(t,i)$ is an adaptive iterative learning term for compensating the repetitive uncertainties and $g_f(t,i)=Ke(t,i)$, $K>0$, is a linear feedback term to cope with unrepeatable disturbances.

If we do not have the prior knowledge about CB, a coarse exploration of the gain direction must be carried out. In the paradigm of adaptive control, the Nussbaum gains were proposed for the exploration based on observation of a performance index. A Nussbaum gain can be considered as a control-direction selector that can swing from positive to negative according to control performance accumulation ζ, e.g. $h(y)=\cos(\frac{1}{2}\pi\xi)\exp(\xi^2)$, $\xi^2\cdot\cos(\xi)$, and $\ln(\xi)\cdot\cos(sqrt(\ln(\xi)))$, etc. Namely, a poor control performance, corresponding to a bigger $d\xi/dt$, tends to change the control gain to its opposite direction but a good control performance ceases this change. A Nussbaum gain has an increased amplitude due to the fixed probing period in the Nussbaum functions. It gives a chance to the system to correct deviation caused by the previous control using a wrong gain. This may cause poor transients during the process of probing, which are the expense of exploration and may happen in any trial run for gaining unknown knowledge, e.g. movement excitation for identification, human learning to reverse a car. Notice that this does not mean the control gain may always increase. Because probing of each direction using a Nussbaum gain has a whole sine curve, the gain could be stabilised at a suitable value depending on the current tracking performance.

Based on the above analysis, an adaptive ILC with less model knowledge is proposed below:

$$u(t,i)=\nu(k(t,i))g(t,i)\ , \tag{5}$$

where $g(t,i)=g_l(t,i)+g_f(t,i)$ and $\nu(k(t,i))$ is a Nussbaum-type function with

$$\dot{k}(t,i)=g^T(t,i)e(t,i); \tag{6}$$

$$k(0,i+1)=k(T_f,i),k(0,0)=0. \tag{7}$$

The control (5) integrates the adaptive ILC with automatic gain selection using a Nussbaum gain. In this paper, we choose a Nussbaum-type function of $\nu(\xi):=k^2\cos(k)$, $k\in R$, which has the following property:

***Property* 1**[6]. The Nussbaum-type function $\nu(\cdot)$ has the properties of

$$\lim_{s\to\infty}\sup\frac{1}{s}\int_0^s\nu(\xi)d\xi=+\infty, \tag{8}$$

$$\lim_{s\to\infty}\inf\frac{1}{s}\int_0^s\nu(\xi)d\xi=-\infty. \tag{9}$$

The control $g(t,i)$ in (5) consists of a learning control term $g_l(t,i)$ and a feedback control term $g_f(t,i)$:

$$g(t,i)=g_l(t,i)+g_f(t,i), \tag{10}$$

with

$$\begin{cases} g_l(t,i)=\hat{b}(t,i)+Y(x(t,i))\hat{a}(t,i) \\ \hat{b}(t,i)=\sum_{q=0}^{i}F_1(i-q)e(t,q) \\ \hat{a}(t,i)=\sum_{q=0}^{i}F_2(i-q)Y^T(x(t,q))e(t,q) \end{cases} \tag{11}$$

$$g_f(t,i)=Ke(t,i) \tag{12}$$

where $F_m(\cdot),m=1,2$ are any *positive definite discrete matrix kernels*; K is a positive constant.

The learning control (11) in a form of *positive definite discrete matrix kernels* is motivated by the discrete model reference adaptive system design using the hyperstability approach[12], where a sub-problem is to find the most general solutions for ϕ_1 such that the following inequality holds:

$$\sum_{k=0}^{k_1} v^T(k+1)w(k+1) \geq -\gamma_0^{\ 2}, \forall k_1 > 0 \quad (13)$$

with $w(k+1) = (\sum_{l=0}^{k} \phi_1(v,k,l) + A_0)y(k)$

where $\gamma_0^{\ 2}$ is an arbitrary positive finite constant; *v* and *w* are the n-dimensional input and output of a block; $\phi_1 \in R^{n\times q}$ represents the adaptive mechanism; *y* is a finite q-dimensional vector and $A_0 \in R^{n\times q}$ is an unknown but constant matrix.

***Property* 2**[12]. The inequality of (13) is satisfied by:

$\phi_1(v,k,l) = F(k-l)v(l+1)[Gy(l)]^T$

where *F*(*k-l*) is a *positive definite discrete matrix kernel* whose z-transformation is a positive real discrete transfer matrix with a pole at $z=1$, and *G* is a positive definite matrix.

Although the inequality (13) can be held only for any unknown constant matrix A_0, the property 2 is able to be extended as a general solution of ILC's in a form of *positive definite discrete matrix kernels* for repeatable uncertainties, e.g. $a(i,t) = a(t)$, if the learning is conducted along the iterative horizon *i*.

If let $n(k) = v(k+1)$, $a(t)$= $-A_0$, $y(k)$=1, and *G*=1 in Property 2, we can obtain the following property along iterative horizon *i*:

***Property* 3**[5]. For any vector *n*(*i*) and any repeatable constant vector *a*(*t*), a positive definite discrete matrix kernel $F(i-q)$ ensures that the following accumulation along iterative horizon *i* is always upper bounded:

$$\sum_{i=0}^{N} n^T(i)(a(t) - \sum_{q=0}^{i} F(i-q)n(q)) \leq \gamma_0^{\ 2}, \forall N$$

where $\gamma_0^{\ 2}$ is any positive finite constant.

In the proposed ILC (11), if F_1 and F_2 are selected to be β_1>0 and β_2>0, which are *positive definite discrete matrix kernels* because their z-transformations are positive real discrete transfer matrices with a pole at z=1. They lead to the P-type learning using current tracking errors:

$$\hat{b}(t,i) = \sum_{q=0}^{i} F_1(i-q)e(t,q) = \hat{b}(t,i-1) + \beta_1 e(t,q)$$

$$\hat{a}(t,i) = \sum_{q=0}^{i} F_2(i-q)Y^T(x(t,q))e(t,q)$$

$$= \hat{a}(t,i-1) + \beta_2 Y^T(x(t,q))e(t,q)$$

Classical ILC's use only the previous tracking errors for current learning. It has an advantage that feedforward compensation can be calculated offline and be implemented online through a simple mechanism of retrieval-from-memory. Some researchers introduced current iterative tracking errors into learning, called feedback ILC, and argued that could improve robustness to uncertainties, the tracking error bound and the ILC convergence rate[13][14][15]. The *positive definite discrete kernels* can provide a general class of feedback ILC's.

B. *Stability analysis*

***Theorem* 1.** Under the two assumptions and the control law (5), $k(t,i)$ is bounded and the repetitive tracking error sequence is asymptotic to zero when iterations go to infinity, i.e. $\|e(t,i)\| \to 0$ as $i \to \infty$.

Proof:

By Assumption 1 there exists $\beta \in \{-1,1\}$ such that βCB is symmetrically and positively definite. We define $Q := (\beta CB)^{-1} = \beta (CB)^{-1}$, then Q is symmetrically and positively definite and $QCB = \beta I$. We define a Lyapunov equation:

$$V(t,i) := \frac{1}{2} < e(t,i), Qe(t,i) >, t \in [0, T_f].$$

From (3) and (10), we have

$$\begin{aligned}&\dot{V}(t,i) = < Qe(t,i), \dot{e}(t,i) >\\&= < Qe(t,i), (CB)[b(t) + Y(x(t,i))a(t) + u(t,i)] > \quad (14)\\&= < e(t,i), \beta b(t) + \beta Y(x(t,i))a(t) - \hat{b}(t,i) -\\&Y(x(t,i))\hat{a}(t,i) - g_f(t,i) + g(t,i) + \beta u(t,i) >\end{aligned}$$

Because of Assumption 2, i.e. *V*(0,*i*)=0, the energy accumulation of the repetitive tracking from (0,0) to (*t*,*i*) can be obtained by:

$$\sum_{j=0}^{i-1} V(T_f, j) + V(t,i) = \sum_{j=0}^{i-1} \int_0^{T_f} \dot{V}(t,j)dt + \int_0^t \dot{V}(t,i)dt$$

$$= \int_0^t \sum_{j=0}^{i} \dot{V}(t,j)dt + \int_t^{T_f} \sum_{j=0}^{i-1} \dot{V}(t,j)dt$$

Substituting (14) into it, we have

$$\sum_{j=0}^{i-1} V(T_f, j) + V(t,i) = A(t,i) + B(t,i),$$

where

$$\begin{aligned}A(t,i) &= \int_0^t \sum_{j=0}^{i} < e(t,j), \beta b(t) - \hat{b}(t,j) > dt\\&+ \int_t^{T_f} \sum_{j=0}^{i-1} < e(t,j), \beta b(t) - \hat{b}(t,j) > dt\\&+ \int_0^t \sum_{j=0}^{i} < e(t,j), Y(x(t,j))[\beta a(t) - \hat{a}(t,j)] > dt\\&+ \int_t^{T_f} \sum_{j=0}^{i-1} < e(t,j), Y(x(t,j))[\beta a(t) - \hat{a}(t,j)] > dt\end{aligned}$$

$$\begin{aligned}B(t,i) &= \sum_{j=0}^{i-1} \int_0^{T_f} < e(t,j), -g_f(t,j) > dt\\&+ \int_0^t < e(t,i), -g_f(t,i) > dt\\&+ \sum_{j=0}^{i-1} \int_0^{T_f} < e(t,j), \beta u(t,j) + g(t,j) > dt\\&+ \int_0^t < e(t,i), \beta u(t,i) + g(t,i) > dt\end{aligned}$$

From the property 3 of a *positive definite discrete matrix* *kernel*, the updating law (11) of $\hat{b}(t,i)$ and $\hat{a}(t,i)$ along iterative horizon can guarantee upper-boundedness of $A(t,i)$:

$$A(t,i) \le 2\int_0^{T_f} \max(\gamma_1^2,\gamma_2^2,\gamma_3^2,\gamma_4^2)dt \le 2\gamma^2 \cdot T_f,$$

where $\gamma^2 = \max(\gamma_m{}^2), m = 1,2,3,4$.

$$B(t,i) = -\sum_{j=0}^{i-1}\int_0^{T_f} Ke^T(t,j)e(t,j)dt$$

$$-\int_0^{t} Ke^T(t,i)e(t,i)dt$$

$$+\sum_{j=0}^{i-1}\int_0^{T_f}(\beta\nu(k)+1)g^T(t,j)e(t,j)dt$$

$$+\int_0^{t}(\beta\nu(k)+1)g^T(t,i)e(t,i)dt$$

$$\le \sum_{j=0}^{i-1}\int_{k(0,j)}^{k(T_f,j)}(\beta\nu(k)+1)dk + \int_{k(0,i)}^{k(t,i)}(\beta\nu(k)+1)dk$$

$$= \int_{k(0,0)}^{k(t,i)}(\beta\nu(k)+1)dk,$$

where equation (6) and equation (7) have been applied.

Therefore,

$$0 \le \sum_{j=0}^{i-1}V(T_f,j)+V(t,i)$$

$$\le 2\gamma^2 \cdot T_f + \int_{k(0,0)}^{k(t,i)}(\beta\nu(k)+1)dk \tag{15}$$

Thus

$$\int_{k(0,0)}^{k(t,i)}(\beta\nu(k)+1)dk \ge -2\gamma^2 \cdot T_f, \tag{16}$$

which is lower bounded. It can be rewritten as

$$\int_{k(0,0)}^{k(t,i)}\beta\nu(k)dk \ge -(k(t,i)-k(0,0)) - 2\gamma^2 \cdot T_f \tag{17}$$

Suppose, at any time instant, $k(t,i)$ becomes divergent. We consider the cases of positive and negative divergence:

1) $k(t,i) \to +\infty$

Equation (17) implies

$$\lim_{k(t,i)\to+\infty}\frac{1}{k(t,i)}\int_{k(0,0)}^{k(t,i)}\beta\nu(k)dk \ge -1$$

where $k(0,0)=0$. It contradicts (9) if β is +1 or contradicts (8) if β is -1;

2) $k(t,i) \to -\infty$

Equation (17) implies

$$\lim_{k(t,i)\to-\infty}\frac{1}{k(t,i)}\int_{k(0,0)}^{k(t,i)}\beta\nu(k)dk \le -1$$

where $k(0,0)=0$. It contradicts (8) if β is +1 or contradicts (9) if β is -1.

So $k(t,i)$ and thus $\int_{k(0,0)}^{k(t,i)}(\beta\nu(k)+1)dk$ must keep bounded for repetitive tracking. Then, from (15), we know that $\sum_{j=0}^{i-1}V(T_f,j)+V(t,i)$ is bounded as a result. For this positive and monotonic series, we know that $V(t,i)\to 0$ as $i\to\infty$, i.e. $\|e(t,i)\| \to 0, \forall t \in [0,T_f]$.

IV. EXAMPLES

Considering the following 2-dimensional system with an unknown gain matrix B and a repeatable uncertainty $f(t)$:

$$\dot{x} = f(t) + Bu, \tag{18}$$

where $f(t) = \begin{bmatrix} 2\sin(t) \\ 3\cos(t)\end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1\end{bmatrix}$ are supposed to be unknown.

Because of the time-varying uncertainty $f(t)$, adaptive control along time-horizon is not adequate for control of this sort of systems but ILC's can be an effective alternative.

Based on the Theorem 1, an adaptive ILC can be constructed for tracking control of a given trajectory:

$$r(t) = \begin{bmatrix} 5\cos(\pi-\pi\cdot t/10)+5 \\ 5\sin(\pi-\pi\cdot t/10)\end{bmatrix}, t\in[0,5](s) \tag{19}$$

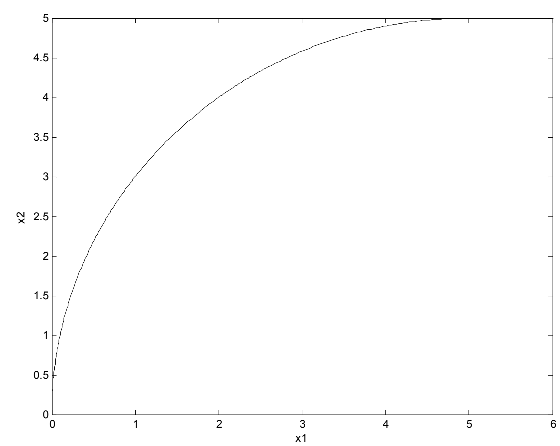

Fig. 1 The desired trajectory

The control law is designed below with a sampling period of 20ms.

$$u(t,i) = v(k(t,i))\left(\sum_{q=0}^{i}F_1(i-q)e_m(t,q)+Ke_m(t,i)\right), \tag{20}$$

where the *discrete positive definite matrix kernel* is set to be a positive definite matrix, i.e. $F_1(.)$=diag(2,2), and the feedback gain K=1.5. The control (20) consists of a control direction selector v in a form of the Nussbaum gain and a history based learning control. In order to verify robustness of the Nussbaum gain, suppose that there exists a random measurement noise n in uniform distribution on the interval of [-0.05, 0.05], i.e., $e_m(t,i)=e(t,i)+n$.

Fig. 2 depicts the first tracking errors. In the first tracking, because the controller did not know its correct control direction and did not have any compensation to the movement, it showed a big tracking error that caused the Nussbaum gain to probe the correct control direction. The evolution of the control performance observation $k(t,i)$ and the corresponding Nussbaum gain are plotted in Fig.3. From the figure, we did not see any evidence of divergence due to the measurement noise and the coarse gain probing reached the correct gain after the first trial. The RMS(Root Mean Square) error of the iterative learning control is illustrated in Fig.4. It clearly indicates the learning and control capabilities of the proposed control law that includes a coarse probing of the control direction and a fine tuning for movement and uncertainty compensation. The control errors of the 30th tracking is depicted in Fig.5. Comparing with the control errors of the 1st tracking in Fig. 1, we can find that the maximum tracking errors in both directions are reduced from $ex1_{max}$=6.189 and $ex2_{\max}$=11.05 to $ex1_{\max}$=0.096 and $ex2_{\max}$=0.086.

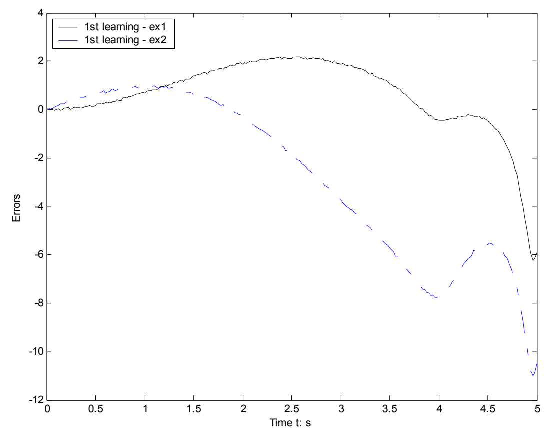

Fig.2. The first tracking errors for case 1

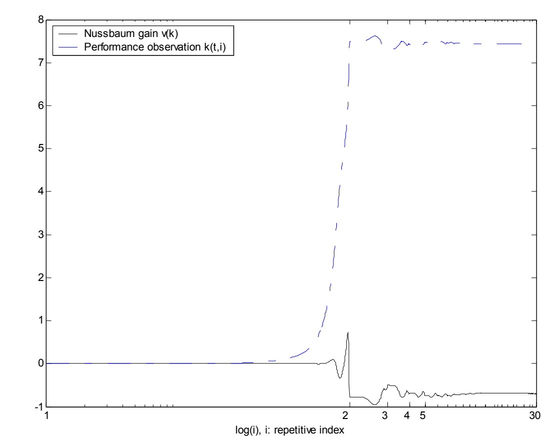

Fig.3. Nussbaum gain v and $k(t,i)$ for case 1

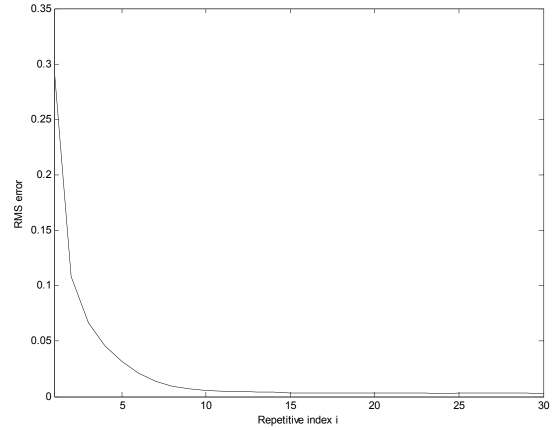

Fig.4. RMS error of the ILC for case 1

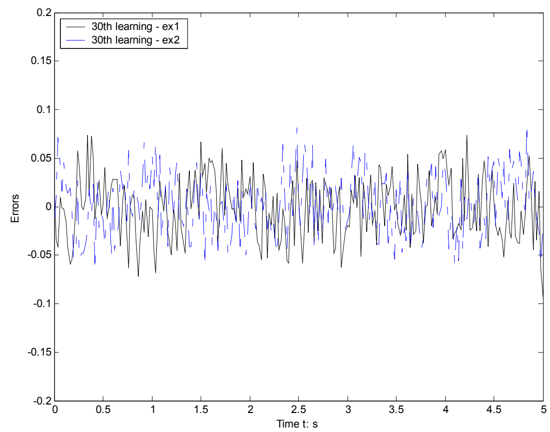

Fig.5. The 30th tracking errors for case 1

Now we suppose an unknown system with a repeatable uncertainty of

$$f(t) = \begin{bmatrix} -0.5e^{0.1t} \\ e^{0.1t} \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix}$$

which is negatively definite. The same control law is applied for the trajectory tracking. Fig. 6 and Fig. 7 show the 1st and 30th tracking errors. The proposed control law can automatically probe the control gain quickly as shown in Fig. 8.

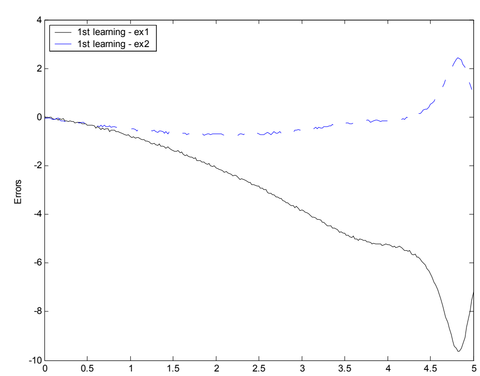

Fig.6 The first tracking errors for case 2

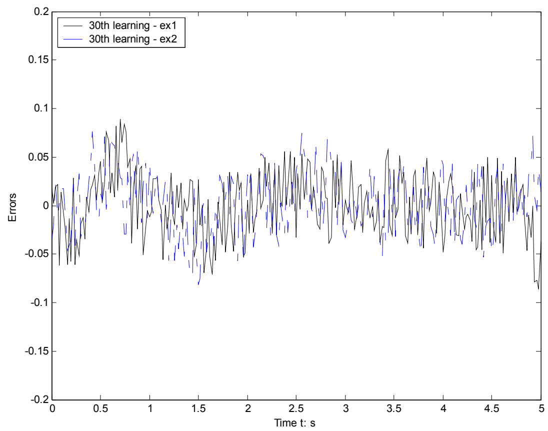

Fig.7 The 30th tracking errors for case 2

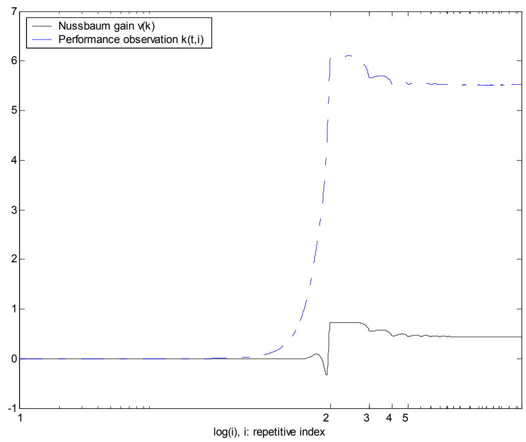

Fig.8 Nussbaum gain v and k(t,i) for case 2

From the evolution of the Nussbaum gain in Fig.3 and Fig.8, we can make the following conclusions:

1) the Nussbaum gain can find out the correct gain rapidly within the first trial and thereafter tracking will contribute to the learning of compensation as typical ILC's;
2) an interesting phenomenon can be observed: a steep increase of $k(t,i)$ and the corresponding Nussbaum gain v usually occur near the end of the first tracking. It is caused by large tracking errors and exhibits more oscillation for gain exploration. However, after the start of the second tracking, both $k(t,i)$ and v are quickly stabilised because of the initial resetting of the ILC's after a short period of local tunings. Consequently, repetitive resetting is a good strategy for Nussbaum function based gain selection, which gives a chance to fine tune the gain. It also insinuates that, in the Nussbaum-gain based adaptive control, a reference rectifying strategy, i.e. one which modifies the desired trajectory to align with the current state and forces the control error to zero regularly, could effectively help stabilise the gain-selection process and avoid extremely high-gain control, which also implies that a better transient performance could be achieved;
3) even with a measurement noise in a range of [-0.05,0.05], both $k(t,i)$ and the Nussbaum gain $v(k(t,i))$ can be stabilised because of the proposed performance-driven gain probing of $\dot{k}(t,i) = g^T(t,i)e(t,i)$ instead of typical norm forms, e.g. $\dot{k}(t,i) = \|e(t,i)\|^2$ in the Nussbaum gain based universal control[8].

V. Conclusions

Based on the idea of the *positive definite discrete matrix kernel* and the *Nussbaum gain*, this paper proposed an iterative learning control to achieve error convergence through repetitive tracking for a class of MIMO nonlinear systems. The system may include unknown but repeatable linear parameters and a symmetric positive definite or negative definite control gain matrix. Without prior knowledge about these uncertainties, the paper proved that the output converges to the desired output based on the previous trials. Simulations showed the validity of the proposed control method.

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