Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Unknown Input Observers Design for Time-Delay Systems Application to An Open-Channel

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Abstract— **This paper deals with the problem of full-order observers design for linear continuous delayed state and inputs systems with unknown input (UI) and time-varying delays. A method to design an Unknown Input Observer (UIO) for such systems is proposed based on a delay-dependent stability conditions of the state estimation error system. A Fault Detection and Isolation (FDI) scheme using a bank of such UIO, is also presented and tested on a (FDI) problem related to irrigation canals.**

I. INTRODUCTION

Irrigation represents more than 80% of world fresh water consumption in the world. However, large water losses occur in irrigation canals due to poor management. Water efficiency can be improved by the integration of automatic control. This automation requires a supervision that inspects the presence of faults. We mean by fault, any malfunction in sensors or actuators (regulation gates). The mode of these faults can be abrupt i.e step-like changes or incipient (slowly developing) like drift. In our study, we suppose that the faults do not occur simultaneously and their modes are unknown. Therefore, this supervision can be achieved by an efficient fault detection and isolation (FDI) scheme using a bank of unknown inputs observers, where each observer is insensitive to only one fault.

Irrigation canals are time-delay systems. It was shown that, in low frequencies, the dynamic of a reach is accurately represented by the integrator-delay model developed in [13]. This model, coupled with the gate equations, gives a delayed state and inputs system.

The observer design problem for systems with unknown inputs has received considerable attention in the last tow decades (see [2], [5]). However, few results have been presented in the case of time-delay systems. Moreover, either state delayed or input delayed systems are considered but rarely both of them. In addition to this, only the UIO design with independent-delay stability criterion has been proposed [12], [6]. Moreover, it was shown that in the state estimation problem such criterion [4],[7] is more conservative than the delay-dependent one [9], [10], [3]. Therefore, we focus in this paper on UIO design with delay-dependent criterion.

The contribution of this paper is to propose a new observer design method for systems with unknown inputs and time-varying delays in both state and inputs. The approach is based on representing the state estimation error system by a descriptor type model [3] and on deriving with the Park's inequality [8] a new delay-dependent stability conditions. The stabilization method of [3] is used and extended to the estimation problem. The new stability conditions of the estimation error system are formulated in terms of LMIs. Finally, a diagnosis scheme is presented at the end of the paper.

To illustrate the method, the diagnosis scheme is tested on a reach of the Canal of Gignac to detect and isolate actuators faults.

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II. PROBLEM FORMULATION

A. Channel hydraulic model

The dynamics of an open-channel can be well approximated at low frequencies around an equilibrium state by the following Laplace transfer matrix[13]:

$$\begin{pmatrix} y_1(s) \\ y_2(s) \end{pmatrix} = \begin{pmatrix} \frac{1}{A_u s} & -\frac{e^{-\tau_u s}}{A_u s} \\ \frac{e^{-\tau_d s}}{A_d s} & -\frac{1}{A_d s} \end{pmatrix} \begin{pmatrix} q_1(s) \\ q_2(s) \end{pmatrix} \quad (1)$$

where y_1 and y_2 are the upstream and downstream water level deviations from the equilibrium water levels, respectively and q_1 and q_2 are the upstream and downstream water flow rate deviations from the equilibrium flow rates, respectively.

If we now introduce the linearized model of the two regulation gates with respect to the equilibrium, we get the following equations:

$$q_1(t) = b_1 y_1(t) + b_2 u_1(t) \quad (2)$$
$$q_2(t) = b_3 y_2(t) + b_4 u_2(t) \quad (3)$$

where u_1 and u_2 are the control inputs.

Combining (1), (2) and (3), the following time-delay state space representation reads:

$$\begin{aligned} \dot{y}_1(t) &= \frac{b_1}{A_u} y_1(t) + \frac{b_2}{A_u} u_1(t) - \frac{b_3}{A_u} y_2(t-\tau_u) - \frac{b_4}{A_u} u_2(t-\tau_u) \\ \dot{y}_2(t) &= \frac{b_1}{A_d} y_1(t-\tau_d) + \frac{b_2}{A_d} u_1(t-\tau_d) - \frac{b_3}{A_d} y_2(t) - \frac{b_4}{A_d} u_2(t) \end{aligned}$$

which can be rewritten as:

$$\begin{aligned} \dot{x}(t) &= \textstyle\sum_{i=0}^{2} A_i x(t-\tau_i) + \sum_{i=0}^{2} B_i u(t-\tau_i) + W w(t) \\ y(t) &= C x(t) \end{aligned} \quad (4)$$

where $x = \begin{pmatrix} y_1 & y_2 \end{pmatrix}^T \in \Re^n$ is the state, $u = \begin{pmatrix} u_1 & u_2 \end{pmatrix}^T \in \Re^m$ is the control, and A_i, B_i, $i = 0,1,2$ and C are known real constant matrices defined by:

$$A_0 = \begin{pmatrix} \frac{b_1}{A_u} & 0 \\ 0 & -\frac{b_3}{A_d} \end{pmatrix},\ A_1 = \begin{pmatrix} 0 & -\frac{b_3}{A_u} \\ 0 & 0 \end{pmatrix},$$

$A_2 = \begin{pmatrix} 0 & 0 \\ \frac{b_1}{A_d} & 0 \end{pmatrix}$, $B_0 = \begin{pmatrix} \frac{b_2}{A_u} & 0 \\ 0 & -\frac{b_4}{A_d} \end{pmatrix}$,

$B_1 = \begin{pmatrix} 0 & -\frac{b_4}{A_u} \\ 0 & 0 \end{pmatrix}$, $B_2 = \begin{pmatrix} 0 & 0 \\ \frac{b_2}{A_d} & 0 \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $W = 0$.

If an additional actuator fault may occur then $u(t) = u^*(t) + \delta u(t)$ where u^* is the nominal actuator and δu the unknown fault actuator vector. In our application, delays are of about 10 minutes, we can then assume that faults can be detected before. In this case, the matrix W and the vector w can be written as follows:

$$W = B_0$$
$$w = \delta u(t)$$

The resulting model (4) is a delayed state and inputs system. In the following section, we develop an UIO for such system.

III. UIO DESIGN

In this section, we propose a full-order UIO structure. Then, we give the conditions to satisfy the asymptotic stability of the estimation error in spite of the presence of unknown inputs.

For the sake of generality, we consider the linear system (4) with time-varying delays, with the known history $x(t) = \varphi_1(t)$ $\forall t \in [-h, 0]$ and the initial known input $u(t) = \varphi_2(t)$, $\forall t \in [-h, 0]$ where the scalar $h > 0$ is an upper bound on the time delays $\tau_i(t)$, $i = 1, 2$ and $\tau_0(t) = 0$. $x \in \Re^n$ is the state, $u \in \Re^m$ is the known control, $w \in \mathbb{R}^q$ is the unknown input (UI), $y \in \Re^p$ is the measurement. A_i, B_i with $i = 0, 1, 2$ and W, C, are real of appropriate dimensions. Without loss of generality it is assumed that $rankC = p$ and $rankW = q$.

Only two delays are considered because of our system'structure but the designed method can be easily extended to the case of multiple delays. In addition to this, as in [3], the following two different cases for time-varying delay are considered.

Case I): $\tau_i(t)$ are known differentiable functions satisfying for all $t \geq 0$, $0 < \tau_i(t) \leq h_i$, $\dot{\tau}_i(t) \leq d_i < 1$, $\quad i = 1, 2$

Case II): $\tau_i(t)$ are known continuous functions satisfying for all $t \geq 0$, $0 < \tau_i(t) \leq h_i$, $\quad i = 1, 2$. In this case, very fast changes in time delay are allowed.

Consider the following full-order UIO for system (4):

$$\begin{cases} \dot{z}(t) = \sum_{i=0}^{2} F_i z(t - \tau_i(t)) + \sum_{i=0}^{2} T B_i u(t - \tau_i(t)) \\ + \sum_{i=0}^{2} G_i y(t - \tau_i(t)) \\ \hat{x}(t) = z(t) + N y(t) \end{cases} \quad (5)$$

with the initial state $z(t) = \varphi_3(t)$ $\forall t \in [-h, 0]$ where $z \in \mathbb{R}^n$ is the observer state, F_i, G_i, $i = 0, 1, 2$, T and N are constant matrices of appropriate dimensions which must be determined such that $\hat{x}(t)$ asymptotically converges to $x(t)$ for any $\varphi_1(t), \varphi_2(t), \varphi_3(t)$, y, u and w.

Define the estimation error state $e(t)$ as:

$$e(t) = \hat{x}(t) - x(t) \quad (6)$$

then, we can establish the following results

Theorem 1: The n th-order observer (5) will asymptotically estimate x if and only if the following conditions hold

1) $\dot{e}(t) = \sum_{i=0}^{2} F_i e(t - \tau_i(t))$ is asymptotically stable
2) $T + NC = I_n$
3) $TW = 0$
4) $\bar{G}_i = G_i - F_i N, \qquad i = 0, 1, 2$
5) $F_i = T A_i - \bar{G}_i C, \qquad i = 0, 1, 2$

Proof: From condition 2 of theorem 1, the estimation error $e(t)$ (6) becomes

$$e = z - Tx \quad (7)$$

and the dynamic of the state estimation error (7) satisfies the differential-delay equation:

$$\dot{e}(t) = \sum_{i=0}^{2} F_i e(t - \tau_i(t)) - TWw(t) + \sum_{i=0}^{2} \left(F_i + \bar{G}_i C - T A_i\right) x(t - \tau_i(t))$$

Now, if the conditions 3, 4 and 5 of theorem 1 hold, then (8) becomes an autonomous system:

$$\dot{e}(t) = \sum_{i=0}^{2} F_i e(t - \tau_i(t)) \quad (8)$$

and if condition 1 of theorem1 holds, then $e(t) \to 0$ as $t \to \infty$ for any $\varphi_1(t), \varphi_2(t), \varphi_3(t)$, y, u and w. ■

IV. OBSERVER PARAMETERS SYNTHESIS

In the sequel, we give the necessary and sufficient conditions to the existence of such UIO. Then, we present a new method to compute the observer parameters, using generalized inverse matrix and leading, by duality, to a problem of stabilization. This latter has already been treated in [3]. Therefore, the corresponding results are used and extended to the estimation problem.

The first step of the observer design consists to rewrite conditions 2), 3) and 5) of theorem 1) like

$$\begin{bmatrix} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 & F_2 & \bar{G}_2 \end{bmatrix} \Theta_1 = \Psi_1 \quad (9)$$

where

$$\Theta_1 = \begin{bmatrix} I_n & W & A_0 & A_1 & A_2 \\ C & 0 & 0 & 0 & 0 \\ 0 & 0 & -I_n & 0 & 0 \\ 0 & 0 & -C & 0 & 0 \\ 0 & 0 & 0 & -I_n & 0 \\ 0 & 0 & 0 & -C & 0 \\ 0 & 0 & 0 & 0 & -I_n \\ 0 & 0 & 0 & 0 & -C \end{bmatrix} \in \Re^{(4n+4p)\times(4n+q)}$$

and

$$\Psi_1 = \begin{bmatrix} I_n & 0 & 0 & 0 & 0 \end{bmatrix} \in \Re^{n\times(4n+q)}.$$

Now, we can give the necessary and sufficient condition for the existence of the observer (5).

Lemma 1: The necessary and sufficient condition for the existence of the observer (5) for system (4) is

$$rankCW = rankW \quad (10)$$

where (10) is the UI decoupled condition needed in standard UIO design [2].

Proof: The solution $\left[\begin{array}{cccccccc} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 & F_2 & \bar{G}_2 \end{array}\right]$ of (9) depends on the rank of matrix Θ_1. A solution exists if and only if[11]

$$rank\left[\begin{array}{c} \Theta_1 \\ \Psi_1 \end{array}\right] = rank\Theta_1 \qquad (11)$$

Using relation (11) and the definition of matrix Θ_1 and Ψ_1, we obtain

$$rank\left[\begin{array}{cc} I_n & W \\ C & 0 \end{array}\right] = n + rankW \qquad (12)$$

Define the nonsingular matrix $V_0 = \left[\begin{array}{cc} I_n & 0 \\ C & -I_p \end{array}\right]$,then

$$\begin{array}{rl} (12) \Leftrightarrow & rankV_0\left[\begin{array}{cc} I_n & W \\ C & 0 \end{array}\right] = n + rankW \\ \Leftrightarrow & rankCW = rankW \end{array}$$

That completes the proof. ■

From [11], under (10), the general solution of (9) is

$$S = \Psi_1\Theta_1^+ - K\left(I_{4(n+p)} - \Theta_1\Theta_1^+\right) \qquad (13)$$

with $S = \left[\begin{array}{cccccccc} T & N & F_0 & \bar{G}_0 & F_1 & \bar{G}_1 & F_2 & \bar{G}_2 \end{array}\right]$ and where K is an arbitrary matrix of appropriate dimension and Θ_1^+ is the generalized inverse matrix of Θ_1.

Solution (13) inserted in condition 5) of theorem 1), i.e.,

$$\left[\begin{array}{ccc} F_0 & F_1 & F_2 \end{array}\right] = S\left[\begin{array}{ccc} A_0 & A_1 & A_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -C & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -C & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -C \end{array}\right]$$

gives

$$F_i = \chi_i - K\beta_i, \qquad i = 0,1,2 \qquad (14)$$

where

$$\chi_0 = \Psi_1\Theta_1^+\left[\begin{array}{cccccccc} A_0^T & 0 & 0 & -C^T & 0 & 0 & 0 & 0 \end{array}\right]^T$$
$$\chi_1 = \Psi_1\Theta_1^+\left[\begin{array}{cccccccc} A_1^T & 0 & 0 & 0 & 0 & -C^T & 0 & 0 \end{array}\right]^T$$
$$\chi_2 = \Psi_1\Theta_1^+\left[\begin{array}{cccccccc} A_2^T & 0 & 0 & 0 & 0 & 0 & 0 & -C^T \end{array}\right]^T$$

and

$$\beta_0 = \left(I - \Theta_1\Theta_1^+\right)\left[\begin{array}{cccccccc} A_0^T & 0 & 0 & -C^T & 0 & 0 & 0 & 0 \end{array}\right]^T$$
$$\beta_1 = \left(I - \Theta_1\Theta_1^+\right)\left[\begin{array}{cccccccc} A_1^T & 0 & 0 & 0 & 0 & -C^T & 0 & 0 \end{array}\right]^T$$
$$\beta_2 = \left(I - \Theta_1\Theta_1^+\right)\left[\begin{array}{cccccccc} A_2^T & 0 & 0 & 0 & 0 & 0 & 0 & -C^T \end{array}\right]^T$$

Now, the dynamic of the estimation error state (8) can be written as

$$\dot{e}\left(t\right) = \sum_{i=0}^{2}\left(\chi_i - K\beta_i\right)e\left(t - \tau_i\left(t\right)\right) \qquad (15)$$

The second step of the UIO design is then reduced to the determination of the free matrix gain K, such that condition 1) of theorem 1) holds.

A. Delay-dependent stability: case I

By duality (15) is asymptotically stable if and only if the following adjoint system

$$\dot{\zeta}\left(t\right) = \sum_{i=0}^{2}\left(\chi_i^T - \beta_i^T K^T\right)\zeta\left(t - \tau_i\left(t\right)\right) \qquad (16)$$

is asymptotically stable too. Therefore, rewriting (16) in the following equivalent descriptor form [3]

$$\left\{\begin{array}{c} \dot{\zeta}\left(t\right) = g\left(t\right) \\ 0 = -g(t) + \sum_{i=0}^{2}\left(\chi_i^T - \beta_i^T K^T\right)\zeta(t) \\ -\sum_{i=1}^{2}\left(\chi_i^T - \beta_i^T K^T\right)\int_{t-\tau_i(t)}^{t} g(s)ds \end{array}\right.$$

and from lemma 1 in [3], we can derive the delay-dependent stability conditions for case I).

Theorem 2: Under (10) there exists a matrix K, for some values of ϵ_i, $i = 1,2$ such that the observer (5) is asymptotically stable for any differentiable functions $\tau_i\left(t\right)$ satisfying for all $t \geq 0$, $0<\tau_i(t)\leq h_i$, $\dot{\tau}_i\left(t\right) \leq d_i < 1$, $i = 1,\ 2$ if there exist $n \times n$ matrices $Q_1 = Q_1^T > 0$, Q_2, Q_3, $\bar{Z}_{i_1}$, $\bar{Z}_{i_2}$, $\bar{Z}_{i_3}$, $\bar{S}_i = \bar{S}_i^T > 0$, $\bar{R}_i = \bar{R}_i^T > 0$ for $i = 1,\ 2$ and $U \in \Re^{n\times(4n+4p)}$ that satisfy the following LMIs:

$$\left[\begin{array}{cccccc} \bar{\alpha}_{11} & \bar{\alpha}_{12} & 0 & 0 & Q_2^T & Q_2^T \\ * & \bar{\alpha}_{22} & \bar{\alpha}_{23} & \bar{\alpha}_{24} & Q_3^T & Q_3^T \\ * & * & \bar{\alpha}_{33} & 0 & 0 & 0 \\ * & * & * & \bar{\alpha}_{44} & 0 & 0 \\ * & * & * & * & -h_1^{-1}\bar{R}_1 & 0 \\ * & * & * & * & * & -h_2^{-1}\bar{R}_2 \end{array}\right] < 0 \qquad (17)$$

$$\left[\begin{array}{ccc} \bar{R}_i & 0 & \epsilon_i\bar{R}_iA_i \\ * & \bar{Z}_{i_1} & \bar{Z}_{i_2} \\ * & * & \bar{Z}_{i_3} \end{array}\right] \geq 0 \quad ; \quad i = 1,2 \qquad (18)$$

where

$$\begin{array}{l} \bar{\alpha}_{11} = Q_2 + Q_2^T + \sum_{i=1}^{2} h_i\bar{Z}_{i_1} + \sum_{i=1}^{2}\bar{S}_i \\ \bar{\alpha}_{12} = Q_3 - Q_2^T + \sum_{i=0}^{2}\epsilon_i\left(Q_1^T\chi_i - U\beta_i\right) + \sum_{i=1}^{2} h_i\bar{Z}_{i_2} \\ \bar{\alpha}_{22} = -Q_3 - Q_3^T + \sum_{i=1}^{2} h_i\bar{Z}_{i_3} \\ \bar{\alpha}_{23} = \left(\chi_1^T - A_1^T\epsilon_1\right)Q_1 - \beta_1^T U^T \\ \bar{\alpha}_{33} = -\left(1 - d_1\right)\bar{S}_1 \\ \bar{\alpha}_{24} = \left(\chi_2^T - A_2^T\epsilon_2\right)Q_1 - \beta_2^T U^T \\ \bar{\alpha}_{44} = -\left(1 - d_2\right)\bar{S}_2 \end{array}$$

The parameter matrix K is given by

$$K = Q_1^{-1}U \qquad (19)$$

Proof: The dynamics of the estimation error state (8) ($\Leftrightarrow$15) is asymptotically stable if and only if the adjoint system (16) is asymptotically stable. This system (16) is different to the feedback matrix $\left(A_0 + BK\right)$ or $\left(A_i + BK_i\right)$, $i = 0,1,2$ described in [3], hence the LMI (19a,b) or (22a,b) defined in [3] can not be directly used. However the results of Lemma 1 defined in [3] can be used to verify the stability of (16). Firstly, substituting $F_i^T = A_i$, i =0,1,2 in LMI (8a) defined in [3]. Secondly, defining $P^{-1} = Q = \left[\begin{array}{cc} Q_1 & 0 \\ Q_2 & Q_3 \end{array}\right]$, $\Delta_1 = diag\left\{Q, Q_1, Q_1\right\}$, $\Delta_2 = diag\left\{R_i^{-1}, Q\right\}$, $R_i^{-1} = \bar{R}_i$, $Y_i = \left[\begin{array}{cc} 0 & \epsilon_iA_i \end{array}\right]P$, $Q^TZ_iQ = \left[\begin{array}{cc} \bar{Z}_{i_1} & \bar{Z}_{i_2} \\ * & \bar{Z}_{i_3} \end{array}\right]$, $Q_1^TS_iQ_1 = \bar{S}_i$, $U^T = K^TQ_1$

where $\varepsilon_i \in \Re^1$, $i = 1, 2$. Thirdly, the LMI (8a) (resp. (8b)) defined in [3], is multiplied by Δ_1^T and Δ_1 (resp. by Δ_2^T and Δ_2) on the left and right sides, respectively. ■

Remark 1: For $\bar{R}_i = \rho I$, $\bar{Z}_i = 0$, $i = 1, 2$, LMI's (17) and (18) implies for $\rho \to \infty$ the following delay-independent/delay-derivative-dependent LMI

$$\begin{bmatrix} \widehat{\alpha}_{11} & \widehat{\alpha}_{12} & 0 & 0 \\ * & \widehat{\alpha}_{22} & \widehat{\alpha}_{23} & \widehat{\alpha}_{24} \\ * & * & -(1-d_1)\bar{S}_1 & 0 \\ * & * & * & -(1-d_2)\bar{S}_2 \end{bmatrix} < 0$$

where:

$$\begin{aligned} &\widehat{\alpha}_{11} = Q_2 + Q_2^T + \textstyle\sum_{i=1}^2 \bar{S}_i \\ &\widehat{\alpha}_{12} = Q_3 - Q_2^T + Q_1^T \chi_0 - U\beta_0 + \textstyle\sum_{i=1}^2 Q_1^T \epsilon_i A_i \\ &\widehat{\alpha}_{22} = -Q_3 - Q_3^T \\ &\widehat{\alpha}_{23} = \left(\chi_1^T - A_1^T \epsilon_1\right) Q_1 - \beta_1^T U^T, \\ &\widehat{\alpha}_{24} = \left(\chi_2^T - A_2^T \epsilon_2\right) Q_1 - \beta_2^T U^T \end{aligned}$$

Proof: Similar to remark4 in [3]. ■

B. Delay-dependent stability: case II

Like corollary 1 in [3], choosing in theorem 2, $\bar{S}_i \to 0$ we obtain the following result for case II)

Theorem 3: Under (10) there exists a matrix K for some values of ϵ_i, $i = 1, 2$ such that the observer (5) is asymptotically stable for any continuous functions $\tau_i(t)$ satisfying for all $t \geq 0$, $0 < \tau_i(t) \leq h_i$, $i = 1, 2$ if there exist $n \times n$ matrices $Q_1 = Q_1^T > 0$, Q_2, Q_3, $\bar{Z}_{i_1}$, $\bar{Z}_{i_2}$, $\bar{Z}_{i_3}$, $\bar{R}_i = \bar{R}_i^T$, $i = 1, 2$ and $U \in \Re^{n \times (4n+4p)}$ that satisfy (18) and the following LMI

$$\begin{bmatrix} \breve{\alpha}_{11} & \bar{\alpha}_{12} & Q_2^T & Q_2^T \\ * & \bar{\alpha}_{22} & Q_3^T & Q_3^T \\ * & * & -h_1^{-1}\bar{R}_1 & 0 \\ * & * & * & -h_2^{-1}\bar{R}_2 \end{bmatrix} < 0 \qquad (20)$$

where

$$\breve{\alpha}_{11} = Q_2 + Q_2^T + \sum_{i=1}^{2} h_i \bar{Z}_{i_1}$$

The parameter matrix K is given by (19).

C. Practical view point

Here, we show how to fixe the modes of the observer in the case of very small delays. It is usual that the eigenvalues of $F = \sum_{i=0}^2 F_i$ must be fixed in a specified region in order to obtain a good estimation performance of the state dynamic (15). For simplicity, the region considered here, is the vertical strip defined by

$$D = \{x + jy \in C, \quad -\lambda_1 < x < -\lambda_2 < 0\} \qquad (21)$$

Before giving the additional LMIs for fixing the eigenvalues of F in region D, the following lemma is done.

Lemma 2: There exists a matrix K such that F is Hurwitz if and only if

$$rank \begin{bmatrix} sI_n - \sum_{i=0}^2 A_i & W \\ C & 0 \end{bmatrix} = n + q, \forall s \in C, \Re(s) \geq 0 \qquad (22)$$

Proof: From (14), the matrix F can be written as

$$F = \chi - K\beta$$

where:

$$\begin{aligned} \chi &= \chi_0 + \chi_1 + \chi_2 = \Psi_1 \Theta_1^+ \varphi, \\ \beta &= \beta_0 + \beta_1 + \beta_2 = \left(I - \Theta_1 \Theta_1^+\right) \varphi \ and \\ \varphi^T &= \begin{bmatrix} \sum_{i=0}^2 A_i^T & 0 & 0 & -C^T & 0 & -C^T & 0 & -C^T \end{bmatrix} \end{aligned}$$

Then, there exists K such that F is Hurwitz if and only if the pair $(\ \chi, \ \ \beta\ )$ is detectable i.e.,

$$rank \begin{bmatrix} sI_n - \chi \\ \beta \end{bmatrix} = n, \qquad \forall s \in C, \Re(s) \geq 0$$

Define the following nonsingular matrices:

$$V_1 = \begin{bmatrix} I_n & 0 \\ -\Theta_1^+ \varphi & I \end{bmatrix}, \qquad V_2 = \begin{bmatrix} -I_n & 0 & 0 \\ sI_n & I_n & 0 \\ 0 & 0 & I_{q+3n} \end{bmatrix}$$

and the full-column rank matrix

$$V_3 = \begin{bmatrix} I & -\Psi_1 \Theta_1^+ \\ 0 & I - \Theta_1 \Theta_1^+ \\ 0 & \Theta_1 \Theta_1^+ \end{bmatrix}$$

Then

$$\begin{aligned} rank \begin{bmatrix} sI_n - \sum_{i=0}^2 A_i & W \\ C & 0 \end{bmatrix} &= rank \begin{bmatrix} sI_n - \sum_{i=0}^2 A_i & W \\ sC & 0 \\ 0 & 0 \\ 0 & 0 \\ C & 0 \\ 0 & 0 \\ C & 0 \\ 0 & 0 \\ C & 0 \end{bmatrix} \\ &= rank \begin{bmatrix} sI_n & \begin{bmatrix} I_n & 0 & 0 & 0 & 0 \end{bmatrix} \\ \varphi & \Theta_1 \end{bmatrix} V_2 - 4n \\ &= rank \begin{bmatrix} sI_n & \Psi_1 \\ \varphi & \Theta_1 \end{bmatrix} - 4n \\ &= rank V_3 \begin{bmatrix} sI_n & \Psi_1 \\ \varphi & \Theta_1 \end{bmatrix} V_1 - 4n \\ &= rank \begin{bmatrix} sI_n - \chi & \times \\ \beta & \times \\ 0 & \Theta_1 \end{bmatrix} - 4n \\ &= rank \begin{bmatrix} sI_n - \chi \\ \beta \end{bmatrix} + rank \Theta_1 - 4n \\ &= rank \begin{bmatrix} sI_n - \chi \\ \beta \end{bmatrix} + rank \begin{bmatrix} I_n & W \\ C & 0 \end{bmatrix} - n \\ &= n + q \end{aligned}$$

■

Now, from the following lemma, under Lemma 1 and 2 the eigenvalues of F can be fixed in specified region.

Lemma 3: Under (10) and (22), matrix F has all its eigenvalues in the region D if and only if there exist $Q_1 = Q_1^T > 0$ and U such that

$$\begin{aligned} &Q_1 \textstyle\sum_{i=0}^2 \chi_i + \left(Q_1 \sum_{i=0}^2 \chi_i\right)^T - U \sum_{i=0}^2 \beta_i - \left(U \sum_{i=0}^2 \beta_i\right)^T + 2\lambda_2 Q_1 < 0 \\ &Q_1 \textstyle\sum_{i=0}^2 \chi_i + \left(Q_1 \sum_{i=0}^2 \chi_i\right)^T - U \sum_{i=0}^2 \beta_i - \left(U \sum_{i=0}^2 \beta_i\right)^T + 2\lambda_1 Q_1 > 0 \end{aligned} \qquad (23)$$

where λ_1 and λ_2 are fixed for a good estimation performance and such that the eventual unobservable stable modes of the pair $\left(\begin{array}{cc}\chi, & \beta\end{array}\right)$ are included in the D region.

Proof: Using the [1] results, the lemma is proved. ■

D. Summary

The procedure for designing the UIO (5) for system (4) can be summarized by the following algorithms.

Algorithm 1: Case I) Under (10) and (22) choose $\epsilon_i \in \Re^1$, $i = 1, 2$ and solve the LMIs (17), (18) and (23) on $Q_1 = Q_1^T > 0$, Q_2, Q_3, $\bar{S}_i = \bar{S}_i^T > 0$, $\bar{Z}_{i_1}$, $\bar{Z}_{i_2}$, $\bar{Z}_{i_3}$, U, and $\bar{R}_i = \bar{R}_i^T > 0$, i =1,2 where λ_1 and λ_2 are fixed for a well estimation performance. From (19), (13) and condition 4) of theorem 1 deduce the matrices K, T, N, F_i, $\bar{G}_i$, G_i, $i = 0, 1, 2$ respectively. ▽

Algorithm 2: Case II) under (10) and (22), choose $\epsilon_i \in \Re^1$, $i = 1, 2$ and solve the LMIs (20), (18) and (23) on $Q_1 = Q_1^T > 0$, Q_2, Q_3, $\bar{Z}_{i_1}$, $\bar{Z}_{i_2}$, $\bar{Z}_{i_3}$, U and $\bar{R}_i = \bar{R}_i^T > 0$, $i = 1, 2$ ...(the end is similar of the above algorithm).
Using the previous algorithm, a fault detection and isolation (FDI) scheme can be implemented as it is shown in the next section.

V. Diagnosis scheme

This consists of generating a set of m observers related to a residual

$$r_j = y - C\hat{x}_j, \quad j = 1, 2, ..., m$$

which is insensitive (i.e., robust) to one element of the vector w (represented by 0) and sensitive to the $m-1$ other components of w (represented by 1). That is summarized in the following table

If	r_1	r_2	$\cdots$	r_m
$\delta u_1 \neq 0$	0	1	$\cdots$	1
$\delta u_2 \neq 0$	1	0		$\vdots$
$\vdots$	$\vdots$	$\vdots$		1
$\delta u_m \neq 0$	1	$\cdots$	1	0

The state $\hat{x}_j$ is the output of the j^{th} observer

$$\begin{array}{l}\dot{z}(t) = \sum_{i=0}^{2} F_i z(t-\tau_i(t)) + \sum_{i=0}^{2} TB_i u(t-\tau_i(t)) \\ +\sum_{i=0}^{2} G_i y(t-\tau_i(t)) \\ \hat{x}_j(t) = z(t) + Ny(t)\end{array}$$

which is designed for the j^{th} fault model (24) according to the above table

$$\begin{array}{l}\dot{x}(t) = \sum_{i=0}^{2} A_i x(t-\tau_i(t)) + \sum_{i=0}^{2} B_i u(t-\tau_i(t)) \\ +W_j w_j(t) + W_{\bar{j}} w_{\bar{j}} \\ y(t) = Cx(t)\end{array} \tag{24}$$

where $W_j = B_{0_j}$, $W_{\bar{j}} = \left[\begin{array}{cccc}... & B_{0_{j-1}} & B_{0_{j+1}} & ...\end{array}\right]$, $w_j(t) = \delta u_j$, $w_{\bar{j}} = \left[\begin{array}{cccc}... & \delta u_{j-1} & \delta u_{j+1} & ...\end{array}\right]^T$ and B_{0_j} the j^{th} column of B_0. Under $w_{\bar{j}} = 0$, the unknown matrices F_i, G_i, $i = 0, 1, 2$, T and N are determined such that $\hat{x}_j(t)$ asymptotically converges to $x(t)$ for any $\varphi_1(t), \varphi_2(t), \varphi_3(t)$, y, u, w_j. Since $W_{\bar{j}}$ is linearly independent of the UI distribution matrix W_j then

$$\left\{\begin{array}{c}\dot{e}(t) = \sum_{i=0}^{2} F_i e(t-\tau_i(t)) + TW_{\bar{j}} w_{\bar{j}} \\ r_j(t) = Ce(t)\end{array}\right.$$

and $r_j(t) \neq 0$ for $w_{\bar{j}} \neq 0$. Now using the m residual, the following j^{th} alarm can be designed

$$\begin{array}{rcl}a_j & = & r_1 \cap ... \cap r_{j-1} \cap r_{j+1} \cap .. \cap r_m \quad , \quad j = 1, 2, ..., m \\ & = & 1 \text{ if } \delta u_j \neq 0 \\ & = & 0 \text{ else}\end{array}$$

Note that, in order to increase the UI insensitivity only a single actuator fault is considered. Practically, the probability that an other fault occurs simultaneously is infinitesimal.

VI. Application to open-channel

In our application, we suppose that unknown faults occur on each regulation gate. It means that $u_1(t) = u_1^*(t) + \delta u_1(t)$ and $u_2(t) = u_2^*(t) + \delta u_2(t)$. Therefore, the diagnosis scheme can be achieved with a bank of two observers where observer$_1$ is insensitive to $\delta u_1(t)$ and observer$_2$ is insensitive to $\delta u_2(t)$. The table decision is as follows:

If	r_1	r_2
$\delta u_1 \neq 0$	0	1
$\delta u_2 \neq 0$	1	0

The dynamic model of the considered canal is given by:

$$\begin{array}{l}A_0 = 10^{-3}\left(\begin{array}{cc}-0.4084 & 0 \\ 0 & -0.2787\end{array}\right), W = B_0 \\ A_1 = 10^{-3}\left(\begin{array}{cc}0 & -0.2432 \\ 0 & 0\end{array}\right), B_0 = 10^{-3}\left(\begin{array}{cc}0.55 & 0 \\ 0 & -0.42\end{array}\right) \\ A_2 = 10^{-3}\left(\begin{array}{cc}0 & 0 \\ -0.4675 & 0\end{array}\right), B_1 = 10^{-3}\left(\begin{array}{cc}0 & -0.3714 \\ 0 & 0\end{array}\right) \\ C = \left(\begin{array}{cc}1 & 0 \\ 0 & 1\end{array}\right), B_2 = 10^{-3}\left(\begin{array}{cc}0 & 0 \\ 0.6320 & 0\end{array}\right)\end{array}$$

The UI decoupled condition (10) is satisfied and the observer (5) can be designed for our system. According to the fault model (24), we have $w_1 = \delta u_1(t) = w_{\bar{2}}$, $w_2 = \delta u_2(t) = w_{\bar{1}}$, $W_1 = B_{0_1} = W_{\bar{2}}$ and $W_2 = B_{0_2} = W_{\bar{1}}$ where B_{0_1} and B_{0_2} are the first and the second column of B_0 respectively.

For the considered canal, the delays are assumed to be constant (i.e. $\dot{\tau}_u = \dot{\tau}_d = 0$) and equal to $\tau_u = 846.5$ s and $\tau_d = 707.5$ s. The delay-dependent criterion (Case I) is then applied for $h_1 = \tau_u$, $h_2 = \tau_d$, $d_1 = d_2 = 0$ and the region D is defined by $\lambda_1 = 0.0020$, $\lambda_2 = 0.0014$.

Using algorithm 1, the following results are obtained

a) Observer$_1$ is designed to be insensitive to $\delta u_1(t)$, the corresponding parameter matrix T_1 obtained leads to:

$$T_1 W_1 = 10^{-15}\left(\begin{array}{c}0.0019 \\ -0.1163\end{array}\right) \approx 0 \text{ while}$$

$$T_1 W_2 = 10^{-3}\left(\begin{array}{c}-0.0000 \\ -0.2125\end{array}\right)$$

b) Observer$_2$ is insensitive to $\delta u_2(t)$ and its parameter matrix T_2 gives:

$T_2W_2 = 10^{-16}\begin{pmatrix} 0.0028 \\ -0.7116 \end{pmatrix} \approx 0$ while
$T_2W_1 = 10^{-3}\begin{pmatrix} 0.2761 \\ -0.0000 \end{pmatrix}$

The eigenvalues of the matrix F found for each observer are $\begin{pmatrix} -0.0020 + 0.0000i & -0.0020 - 0.0000i \end{pmatrix}$.

We can notice that the eigenvalues of the matrix F are included in the desired region D and that the decoupling objective is satisfied.

To illustrate the (FDI) scheme, we generate non simultaneous biased faults of 0.01 meter on the regulation gates. Figures (1), (2) and (3) show respectively the first, second and third simulation, where

Simulation 1	$\delta u_1 \neq 0$	$\delta u_2 = 0$
Simulation 2	$\delta u_1 = 0$	$\delta u_2 \neq 0$
Simulation 3	$\delta u_1 = 0$	$\delta u_2 = 0$

The figure (1) (resp. figure (2)) shows that in the presence

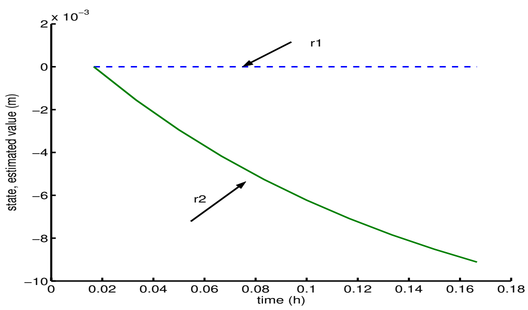

Fig. 1. case I: a) $\delta u_1 \neq 0$, $\delta u_2 = 0$

of the fault δu_1 (resp. δu_2), the observer$_1$(resp. observer$_2$) generates a null residual r_1 (resp.r_2) while the observer$_2$ (resp. observer$_1$) generates a non null one $r_2 \neq 0$ (resp. $r_1 \neq 0$) which corresponds to the table decision. Without faults $\delta u_1 = \delta u_2 = 0$, the state is well estimated as it is shown in figure 3.

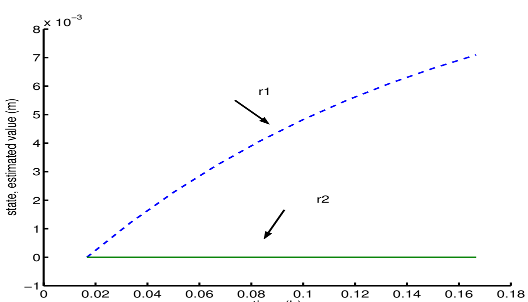

Fig. 2. b) $\delta u_2 \neq 0, \delta u_1 = 0$

VII. CONCLUSION

In this paper, we have presented a rigorous method for the design of a full-order UIO for time-varying delayed state and inputs system. Existence conditions, delay-dependent stability conditions of the observer have been given and proved. Then, a fault detection and isolation scheme (FDI) based on a bank of such UIO has been proposed and tested on the Canal of Gignac in order to detect and isolate non simultaneous regulation gate faults. The results obtained showed that the diagnosis objective has been achieved.

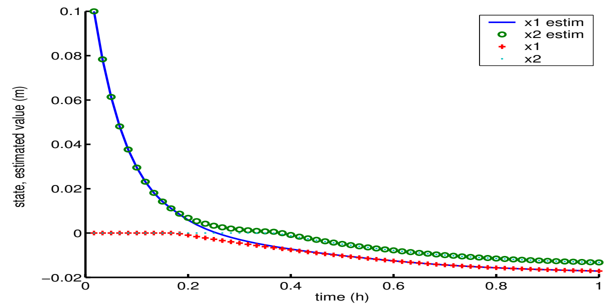

Fig. 3. $\delta u_1 = \delta u_2 = 0$

VIII. ACKNOWLEDGMENTS

This work is supported by the Région Languedoc-Roussillon, Conseil Général de l'Hérault et l'Agence de l'Eau through the Gignac project.

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