

Decentralized Constrained Optimal Control Approach to Distributed Paper Machine Control

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Abstract—A decentralized control scheme is proposed to address a spatially distributed paper machine control problem under industrial investigation. The methodology is based on constrained optimal control and tackles the complexity of the problem by solving local problems of small size, which are coupled through their performance objectives and constraints. Properties of the decentralized scheme are analyzed in detail and compared to the existing centralized receding horizon controller currently used in industry. In terms of the simulated example the decentralized controller can achieve similar performance to the centralized controller but at a fraction of the computational cost.

I. INTRODUCTION

In this paper we apply the decentralized control scheme proposed in [1] to a paper machine control problem. We will focus on the dry weight property of the paper. A paper machine problem has the following characteristics: (i) it involves large number of actuators (later referred to as subsystems) which can be independently actuated, (ii) the control objective can only be achieved through a collective behavior of the subsystems, and (iii) the feasible set of states of each subsystem is a function of other subsystems' states. Applications with these kind of idiosyncrasies fall under the general class of optimal control problems for a set of decoupled dynamical systems where cost function and constraints couple the dynamical behavior of the systems.

In the recent paper [1] we have proposed a method for designing decentralized receding horizon controllers (RHC). A centralized RHC controller is broken into distinct RHC controllers of smaller sizes. Each RHC controller is associated to a different node and computes the local control inputs based only on the states of the node and of its neighbors. More into details, the problem of decentralized control for decoupled systems can be formulated as follows. A dynamical system is composed of (or can be decomposed into) distinct dynamical subsystems that can be independently actuated. The subsystems are dynamically decoupled but have common objectives and constraints which make them interact between each other. Typically the *interaction* is local, i.e. the objective and the constraints of a subsystem are function of only a subset of other subsystems' states.

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The interaction will be represented by an “interaction graph”, where the nodes represent the subsystems and an arc between two nodes denotes a coupling term in the objectives and/or in the constraints associated to the nodes. Also, typically it is assumed that the *exchange of information* has a special structure, i.e., it is assumed that each subsystem can sense and/or exchange information with only a subset of other subsystems. Often the *interaction graph* and the *information exchange graph* coincide. A decentralized control scheme consists of distinct controllers, one for each subsystem, where the inputs to each subsystem are computed only based on local information, i.e. on the states of the subsystem and its neighbors.

The paper is structured as follows. In Section II we introduce a class of independently actuated systems and the optimal control problem which couple the dynamical behavior of the systems. In Section III we present the paper machine application example and show why it falls under this class of problems. Section IV introduces the decentralized control scheme used to approach this problem. Comparison of centralized and decentralized simulation results concludes the paper.

II. PROBLEM FORMULATION

Consider a set of N_v linear decoupled dynamical systems, where the i -th system is described by the discrete-time time-invariant state equations:

$$x_{k+1}^i = f^i(x_k^i, u_k^i) \quad (1)$$

where $x_k^i \in \mathbb{R}^{n^i}$, $u_k^i \in \mathbb{R}^{m^i}$, $f^i : \mathbb{R}^{n^i} \times \mathbb{R}^{m^i} \rightarrow \mathbb{R}^{n^i}$ are states, inputs and state update function of the i -th system, respectively. Let $\mathcal{U}^i \subseteq \mathbb{R}^{m^i}$ and $\mathcal{X}^i \subseteq \mathbb{R}^{n^i}$ denote the set of feasible inputs and states of the i -th system

$$x_k^i \in \mathcal{X}^i, \quad u_k^i \in \mathcal{U}^i, \quad k \geq 0 \quad (2)$$

where \mathcal{X}^i and \mathcal{U}^i are given polytopes.

We will refer to the set of N_v constrained systems as a *team system*. Let $\tilde{x}_k \in \mathbb{R}^{N_v \times n^i}$ and $\tilde{u}_k \in \mathbb{R}^{N_v \times m^i}$ be the vectors which collect the states and inputs of the team system at time k , i.e. $\tilde{x}_k = [x_k^1, \dots, x_k^{N_v}]$, $\tilde{u}_k = [u_k^1, \dots, u_k^{N_v}]$, with

$$\tilde{x}_{k+1} = f(\tilde{x}_k, \tilde{u}_k) \quad (3)$$

We denote by (x_e^i, u_e^i) the equilibrium pair of the i -th system and $(\tilde{x}_e, \tilde{u}_e)$ the corresponding equilibrium for the team system.

So far the individual systems belonging to the team system are completely decoupled. We consider an optimal

control problem for the team system where cost function and constraints couple the dynamic behavior of individual systems. We use a graph topology to represent the coupling in the following way. We associate the i -th system to the i -th node of the graph, and if an edge (i, j) connecting the i -th and j -th node is present, then the cost and the constraints of the optimal control problem will have a component which is a function of both x^i and x^j . The graph will be *undirected*, i.e. $(i, j) \in \mathcal{A} \Rightarrow (j, i) \in \mathcal{A}$ and furthermore, the edges representing coupling change with time. Therefore, before defining the optimal control problem, we need to define a graph (which can be time-varying)

$$\mathcal{G} = \{\mathcal{V}, \mathcal{A}\} \quad (4)$$

where \mathcal{V} is the set of nodes $\mathcal{V} = \{1, \dots, N_v\}$ and $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$ the set of time-varying arcs (i, j) with $i \in \mathcal{V}$, $j \in \mathcal{V}$.

Once the graph structure has been fixed, the optimization problem is formulated as follows. Denote with \tilde{x}_k^i the states of all neighbors of the i -th system at time k , i.e. $\tilde{x}_k^i = \{x_k^j \in \mathbb{R}^{n^j} \mid (j, i) \in \mathcal{A}\}$, $\tilde{x}_k^i \in \mathbb{R}^{\tilde{n}_k^i}$ with $\tilde{n}_k^i = \sum_{j \mid (j, i) \in \mathcal{A}} n^j$. Analogously, $\tilde{u}_k^i \in \mathbb{R}^{\tilde{m}_k^i}$ denotes the inputs to all the neighbors of the i -th system at time k . Let

$$g^{i,j}(x^i, x^j) \leq 0 \quad (5)$$

define interconnection constraints between the i -th and the j -th systems, with $g^{i,j} : \mathbb{R}^{n^i} \times \mathbb{R}^{n^j} \rightarrow \mathbb{R}^{nc_{i,j}}$. We will often use the following shorter form of the interconnection constraints defined between the i -th system and all its neighbors:

$$g_k^i(x_k^i, \tilde{x}_k^i) \leq 0 \quad (6)$$

with $g_k^i : \mathbb{R}^{n^i} \times \mathbb{R}^{\tilde{n}_k^i} \rightarrow \mathbb{R}^{nc_{i,k}}$.

Consider the following cost

$$l(\tilde{x}, \tilde{u}) = \sum_{i=1}^{N_v} l_k^i(x^i, u^i, \tilde{x}_k^i, \tilde{u}_k^i) \quad (7)$$

where $l^i : \mathbb{R}^{n^i} \times \mathbb{R}^{m^i} \times \mathbb{R}^{\tilde{n}_k^i} \times \mathbb{R}^{\tilde{m}_k^i} \rightarrow \mathbb{R}$ is the cost associated to the i -th system and is a function of its states and the states of its neighbor nodes. Assume that l is a positive convex function with $l(\tilde{x}_e, \tilde{u}_e) = 0$.

Consider the infinite time optimal control problem

$$\tilde{J}_\infty^*(\tilde{x}) \triangleq \min_{\{\tilde{u}_0, \tilde{u}_1, \dots\}} \sum_{k=0}^{\infty} l(\tilde{x}_k, \tilde{u}_k) \quad (8a)$$

$$\text{subj. to} \begin{cases} x_{k+1}^i = f^i(x_k^i, u_k^i), \\ i = 1, \dots, N_v, \quad k \geq 0 \\ g^{i,j}(x_k^i, x_k^j) \leq 0, \\ i = 1, \dots, N_v, \quad k \geq 0, \\ (i, j) \in \mathcal{A} \\ x_k^i \in \mathcal{X}^i, \quad u_k^i \in \mathcal{U}^i, \\ i = 1, \dots, N_v, \quad k \geq 0 \\ \tilde{x}_0 = \tilde{x} \end{cases} \quad (8b)$$

For all $\tilde{x} \in \mathbb{R}^{N_v \times n^i}$, if problem (8) is feasible, then the optimal input $\tilde{u}_0^*, \tilde{u}_1^*, \dots$ will drive the N_v systems to their equilibrium points x_e^i while satisfying state, input and

interconnection constraints. The following assumption will be valid throughout the whole paper.

Assumption 1: We assume the systems f_i and constraints $g^{i,j}$ to be linear functions of states and inputs variables. Moreover we assume the cost function l to be a convex and quadratic function of states and inputs variables.

Remark 1: Since we assumed that the graph is undirected, there will be redundant constraints in problem (8). Note the form of constraints (6) is rather general and it will include the case when only partial information about states of neighboring nodes is involved.

With the exception of a few cases, solving an infinite horizon optimal control problem is computationally prohibitive. This can be approximated by repeatedly solving finite time optimal control problems in a receding horizon fashion as described next. At each sampling time, starting at the current state, an open-loop optimal control problem is solved over a finite horizon. The optimal command signal is applied to the process only during the following sampling interval. At the next time step a new optimal control problem based on new measurements of the state is solved over a shifted horizon. The resultant controller is often referred to as Receding Horizon Controller (RHC). Assume at time t the current state \tilde{x}_t to be available. Consider the following constrained finite time optimal control problem

$$\tilde{J}_N^*(\tilde{x}_t) \triangleq \min_{\{U_t\}} \sum_{k=0}^{N-1} l(\tilde{x}_{k,t}, \tilde{u}_{k,t}) + l_N(\tilde{x}_{N,t}) \quad (9a)$$

$$\text{subj. to} \begin{cases} x_{k+1,t}^i = f^i(x_{k,t}^i, u_{k,t}^i), \\ i = 1, \dots, N_v, \quad k \geq 0 \\ g^{i,j}(x_{k,t}^i, x_{k,t}^j) \leq 0, \\ i = 1, \dots, N_v, \quad (i, j) \in \mathcal{A}, \\ k = 1, \dots, N-1 \\ x_{k,t}^i \in \mathcal{X}^i, \quad u_{k,t}^i \in \mathcal{U}^i \\ i = 1, \dots, N_v, \\ k = 1, \dots, N-1 \\ \tilde{x}_{N,t} \in \mathcal{X}_f, \\ \tilde{x}_{0,t} = \tilde{x}_t \end{cases} \quad (9b)$$

where N is the prediction horizon, $\mathcal{X}_f \subseteq \mathbb{R}^{N_v \times n^i}$ is a terminal region, l_N is the cost on the terminal state. In (9) we denote with $U_t \triangleq [\tilde{u}_{0,t}, \dots, \tilde{u}_{N-1,t}]' \in \mathbb{R}^s$, $s \triangleq N_v \times mN$ the optimization vector, $x_{k,t}^i$ denotes the state vector of the i -th node predicted at time $t+k$ obtained by starting from the state x_t^i and applying to system (1) the input sequence $u_{0,t}^i, \dots, u_{k-1,t}^i$. The tilded vectors will denote the prediction vectors associated to the team system.

Let $U_t^* = \{\tilde{u}_{0,t}^*, \dots, \tilde{u}_{N-1,t}^*\}$ be the optimal solution of (9) at time t and $\tilde{J}_N^*(\tilde{x}_t)$ the corresponding value function. Then, the first sample of U_t^* is applied to the team system (3)

$$\tilde{u}_t = \tilde{u}_{0,t}^* \quad (10)$$

The optimization (9) is repeated at time $t+1$, based on the new state x_{t+1} .

We assume that the reader is familiar with the basic concept of RHC and its main issues, we refer to [2] for

a comprehensive treatment of the topic. In this paper we will assume that terminal cost l_N and the terminal constraint set \mathcal{X}_f can be appropriately chosen in order to ensure the stability of the closed-loop system.

In general, the optimal input u_t^i to the i -th system computed by solving (9) at time t , will be a function of the overall state information \tilde{x}_t .

III. THE PAPER MACHINE PROBLEM

An industrial paper machine is a enormous and complex piece of equipment designed to convert a one-dimensional slurry of 99.5% water and 0.5% pulp fibres into a dry two-dimensional paper sheet up to 12metres wide with tight requirements on the variability of quality variables which will include dry weight, moisture content, caliper (thickness). The paper machine can operate at speeds over 100km/hour and the various paper properties are measured by sensors installed on a scanner typically (but not necessarily) installed at the extreme dry end of the paper machine that traverses the sheet measuring up to 2000 locations in each pass of the scanner once every 15-45 seconds.

The variability of the quality of the sheet must be controlled in both the machine direction (MD) and also the cross-direction (CD) perpendicular to the direction of sheet travel. The CD performance target will be to achieve a (often flat) target profile in the cross-direction. The cross-directional profiles are controlled using various actuator arrays distributed across the width of the paper machine. Each array will consist of 30-300 identically constructed and individually controlled actuators which must work together to achieve the control goals. The physical mechanisms of operation for different actuator arrays are quite wide ranging and include mechanical force, fluid flow, steam flow, induction heating, infrared heating and the interested reader is referred to [3]–[5].

Figure 1 illustrates a paper machine outfitted with several actuator arrays as indicated. This paper will concentrate on the functioning of a slice lip actuator array.¹ The wet pulp slurry enters from the left of Figure 1 into the headbox which is the width of the paper machine. The slurry is then extruded through a narrow opening (1-6cm tall) and has a width equal to that of the paper machine. The conversion of the one-dimensional pulp flow into a two-dimensional flow is a complex fluid dynamical problem requiring knowledge of flows as well as local consistency variations to understand the distribution of fibres that contribute to the dry weight. Deviations in the paper sheet's weight from its target will affect most other properties. Industrial headboxes require feedback control from the measured weight profile in order to achieve the correct weight profile for the paper sheet. An array of 50-120 force actuators are distributed across the width of the upper lip of the headbox opening - known as the slice lip. The pulp stock profile at the headbox can then be altered by using the array of force actuators to deform

¹Although it is important to remember that the vast majority of paper machines have several sets of actuators and measurements - the interactions of which are important [6] but beyond the scope of this paper.

the slice lip. Slice lip control is surprisingly delicate and successful weight control is achieved by moving the lip at most 0.17mm-0.75mm. Slice lips are very expensive and any control strategy must respect maximum and minimum actuator limits as well as 'bend limits' on the slice lip to prevent damage.

We will consider a model of the slice lip process obtained from a working paper machine using the model identification tool in [7]. The model describes the spatial response of the weight properties as a function of the actuator profile. The dynamics of each actuator in the headbox is modeled as a first order system with deadtime. The deadtime models the transport delay equivalent to the time taken for the paper to travel from the actuators to the scanning sensor. The model of each actuator system is described by the linear state update function

$$x_{k+1}^i = Ax_k^i + Bu_k^i \quad (11)$$

where x_k^i represents the position of the i -th actuator at time k and at previous time instants $k-1, \dots, k-\Delta$ (where Δ is the order of the deadtime), and u_k^i is the desired position of the actuators. We assume p measurements of the cross-directional paper weight. The variable y_k^j represents the weight at time k measured by the j -th sensor :

$$y_k^j = C\tilde{x}_k + Dd_k^j, \quad j = 1, \dots, p \quad (12)$$

and it is ideally a function of all the actuator positions \tilde{x}_k . The overall system model includes additive disturbances d_k^1, \dots, d_k^m , which act on the measurements and represent an inhomogeneous pulp weight distribution. Typically, the impulse response of individual actuators, also known as the cross-directional (CD) bump response is much narrower than the width of the paper sheet as illustrated in Figure 2. This implies that y_k^j is a function of spatially "close" actuators only. Denoting the set of such actuators with \mathcal{S}^j , the output equation can be written as

$$y_k^j = \sum_{l \in \mathcal{S}^j} C_l \tilde{x}_{l,k} + Dd_k^j \quad (13)$$

where C_l is the l -th element of the row vector C .

The control problem can be arranged in the form (8) by considering independent actuator dynamics and an objective function which minimizes the error between the desired and actual paper weight profile. In particular the term l^i in (7), associated to the i -th node, can be defined as the sum of the errors between the desired and actual weight measures y^j for all sensors j "spatially close" to the i -th actuator. In summary, the paper machine application example can be described by the following features

- *Subsystems*
Independently actuated elements along the slice lip profile in the paper headbox, where the inputs are the desired actuator movements and the outputs are the actual actuator positions.
- *Subsystem Constraints*
Maximum and minimum bounds on actuator positions.

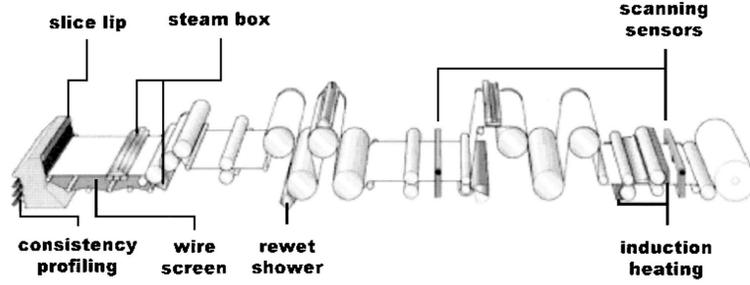


Fig. 1. Wide view of the paper machine showing typical positions of the various actuator arrays and scanning sensor(s) (Artwork courtesy of Honeywell).

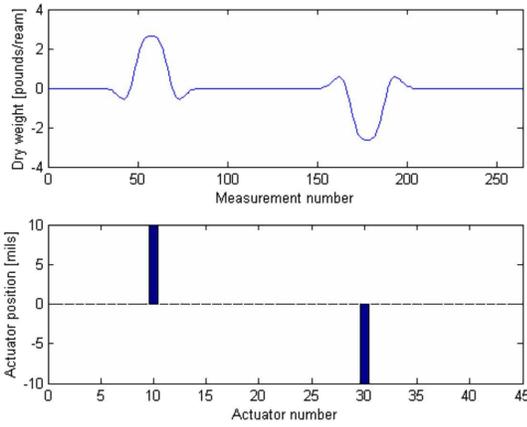


Fig. 2. Dry weight spatial response to slice lips.

- **Interaction Constraints**

Bounded deviation between neighboring actuator movements to prevent excessive bending and permanent deformation of the slice lip $u_k^{i-1} - 2u_k^i + u_k^{i+1} \leq B$ for all actuators i .

- **Objective Function**

Tracking of paper weight profiles in the presence of changing fluid flow properties of the pulp slurry through the headbox. The paper weight measured by downstream sensors is a function of neighboring actuator positions.

- **Graph**

Depending on the bending restrictions for the slice lip, a time-invariant line graph or n closest neighbor interconnection gives the underlying topology such as the one shown in Figure 3.



Fig. 3. Typical interconnection topology of actuators in the paper machine example.

IV. DECENTRALIZED CONTROL SCHEME

We tackle the complexity associated with the design of controllers for such a class of large scale systems by using

decentralized optimal control schemes. In this section we present a possible way to decentralize the RHC problem (9). Decentralization of RHC problems raises issues of stability and feasibility to be addressed, which are hot topics of current research in decentralized control design [1], [8], [9].

We decompose problem (9) into N_v finite time optimal control problems [1], each one associated to a different node as detailed next. Each node has information about its current states and its neighbors' current states. Based on such information, each node computes its optimal inputs and its neighbors' optimal inputs. The input to the neighbors will only be used to predict their trajectories and then discarded, while the first component of the optimal input to the node will be implemented where it was computed.

Considering the overall problem description given by systems (1), graph \mathcal{G} , and RHC policy (9)-(10), let the following finite time optimal control problem \mathcal{P}_i be associated to the i -th system at time t

$$\begin{aligned}
 & \min_{\tilde{U}_t^i} \sum_{k=0}^{N-1} l_k^i(x_{k,t}^i, u_{k,t}^i, \tilde{x}_{k,t}^i, \tilde{u}_{k,t}^i) + l_N^i(x_{N,t}^i, \tilde{x}_{N,t}^i) \\
 \text{subj. to} \quad & x_{k+1,t}^i = f^i(x_{k,t}^i, u_{k,t}^i), \\
 & x_{k,t}^i \in \mathcal{X}^i, \quad u_{k,t}^i \in \mathcal{U}^i, \\
 & k = 1, \dots, N-1 \\
 & x_{k+1,t}^j = f^j(x_{k,t}^j, u_{k,t}^j), \quad (j, i) \in \mathcal{A}, \\
 & x_{k,t}^j \in \mathcal{X}^j, \quad u_{k,t}^j \in \mathcal{U}^j, \quad (j, i) \in \mathcal{A}, \\
 & k = 1, \dots, N-1 \\
 & g^{i,j}(x_{k,t}^i, u_{k,t}^i, x_{k,t}^j, u_{k,t}^j) \leq 0, \quad (i, j) \in \mathcal{A}, \\
 & k = 1, \dots, N-1 \\
 & g^{q,r}(x_{k,t}^q, u_{k,t}^q, x_{k,t}^r, u_{k,t}^r) \leq 0, \\
 & (q, i) \in \mathcal{A}, \quad (r, i) \in \mathcal{A}, \\
 & k = 1, \dots, N-1 \\
 & x_{N,t}^i \in \mathcal{X}_f^i, \quad x_{N,t}^j \in \mathcal{X}_f^j, \quad (i, j) \in \mathcal{A} \\
 & x_{0,t}^i = x_t^i, \quad \tilde{x}_{0,t}^i = \tilde{x}_t^i
 \end{aligned}$$

where $\tilde{U}_t^i \triangleq [u_{0,t}^i, \tilde{u}_{0,t}^i, \dots, u_{N-1,t}^i, \tilde{u}_{N-1,t}^i]'$ $\in \mathbb{R}^s$, $s \triangleq (\tilde{m}^i + m^i)N$ denotes the optimization vector, $x_{k,t}^i$ denotes the state vector of the i -th node predicted at time $t+k$ obtained by starting from the state x_t^i and applying to system (1) the input sequence $u_{0,t}^i, \dots, u_{k-1,t}^i$. Denote by

$\tilde{U}_t^{i*} = [u_{0,t}^{*i}, \tilde{u}_{0,t}^{*i}, \dots, u_{N-1,t}^{*i}, \tilde{u}_{N-1,t}^{*i}]$ the optimizer of problem \mathcal{P}_i . Note that problem \mathcal{P}_i involves only the state and input variables of the i -th node and its neighbors at time t .

We will define the following decentralized RHC control scheme for a fixed graph connection \mathcal{A} . At time t

- 1) Each node i solves problem \mathcal{P}_i based on measurements of its state x_t^i and the states of all its neighbors \tilde{x}_t^i .
- 2) Each node i implements the first sample of \tilde{U}_t^{i*}

$$u_t^i = u_{0,t}^{*i}. \quad (15)$$

- 3) Each node repeats steps 1 to 3 at time $t + 1$, based on the new state information $x_{t+1}^i, \tilde{x}_{t+1}^i$.

In order to solve problem \mathcal{P}_i each node needs to know its current states, its neighbors' current states, its terminal region, its neighbors' terminal regions and models and constraints of its neighbors. Based on such information each node computes its optimal inputs and its neighbors' optimal inputs assuming a constant set of neighbors over the horizon. The input to the neighbors will only be used to predict their trajectories and then discarded, while the first component of the i -th optimal input of problem \mathcal{P}_i will be implemented on the i -th node. The solution of the i -th subproblem will yield a control policy for the i -th node of the form

$$u_t^i = k^i(x_t^i, \tilde{x}_t^i). \quad (16)$$

where the functions k^i are implicitly defined by the optimal control problems \mathcal{P}_i .

Even if we assume N to be infinite, the decentralized RHC approach described so far does not guarantee that solutions computed locally are globally feasible and stable (i.e. feasible for problem (9)). The reason is simple: At the i -th node the prediction of the neighboring state x^j is done independently from the prediction of problem \mathcal{P}_j . Therefore, the trajectory of x^j predicted by problem \mathcal{P}_i and the one predicted by problem \mathcal{P}_j , based on the same initial conditions, are different (since in general, \mathcal{P}_i and \mathcal{P}_j will be different). This will imply that constraint fulfillment will be ensured by the optimizer u_t^{*i} for problem \mathcal{P}_i but not for the overall problem (9).

A detailed discussion on feasibility and stability issues of decentralized RHC schemes goes beyond the scope of this paper. Some important observations can be found in [1], [8]–[10].

V. REAL-TIME CONTROLLER IMPLEMENTATION

Under Assumption 1, the decentralized RHC controllers designed in the previous section require small size quadratic programs (QP) to be solved on-line for each actuator. A centralized RHC approach would require the online solution of one large size and ill/conditioned QP. Furthermore, if each node has few neighbors, then the decentralized QPs have small dimension, in both the input space and neighbors state-space. Then, their explicit solution can be computed off-line by means of multiparametric quadratic programming [11] yielding continuous and piecewise affine state feedback functions k^i in (16), i.e.

$$k^i(x_t^i, \tilde{x}_t^i) = F^i[x_t^i, \tilde{x}_t^i] + g^i \quad \text{if} \quad H^i[x_t^i, \tilde{x}_t^i] \leq K^i, \quad (17)$$

where $i = 1, \dots, N^r$. In the RHC law (17), the number of polyhedral regions N^r depends on the tuning of the RHC problem.

The use of equivalent PWA form of the RHC law has several advantages. It is immediate to implement any hardware platform as a simple look-up table of gain-scheduled controllers without solving online optimization. It can also be easily verified (an on-line optimization solver is impossible to verify). Its worst case computational time can also be computed immediately.

VI. SIMULATION RESULTS

The decentralized scheme presented in Section IV has been implemented on the application example discussed in Section III. Simulations were performed with a model having 45 inputs, 250 outputs and a two time-step delay. The graph topology shown in Figure 3 was used and a cost function which minimizes the two norm error between the measured weight profile and a desired constant weight profile of 116.5pounds/ream. We have simulated the control systems with the experimental disturbance profile d depicted in Figure 8 and with a disturbance profile equal to 17.5 times the one depicted in Figure 8.

We have compared the results with the centralized control design presented in [6]. We have used a prediction horizon of six steps and no terminal constraint.

Figures 4 and 5 shows the steady-state profile of the inputs and outputs, respectively, for the centralized approach and decentralized RHC solution associated to the disturbance profile. Figures 6 and 7 represents the same quantities associated to the second disturbance profile.

The performance of the centralized and of the decentralized controllers is almost identical from a practical point of view. The horizons and optimization weights of the decentralized controller were tuned to achieve similar performance as the centralized RHC. Remaining differences in performance may be due as to tuning differences and not necessarily attributable to fundamental differences in decentralized versus centralized RHC. The computational advantages of the decentralized scheme are immediate as discussed in Section V. The online algorithms resorts to the implementation the a lookup table (17) with 44 entries.

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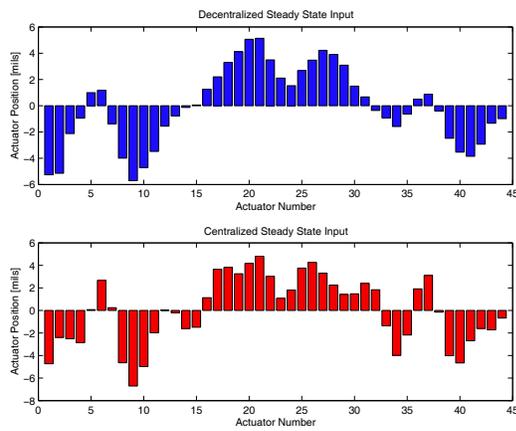


Fig. 4. Steady state input profiles. Disturbance profile depicted in Figure 8

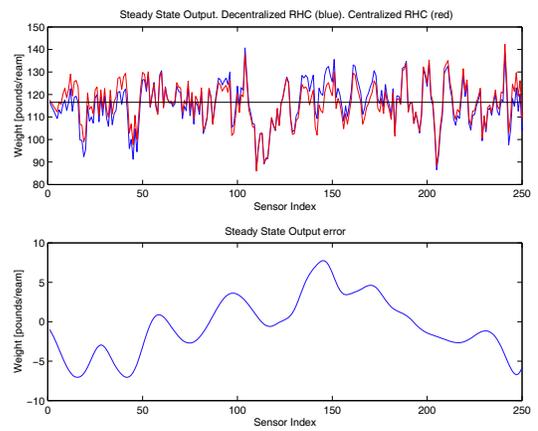


Fig. 7. Steady state output profiles. Disturbance profile = 17.5 * disturbance in Figure 8. The upper plot depicts the comparison between the centralized and the decentralized solutions. The lower one depicts the difference between the two.

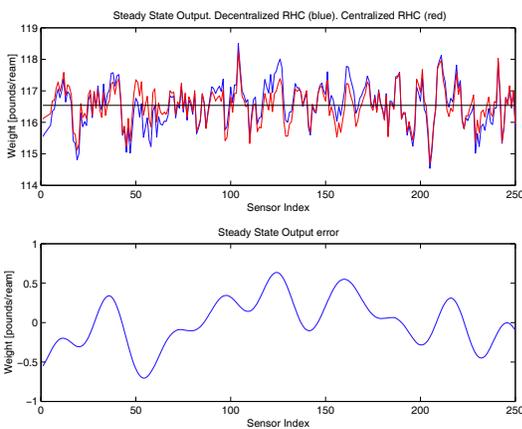


Fig. 5. Steady state output profiles for the disturbance depicted in Figure 8. The upper plot depicts the comparison between the centralized and the decentralized solutions. The lower one depicts the difference between the two.

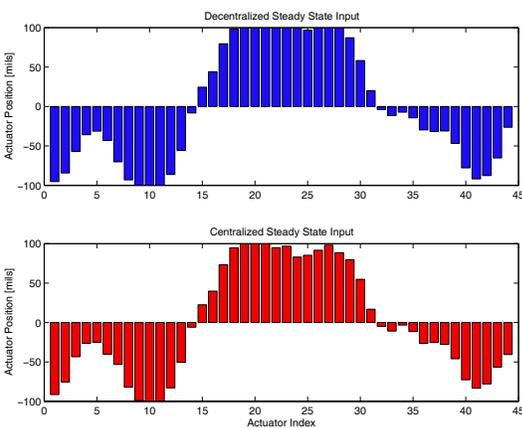


Fig. 6. Steady state input profiles. Disturbance profile = 17.5 * disturbance in Figure 8

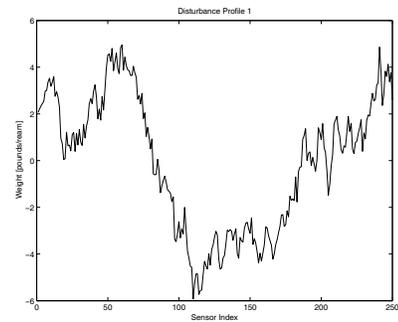


Fig. 8. Disturbance profile 1

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