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Robust Trajectory Control of Robot Manipulators Using Time Delay Estimation and Internal Model Concept

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Abstract— **This paper proposes an enhanced controller, using Time Delay Estimation(TDE) and Internal Model Control(IMC) concept for robot manipulators. The proposed controller has a compensator based on IMC to improve robustness of Time Delay Control (TDC) against friction; it is effective enough to handle bad effect of friction, moreover, simple and efficient as to match positive attribute of TDC. The controller with TDE, does not need whole model of plants, thus, it is easily applicable. The analysis and experimental results show the effectiveness of the proposed controller against the friction effects of the robot manipulators.**

I. Introduction

In this paper, An enhanced controller to improve the robustness of *Time Delay Control(TDC)* by using *Internal Model Control(IMC)* concept is proposed and analyzed. And it is applied to control a robot manipulator.

TDC is the control technique which compensates the unpredicted disturbances of the plant and/or the unknown dynamics of the plant with *Time Delay Estimation (TDE)* [1,4]. Although TDC has simpler structure than other advanced control algorithms developed until now, TDC has a distinct robustness for the disturbances and the parameter variations. Its effectiveness to robot manipulators has been showed through many successful applications[3,4,9].

It has been observed, however, that in the presence of so-called hard nonlinearity such as coulomb friction/static friction, TDC reveals some problems commonly founded in other methods like PID control or disturbance observer[8]. More specially, in the system having coulomb friction, its tracking error increases at zero velocity. Moreover, in the system having static friction and stribeck effect, slow motions often involve stick-slip phenomena expressed in the form of limit cycle in the position control and oscillation in trajectory tracking control. Therefore, the compensator to remedy these problems is necessary in order to achieve better tracking performance.

There are a few researches to solve these problems. Chang and Park proposed TDCSA[8]. It is TDC with a compensator of the switching action based on *Siding Mode Control(SMC)*. On the other hand, Kwon & Chung suggested the MPEC[11], which has multi-loop stages of the perturbation observer known as the similar algorithm to TDC. And, Nam added the perturbation observer as a compensator to TDC[15].

IMC is the control algorithm which uses the plant model directly[5,6]. It has a straight forward design method, and has good control performance as providing perfect control scheme theoretically. It has been widely used to control chemical processes, and used to control a robot manipulator by adding to computed torque control[9].

In IMC, however, it is a weak point that it requires plant model. Generally, many nonlinear plants, such as robot manipulators, are hard and need much time to be modeled. Moreover, IMC is fundamentally applicable to stable system. So, it has been seldom used in the mechanical field[9].

This paper proposes an enhanced controller for robot manipulators by adding two concepts, TDE and IMC, to make up for weak points of each algorithm. It doesn't need whole model of the plant and is better to control friction system than TDC. It is named *Time Delay Control with Internal Model (TDCIM)*.

This paper structured as following. In Section II, TDC and IMC are introduced to and analyzed. In Section III, enhanced controller named TDCIM is proposed and its some properties are analyzed. Section IV provides the analysis why TDCIM is robust to friction system, and Section V shows its robustness through the experimental results. Finally, we summarized the results and draw the conclusions in Section VI.

II. Preliminaries

This section introduces to TDC and IMC, and points out their properties and weak points.

A. Review of TDC

We summarize TDC law for robot manipulators[4] and analyze its problems concerned with TDE error.

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1) TDC law

Dynamics of robot manipulators is generally described as follows.

$$\mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{V}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) + \mathbf{G}(\boldsymbol{\theta}) + \mathbf{F}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) = \boldsymbol{\tau} \quad (1)$$

Where $\mathbf{M}$ denotes inertia matrix, $\mathbf{V}$ the coriolis and the centrifugal forces, $\mathbf{G}$ gravity, $\mathbf{F}$ frictions and unmodeled disturbances and $\boldsymbol{\tau}$ input torque.

Eq. (1) can be rewritten as follows by importing constant matrix $\bar{\mathbf{M}}$ which is selected based on $\mathbf{M}$.

$$\bar{\mathbf{M}}\ddot{\boldsymbol{\theta}} + \mathbf{H}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) = \boldsymbol{\tau} \quad (2)$$

Where, $\mathbf{H}$ denotes nonlinear dynamics of robot manipulators and is described as follows.

$$\mathbf{H}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\ddot{\boldsymbol{\theta}}) = (\mathbf{M}(\boldsymbol{\theta}) - \bar{\mathbf{M}})\ddot{\boldsymbol{\theta}} + \mathbf{V}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) + \mathbf{G}(\boldsymbol{\theta}) + \mathbf{F}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \quad (3)$$

Based on the computed torque control, $\boldsymbol{\tau}$ can be designed as follows.

$$\boldsymbol{\tau} = \bar{\mathbf{M}}\mathbf{u} + \hat{\mathbf{H}} \quad (4)$$

$$\mathbf{u} = \ddot{\boldsymbol{\theta}}_d + \mathbf{K}_D\left(\dot{\boldsymbol{\theta}}_d - \dot{\boldsymbol{\theta}}\right) + \mathbf{K}_P\left(\boldsymbol{\theta}_d - \boldsymbol{\theta}\right) \quad (5)$$

Where $\hat{\mathbf{H}}$ denotes estimation values of $\mathbf{H}$, $\mathbf{K}_D$ and $\mathbf{K}_P$ PD gain diagonal matrices.

TDC uses Time Delay Estimation(TDE) to get $\hat{\mathbf{H}}$. Under the assumption that the time delay L is sufficiently small, following approximation is valid due to (2).

$$\mathbf{H}_{(t)} \cong \hat{\mathbf{H}}_{(t)} = \mathbf{H}_{(t-L)} = \boldsymbol{\tau}_{(t-L)} - \bar{\mathbf{M}}\ddot{\boldsymbol{\theta}}_{(t-L)} \quad (6)$$

Finally, TDC can be summarized as follows.

$$\begin{aligned}\boldsymbol{\tau} = {} & \boldsymbol{\tau}_{(t-L)} - \bar{\mathbf{M}}\ddot{\boldsymbol{\theta}}_{(t-L)} \\ & + \bar{\mathbf{M}}\left[\ddot{\boldsymbol{\theta}}_d + \mathbf{K}_D\left(\dot{\boldsymbol{\theta}}_d - \dot{\boldsymbol{\theta}}\right) + \mathbf{K}_P\left(\boldsymbol{\theta}_d - \boldsymbol{\theta}\right)\right]\end{aligned} \quad (7)$$

If we select $\bar{\mathbf{M}}$ as diagonal constant matrix, TDC can be designed as separated joint controller using only $\bar{\mathbf{M}}$ and PD gains, just like Fig. 1. Thus, TDC is very efficient and has few burden of computation by using TDE, which does not need to compute whole dynamics of robot manipulators.

2) Problems of TDC concerned with TDE error

If we can use infinitesimal time delay L, it is possible to estimate $\mathbf{H}$ perfectly by TDE. However, it's impossible to set L as infinitesimal. In many cases, sampling time is used for the time delay L, and there exists limitation of hardware. Therefore, there exists TDE error due to finite L. From (2), (4) and (6), the following is induced.

$$\bar{\mathbf{M}}\left(\mathbf{u}_{(t)} - \ddot{\boldsymbol{\theta}}_{(t)}\right) = \mathbf{H}_{(t)} - \hat{\mathbf{H}}_{(t)} = \mathbf{H}_{(t)} - \mathbf{H}_{(t-L)} \quad (8)$$

The right term denotes TDE error. Here, we define TDE error as follows.

$$\boldsymbol{\varepsilon}_{(t)} \triangleq \mathbf{u}_{(t)} - \ddot{\boldsymbol{\theta}}_{(t)} = \bar{\mathbf{M}}^{-1}\left(\mathbf{H}_{(t)} - \mathbf{H}_{(t-L)}\right) \quad (9)$$

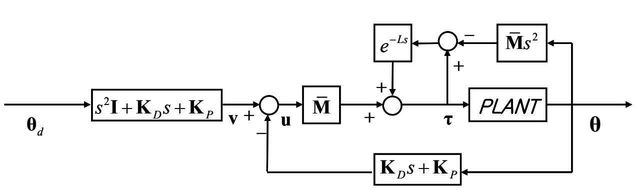

Fig. 1. Block diagram of TDC

The error dynamics of TDC is represented as follows; it shows the influence of TDE error to the tracking error.

$$\ddot{\mathbf{e}}_{(t)} + \mathbf{K}_D\dot{\mathbf{e}}_{(t)} + \mathbf{K}_P\mathbf{e}_{(t)} = \boldsymbol{\varepsilon}_{(t)} \quad (10)$$

Where, $\mathbf{e} \triangleq \boldsymbol{\theta}_d - \boldsymbol{\theta}$.

If time delay L is sufficiently small compared to the changing rate of system dynamics, TDE error can be ignored. In mechanical system, however, there exist fast dynamics such as coulomb friction/static friction, which can not be estimated well by TDE with finite L. Therefore, when TDC is applied to such systems, TDE error become large, and thus large tracking error is occurred.

B. Introduction to IMC

1) Structure of IMC and its properties

IMC, proposed by Garcia & Morari[5], is showed in Fig.2. It is composed of IMC controller Q and internal model G_m.

The effect of the parallel path with the model(G_m) is to subtract the effect of the manipulated variables from the plant output. If we assume that the model represents the plant perfectly, then the feedback signal is equal to the influence of disturbances and is not affected by the action of the manipulated variables. Thus, the system is effectively open-loop[6].

IMC controller Q designed based on G_m^{-1} plays the role of a feed-forward controller. But th IMC controller does not suffer from the disadvantages of feed-forward controllers; it can cancel the influence of disturbances because the feed-back signal is equal to this influence and modifies the controller set-point accordingly[6].

Overall transfer function of IMC is given as follows.

$$y(s) = \frac{GQ}{1+Q\left(G-G_m\right)}y_d + \frac{1-G_mQ}{1+Q\left(G-G_m\right)}Gd \quad (11)$$

From the transfer function, we can easily find several properties of IMC, which can be arranged as follows[5,7].

- **Property I: *Dual Stability Criterion.*** When the model is exact, stability of both controller and plant is sufficient for overall system stability.
- **Property II: *Perfect Controller.*** Assume that $Q=G_m^{-1}$ is realizable and the IMC system is closed-loop-stable, then, perfect reference-tracking control($y=y_d$) can be achieved, for all $t>0$, despite any disturbance d.
- **Property III: *Zero offset.*** A controller Q which satisfies $Q(0)=G_m^{-1}(0)$ yields zero offset.

2) Limitation of IMC

IMC is an intuitive and prominent algorithm. However,

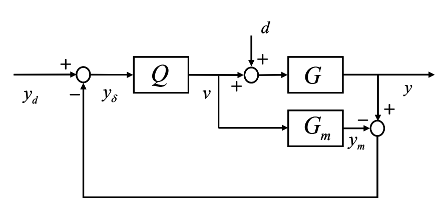

Fig. 2. Block diagram of IMC. Where, Q denotes IMC controller, G plant, G_m internal model.

there exist some limitations of it to be applied solely to mechanical field. One thing is that it requires plant model, which is hard and takes much time to be obtained in complex nonlinear plants such as robot manipulators. Moreover, its scheme, in its original design form, results in an open-loop control, which is applicable only to stable systems[7]. They are maybe main reasons why IMC has been seldom applied to the control of mechanical systems.

III. Proposition of TDCIM

In this section, a robust controller named TDCIM(TDC with Internal Model), which is composed with TDC and compensator based on IMC, is proposed and its properties are analyzed.

A. Derivation of TDCIM

1) Derivation of linear dynamics from TDC system

- ***Plant linearization using TDE.***

In TDC, TDE is used to cancel nonlinear dynamics of plant. And the robot dynamics compensated by TDE can be written as following linear equation.

$$\ddot{\boldsymbol{\theta}}_{(t)} = \mathbf{u}_{(t)} - \bar{\mathbf{M}}^{-1}\left(\mathbf{H}_{(t)} - \mathbf{H}_{(t-L)}\right) = \mathbf{u}_{(t)} - \boldsymbol{\varepsilon}_{(t)} \tag{12}$$

This means that the relationship between joint variable $\boldsymbol{\theta}$ and control input $\mathbf{u}$ is described as a linear equation $\boldsymbol{\theta}=s^{-2}\mathbf{I}\mathbf{u}$. Where, TDE error $\boldsymbol{\varepsilon}$ can be treated as disturbance.

- ***Consideration of PD feed-back***

Control input $\mathbf{u}$ in (5) can be separated reference input and feed-back input.

$$\mathbf{u} = \mathbf{v} - \left(\mathbf{K}_D\dot{\boldsymbol{\theta}} + \mathbf{K}_P\boldsymbol{\theta}\right) \tag{13}$$

Where, $\mathbf{v}$ denotes reference input; $\mathbf{v} \triangleq \ddot{\boldsymbol{\theta}}_d + \mathbf{K}_D\dot{\boldsymbol{\theta}}_d + \mathbf{K}_P\boldsymbol{\theta}_d$.

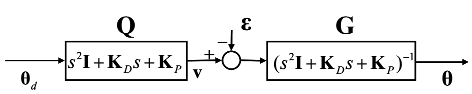

Fig. 3. Simplified block diagram of TDC, with linearized plant by TDE and PD feed-back.

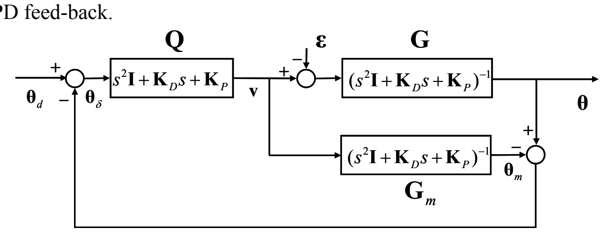

Fig. 4. Simplified block diagram of TDCIM.

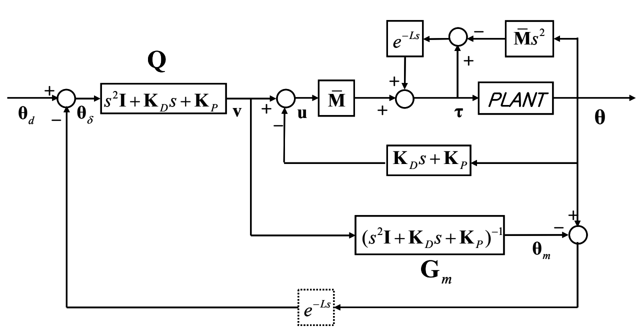

Fig. 5. Overall block diagram of TDCIM.

And then, (12) can be rewritten as follows

$$\ddot{\boldsymbol{\theta}} + \mathbf{K}_D\dot{\boldsymbol{\theta}} + \mathbf{K}_P\boldsymbol{\theta} = \mathbf{v} - \boldsymbol{\varepsilon} \tag{14}$$

Finally, we can find out that the dynamics of $\boldsymbol{\theta}$ can be described as the separated linear dynamics of $\mathbf{v}$, and TDC system can be simplified as Fig. 3.

After all, TDC system can be explained as a feed-forward controller with the plant linearized by TDE and PD feedback. Through the linearization process, however, TDE error is occurred, which is not compensated and affects on the manipulated variables just like as disturbance.

2) Addition Compensator based on IMC

Now, let us add the compensator based on internal model concept to TDC simplified in Fig. 3.

According to design scheme of IMC[5,6], internal model $\mathbf{G}_m$ can be easily selected from Fig. 3 as follows.

$$\mathbf{G}_m(s) = \left(s^2\mathbf{I} + \mathbf{K}_D s + \mathbf{K}_P\right)^{-1} \tag{15}$$

And, IMC controller can be designed as follows based on internal model.

$$\mathbf{Q}(s) = s^2\mathbf{I} + \mathbf{K}_D s + \mathbf{K}_P{}^{1} \tag{16}$$

Fig. 3 shows that the simplified TDC already has the controller such as (16). Therefore, TDCIM can be achieved by adding only the internal model and IMC feed-back as described in Fig. 4.

Fig. 5 represents overall block diagram of TDCIM. In TDCIM, after all, $\boldsymbol{\theta}_\delta$ combined reference and IMC feed-back is used instead of the reference $\boldsymbol{\theta}_d$. The control law of TDCIM is given as follows.

$$\boldsymbol{\theta}_\delta = \boldsymbol{\theta}_d - \boldsymbol{\theta} + \boldsymbol{\theta}_m \tag{17}$$

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{\tau}_{(t-L)} - \bar{\mathbf{M}}\ddot{\boldsymbol{\theta}}_{(t-L)} \\ &\quad + \bar{\mathbf{M}}\left[\ddot{\boldsymbol{\theta}}_\delta + \mathbf{K}_D\left(\dot{\boldsymbol{\theta}}_\delta - \dot{\boldsymbol{\theta}}\right) + \mathbf{K}_P\left(\boldsymbol{\theta}_\delta - \boldsymbol{\theta}\right)\right]\end{aligned} \tag{18}$$

TDCIM has a simple structure as showed in Fig. 5, and does not need any extra gain. The control law described above shows that TDCIM can be designed by choosing only $\bar{\mathbf{M}}$, $\mathbf{K}_D$ and $\mathbf{K}_P$ same as TDC case. Therefore, it is easily applicable and efficient as to match positive attribute of TDC. Moreover, it provides the chance that IMC can be applied easily to robot manipulators, without whole computation of the robot dynamics.

- ***Remark***

Feed-back time delay in IMC feed-back is included in Fig. 5(dashed box). It is not included intentionally, but inevitable in discrete implemented controllers. If $\mathbf{Q}=\mathbf{G}_m^{-1}$, IMC provides the perfect control scheme theoretically, but, it's possible only under the continuous time condition. In many cases, the controllers are designed on the computer systems, and they

[1] The controller needs the velocity and the acceleration of the plants. In many cases, however, they are not measurable, thus, obtained by numerical differentiation. Numerical differentiation causes the delay of responses and degrades tracking performance, but it is not a critical problem to apply. It is also well known that it enlarge the influence of sensing noise, but it can be overcome by using low pass filter[2,6].

must work under the discrete time condition. And, it's impossible to prevent the time delay in feed-back due to sampling. Thus, IMC cannot work as the perfect controller. So, it is necessary to include the time delay due to sampling for precise analysis.

B. Some properties of TDCIM

In this subsection, it is analyzed that what the IMC feed-back value means and how it affects to control systems.

1) The meanings of IMC feed-back values

At first, let us define the control input of TDC as follows, to prevent confusion with control input of TDCIM.

$$\mathbf{u}_{TDC(t)} \triangleq \ddot{\boldsymbol{\theta}}_{d(t)} + \mathbf{K}_D\dot{\mathbf{e}}_{(t)} + \mathbf{K}_P\mathbf{e}_{(t)} \tag{19}$$

Then, control input of TDCIM including the time delay in IMC feed-back can be described as follows.

$$\begin{aligned}\mathbf{u}_{(t)} = \mathbf{u}_{TDC(t)} - \Big[&\ddot{\boldsymbol{\theta}}_{(t-L)} - \ddot{\boldsymbol{\theta}}_{m(t-L)} + \mathbf{K}_D\left(\dot{\boldsymbol{\theta}}_{(t-L)} - \dot{\boldsymbol{\theta}}_{m(t-L)}\right)\\ &+\mathbf{K}_P\left(\boldsymbol{\theta}_{(t-L)} - \boldsymbol{\theta}_{m(t-L)}\right)\Big]\end{aligned} \tag{20}$$

The right terms except $\mathbf{u}_{TDC(t)}$ are the adding parts by IMC feed-back. And, their meanings can be obtained by subtraction from the plant dynamics $\mathbf{G}$ to the internal model dynamics $\mathbf{G}_m$ before one sampling time, i.e. at *t-L*.

$$\begin{aligned}\ddot{\boldsymbol{\theta}}_{(t-L)} - \ddot{\boldsymbol{\theta}}_{m(t-L)} + \mathbf{K}_D\left(\dot{\boldsymbol{\theta}}_{(t-L)} - \dot{\boldsymbol{\theta}}_{m(t-L)}\right)&\\ +\mathbf{K}_P\left(\boldsymbol{\theta}_{(t-L)} - \boldsymbol{\theta}_{m(t-L)}\right) &= -\boldsymbol{\varepsilon}_{(t-L)}\end{aligned} \tag{21}$$

This shows that the IMC feed-back value means the TDE error before one sampling time. Therefore, the control input and the input torque of TDCIM can be rewritten as follows.

$$\mathbf{u}_{(t)} = \mathbf{u}_{TDC(t)} + \boldsymbol{\varepsilon}_{(t-L)} \tag{22}$$

$$\boldsymbol{\tau}_{(t)} = \bar{\mathbf{M}}\mathbf{u}_{(t)} + \hat{\mathbf{H}}_{(t)} = \bar{\mathbf{M}}\mathbf{u}_{TDC(t)} + \mathbf{H}_{(t-L)} + \bar{\mathbf{M}}\boldsymbol{\varepsilon}_{(t-L)} \tag{23}$$

TDCIM can be treated as TDC with the compensator using the TDE error before one sampling time. In addition, error dynamics of TDCIM is given as follows.

$$\ddot{\mathbf{e}}_{(t)} + \mathbf{K}_D\dot{\mathbf{e}}_{(t)} + \mathbf{K}_P\mathbf{e}_{(t)} = \boldsymbol{\varepsilon}_{(t)} - \boldsymbol{\varepsilon}_{(t-L)} \tag{24}$$

In other words, TDCIM can be regard as the controller with an estimator of $\mathbf{H}$ working as follows.

$$\hat{\mathbf{H}}_{TDCIM(t)} = \mathbf{H}_{(t-L)} + \bar{\mathbf{M}}\varepsilon_{(t-L)} = 2\mathbf{H}_{(t-L)} - \mathbf{H}_{(t-2L)} \tag{25}$$

This means that the compensator decreases the influence of TDE error, and improves the estimation performance of $\mathbf{H}$ by using TDE error before one sampling time.

2) Property of TDCIM as the integrator

It is known that TDC has 1st order integrator[8]. Well, how is TDCIM case?

After taking Laplace transform, the input torque of TDCIM include time delay by sampling in the IMC feed-back can be rewritten as follows.

$$\boldsymbol{\tau}(s) = \frac{\bar{\mathbf{M}}}{\left(1-e^{-Ls}\right)^2}\left(\mathbf{u}_{TDC}(s) - 2\boldsymbol{\theta}s^2e^{-Ls} + \boldsymbol{\theta}s^2e^{-2Ls}\right) \tag{26}$$

If L is sufficiently small that holds the approximation $e^{-Ls}\approx 1-Ls$, above equation can be approximated to follows.

$$\boldsymbol{\tau}(s) = \frac{\bar{\mathbf{M}}}{L^2s^2}\left(\mathbf{u}_{TDC}(s) - 2\boldsymbol{\theta}s^2e^{-Ls} + \boldsymbol{\theta}s^2e^{-2Ls}\right) \tag{27}$$

This means that TDCIM has 2nd order integrator with high gain include L^{-2}. Thus, we can conjecture that the input torque of TDCIM changes very fast to reduce the tracking error when it occurs due to disturbances or any other reason.

C. Stability analysis of TDCIM

The sufficient stability conditions have been derived based on the analysis in L_∞^N [14]. The notations used in stablity analysis are arranged in Table I.

- ***Theorem.***

If the assumption and the stability condition presented below are satisfied, then the robot manipulator controlled by TDCIM is L_∞ stable.

Assumption : $\mathbf{w}\,,\boldsymbol{\theta}_d\,,\dot{\boldsymbol{\theta}}_d\,,\ddot{\boldsymbol{\theta}}_d \in L_\infty^N$

Stability condition : $\left\|\mathbf{A'}_{(t)}\right\|_{i2\infty} < 1$ & $\left\|\mathbf{B'}_{(t)}\right\|_{i2\infty} < 1$

Where, $\mathbf{w}$ denotes disturbance, and the matrices $\mathbf{A}$, $\mathbf{B}$ are defined as follows.

$\sqrt{\bar{\mathbf{M}}}$ is the diagonal positive definite $N \times N$ matrix, that satisfies $\sqrt{\bar{\mathbf{M}}}\sqrt{\bar{\mathbf{M}}} = \bar{\mathbf{M}}$.

And $\boldsymbol{\Omega}_{(t)} = \mathbf{I} - \sqrt{\bar{\mathbf{M}}}\mathbf{M}_{(t)}^{-1}\sqrt{\bar{\mathbf{M}}}$ is diagonalizable in the form described below.

$$\boldsymbol{\Omega}_{(t)} = \mathbf{P}_{(t)}\boldsymbol{\Lambda}_{(t)}\mathbf{P}_{(t)}^{-1} \tag{28}$$

Where $\boldsymbol{\Lambda}_{(t)} = diag[\lambda_{1(t)},\cdots,\lambda_{N(t)}]$ and $\mathbf{P}_{(t)}$ is the orthogonal matrix that diagonalizes $\boldsymbol{\Omega}_{(t)}$.

Then $\mathbf{A'}_{(t)}$ and $\mathbf{B'}_{(t)}$ are defined as follows.

TABLE I
NOTATIONS

When $\bullet_{(t)}$ is a $N\times 1$ vector	When $\bullet_{(t)}$ is a matrix	When $\bullet_{(t)}$ is a vector or a matrix
$\left\|\bullet_{(t)}\right\| = \sqrt{\bullet_{(t)}^T\bullet_{(t)}}$. $\left\|\bullet_{(t+c)}\right\|_\infty = \sup_{t\ge 0}\left\|\bullet_{(t+c)}\right\|$ (This means L_∞^N) $\left\|\bullet_{(t+c)}\right\|_{T\infty} = \sup_{0\le t\le T}\left\|\bullet_{(t+c)}\right\|$.	$\left\|\bullet_{(t)}\right\|_{i2}$ is the induced 2 norm of $\bullet_{(t)}$. $\left\|\bullet_{(t)}\right\|_{i2\infty} = \left\|\left\|\bullet_{(t)}\right\|_{i2}\right\|_\infty$.	$\tilde{\bullet}_{(t)} = \bullet_{(t)} - \bullet_{(t-L)}$. $\tilde{\tilde{\bullet}}_{(t)} = \tilde{\bullet}_{(t)} - \tilde{\bullet}_{(t-L)}$. $\breve{\bullet}_{(t)} = \bullet_{(t)} - \bullet_{(t-2L)}$. (where, L denotes the sampling time)

$$\mathbf{A'}_{(t)} = \mathbf{P}^{-1}_{(t)}\mathbf{A}_{(t)}\mathbf{P}_{(t)},\ \mathbf{B'}_{(t)} = \mathbf{P}^{-1}_{(t)}\mathbf{B}_{(t)}\mathbf{P}_{(t)} \tag{29}$$

Where, $\mathbf{A}_{(t)} = diag[a_{1(t)},..,a_{N(t)}]$, $a_{i(t)} = \lambda_{i(t)} + \sqrt{\lambda^2_{i(t)} - \lambda_{i(t)}}$ and $\mathbf{B}_{(t)} = diag[b_{1(t)},..,b_{N(t)}]$, $b_{i(t)} = \lambda_{i(t)} - \sqrt{\lambda^2_{i(t)} - \lambda_{i(t)}}$.

Note that $\mathbf{A'}_{(t)}$ and $\mathbf{B'}_{(t)}$ satisfy the followings.

$$\mathbf{A'}_{(t)} + \mathbf{B'}_{(t)} = 2\mathbf{\Omega}_{(t)},\ \mathbf{A'}_{(t)}\mathbf{B'}_{(t)} = \mathbf{B'}_{(t)}\mathbf{A'}_{(t)} = \mathbf{\Omega}_{(t)} \tag{30}$$

- ***Proof.***

Including time delay, (17) can be expressed as follows.

$$\boldsymbol{\theta}_{\delta(t)} = \boldsymbol{\theta}_{d(t)} - \boldsymbol{\theta}_{(t-L)} + \boldsymbol{\theta}_{\delta(t-L)} \tag{31}$$

From (18) and (31), input torque of TDCIM can be derived as follows.

$$\begin{aligned}\boldsymbol{\tau}_{(t)} &= 2[\boldsymbol{\tau}_{(t-L)} - \bar{\mathbf{M}}\ddot{\boldsymbol{\theta}}_{(t-L)}] - [\boldsymbol{\tau}_{(t-2L)} - \bar{\mathbf{M}}\ddot{\boldsymbol{\theta}}_{(t-2L)}] \\ &\quad + \bar{\mathbf{M}}[\ddot{\boldsymbol{\theta}}_{d(t)} + \mathbf{K}_D\dot{\mathbf{e}}_{(t)} + \mathbf{K}_P\mathbf{e}_{(t)}]\end{aligned} \tag{32}$$

And let $\mathbf{F}$ of equation (1) as (33) where $\mathbf{F}_v$ denotes viscous friction, $\mathbf{w}$ is disturbance including coulomb and static frictions.

$$\mathbf{F} = \mathbf{F}_v + \mathbf{w} \tag{33}$$

Then the closed loop error dynamics of TDCIM can be derived as (34) from (1), (32), (33).

$$\begin{aligned}\tilde{\boldsymbol{\varepsilon}}_{(t)} - 2[\mathbf{I} - \mathbf{M}^{-1}_{(t)}\bar{\mathbf{M}}]\tilde{\boldsymbol{\varepsilon}}_{(t-L)} + [\mathbf{I} - \mathbf{M}^{-1}_{(t)}\bar{\mathbf{M}}]\tilde{\boldsymbol{\varepsilon}}_{(t-2L)} \\ = \boldsymbol{\psi}_{(t)} + \boldsymbol{\varphi}_{(t)}\end{aligned} \tag{34}$$

Where,

$$\begin{aligned}\boldsymbol{\psi}_{(t)} &= [\mathbf{I} - \mathbf{M}^{-1}_{(t)}\bar{\mathbf{M}}][\mathbf{K_D}\tilde{\tilde{\dot{\mathbf{e}}}}_{(t)} + \mathbf{K_P}\tilde{\tilde{\mathbf{e}}}_{(t)}] \\ &\quad - \mathbf{M}^{-1}_{(t)}[-2\tilde{\mathbf{M}}_{(t)}\ddot{\boldsymbol{\theta}}_{(t-L)} + \breve{\mathbf{M}}_{(t)}\ddot{\boldsymbol{\theta}}_{(t-2L)} - \tilde{\tilde{\mathbf{V}}}_{(t)} - \tilde{\tilde{\mathbf{G}}}_{(t)} - \tilde{\tilde{\mathbf{F}}}_{v(t)}]\end{aligned}$$

$$\boldsymbol{\varphi}_{(t)} = \mathbf{M}^{-1}_{(t)}\tilde{\tilde{\mathbf{w}}}_{(t)} + [\mathbf{I} - \mathbf{M}^{-1}_{(t)}\bar{\mathbf{M}}]\tilde{\tilde{\ddot{\boldsymbol{\theta}}}}_{d(t)}$$

Let $\mathbf{e}_{\gamma(t)}$ and $\boldsymbol{\varepsilon}_{\gamma(t)}$ as (35).

$$\mathbf{e}_{\gamma(t)} = \sqrt{\bar{\mathbf{M}}}\mathbf{e}_{(t)},\ \boldsymbol{\varepsilon}_{\gamma(t)} = \sqrt{\bar{\mathbf{M}}}\boldsymbol{\varepsilon}_{(t)} \tag{35}$$

And (36) can be derived by transforming (34) which is of $\mathbf{e}_{(t)}$ and $\boldsymbol{\varepsilon}_{(t)}$ to equation which is of $\mathbf{e}_{\gamma(t)}$ and $\boldsymbol{\varepsilon}_{\gamma(t)}$.

$$\tilde{\boldsymbol{\varepsilon}}_{\gamma(t)} - 2\mathbf{\Omega}_{(t)}\tilde{\boldsymbol{\varepsilon}}_{\gamma(t-L)} + \mathbf{\Omega}_{(t)}\tilde{\boldsymbol{\varepsilon}}_{\gamma(t-2L)} = \sqrt{\bar{\mathbf{M}}}\boldsymbol{\psi}_{(t)} + \sqrt{\bar{\mathbf{M}}}\boldsymbol{\varphi}_{(t)} \tag{36}$$

(36) can be expressed as (37) by (30)

$$\begin{aligned}\tilde{\boldsymbol{\varepsilon}}_{\gamma(t)} - (\mathbf{A'}_{(t)} + \mathbf{B'}_{(t)})\tilde{\boldsymbol{\varepsilon}}_{\gamma(t-L)} + \mathbf{B'}_{(t)}\mathbf{A'}_{(t)}\tilde{\boldsymbol{\varepsilon}}_{\gamma(t-2L)} \\ = \sqrt{\bar{\mathbf{M}}}\boldsymbol{\psi}_{(t)} + \sqrt{\bar{\mathbf{M}}}\boldsymbol{\varphi}_{(t)}\end{aligned} \tag{37}$$

If $\|\mathbf{B'}_{(t)}\|_{i2\infty} < 1$ is satisfied, (38) can be derived from (37) after some norm operation.

$$\begin{aligned}(1 - \|\mathbf{B'}_{(t)}\|_{i2\infty})(1 - \|\mathbf{A'}_{(t)}\|_{i2\infty})\|\tilde{\boldsymbol{\varepsilon}}_{\gamma(t)}\|_{T\infty} \\ \le \left\|\sqrt{\bar{\mathbf{M}}}\boldsymbol{\psi}_{(t)} - \mathbf{B'}_{(t)}\tilde{\mathbf{A}}'_{(t)}\tilde{\boldsymbol{\varepsilon}}_{\gamma(t-2L)}\right\|_{T\infty} + \left\|\sqrt{\bar{\mathbf{M}}}\boldsymbol{\varphi}_{(t)}\right\|_{T\infty}\end{aligned} \tag{38}$$

And if $\|\mathbf{A'}_{(t)}\|_{i2\infty} < 1$ is satisfied, (38) can be derived from (39).

$$\|\tilde{\boldsymbol{\varepsilon}}_{\gamma(t)}\|_{T\infty} \le \frac{\left\|\sqrt{\bar{\mathbf{M}}}\boldsymbol{\psi}_{(t)} - \mathbf{B'}_{(t)}\tilde{\mathbf{A}}'_{(t)}\tilde{\boldsymbol{\varepsilon}}_{\gamma(t-2L)}\right\|_{T\infty} + \left\|\sqrt{\bar{\mathbf{M}}}\boldsymbol{\varphi}_{(t)}\right\|_{T\infty}}{(1 - \|\mathbf{B'}_{(t)}\|_{i2\infty})(1 - \|\mathbf{A'}_{(t)}\|_{i2\infty})} \tag{39}$$

Letting $T \to \infty$, (40) is obtained.

$$\|\tilde{\boldsymbol{\varepsilon}}_{\gamma(t)}\|_{\infty} \le \frac{\left\|\sqrt{\bar{\mathbf{M}}}\boldsymbol{\psi}_{(t)} - \mathbf{B'}_{(t)}\tilde{\mathbf{A}}'_{(t)}\tilde{\boldsymbol{\varepsilon}}_{\gamma(t-2L)}\right\|_{\infty} + \left\|\sqrt{\bar{\mathbf{M}}}\boldsymbol{\varphi}_{(t)}\right\|_{\infty}}{(1 - \|\mathbf{B'}_{(t)}\|_{i2\infty})(1 - \|\mathbf{A'}_{(t)}\|_{i2\infty})} \tag{40}$$

For sufficiently small L , $\left\|\sqrt{\bar{\mathbf{M}}}\boldsymbol{\psi}_{(t)} - \mathbf{B'}_{(t)}\tilde{\mathbf{A}}'_{(t)}\tilde{\boldsymbol{\varepsilon}}_{\gamma(t-2L)}\right\|_{\infty}$ is very small[2]. And if assumption is satisfied, $\left\|\sqrt{\bar{\mathbf{M}}}\boldsymbol{\varphi}_{(t)}\right\|_{\infty} < \infty$.

And $\|\tilde{\boldsymbol{\varepsilon}}_{\gamma(t)}\|_{\infty} \le \infty$, $e_{(t)}$, $\dot{e}_{(t)}$ are bounded.

As $\bar{\mathbf{M}}$ is similar to $\mathbf{M}_{(t)}$, $\|\mathbf{A'}_{(t)}\|_{i2\infty}$ and $\|\mathbf{B'}_{(t)}\|_{i2\infty}$ become smaller, so TDCIM has more possibility to satisfy condition 1 and 2.

It has been detected that stable gain of TDCIM is less then TDC from analysis and simulation.

IV. EFFECT OF TDCIM AGAINST FRICTION OF SYSTEM

In this section, the effect of the proposed compensator is showed. And it is analyzed how it affects against fast dynamics of system, especially including coulomb friction and stribeck effect inciting stick-slip phenomena.

For quantitative analysis, we used the simulation results of TDC and TDCIM applied to 1 DOF manipulator shown Fig. 6. We applied TDC with $\bar{M} = 1.0$ and TDCIM with $\bar{M} = 0.5$. The PD gains are set to K_D=20, K_P=100 for both controllers that make natural frequencies ω_n=10rad/sec and damping ratios ξ=1 of the error dyanmics eq. (10) and (24).

A. Effect of the compensator against slow changes of ***H***

TDCIM has a compensator using TDE error before one sampling time. To show the role of the compensator more visible, let us observe its effect when **H** changes slowly.

Fig. 7 shows the estimation values of **H** in TDC and TDCIM case conceptually. In TDC using TDE $\hat{\mathbf{H}}_{(t)} = \mathbf{H}_{(t-L)}$, TDE error occurs as much as the changes of **H** during one sampling time. In TDCIM, TDE error is compensated by that before one sampling time, thus, more exact estimation is possible. It's a clue that control performance of TDCIM is better than that of TDC.

B. When ***H*** *changes rapidly - Coulomb friction case*

When the velocity of plant passes by 0, coulomb friction changes its direction and the plant has rapid nonlinearity. In TDC, TDE cannot estimate it exactly, thus, tacking error become large. This subsection shows the effect of the compensator in TDCIM against rapid nonlinearity such as coulomb friction.

[2] This assumption, which was also used to prove the stability of TDC in [2,4], is incomplete because $\Psi_{(t)}$ is a function of manipulated variables and affects the stability when $L \neq 0$. This problem has been also presented in [13]. And we will deal this problem in further works.

Generally, coulomb friction is modeled as follows.

$$F = \tau_{slip}\,\mathrm{sgn}\left(\dot{\theta}\right) \tag{41}$$

Where, τ_{slip} denotes coulomb friction coefficient.

The simulation results of the effect of coulomb friction are showed in Fig. 8. (a) and (c) are conceptual figures of the estimation errors in TDC and TDCIM, (b) and (d) results of simulation of controlled system with the 1 DOF manipulator described in Fig. 6.

In TDC, as described in Fig. 8 (a), TDE error jumps when coulomb friction changes its direction at zero velocity. And it incites tracking error to be large. On the other hand, in TDCIM, a reverse action as described in (c) is appeared after occurring TDE error due to the compensator. And it eliminates the influence of TDE error and reduces tracking error considerably as shown in (d).

C. Stick-Slip case

When the controller with the integrator is applied to the system with stribeck effect, stick-slip phenomena occurred.

TDC and TDCIM have the integrators, and can't avoid them. In this subsection, stick-slip phenomena of TDC and TDCIM are observed and analyzed.

- ***Friction model***

There exist several friction models to represent stribeck effect[10]. Among them, we simulate and analyze the stick-slip phenomena based on Karnopp's model.

Karnopp's model, one of the friction model used widely, is represented in Fig. 9. The small zero velocity zone($\pm\alpha$) in Kornopp's model surrounding $\dot{\theta} = 0$ is necessary for numerical computation and digital implementation since an exact value of zero is hard to be computed[10].

- ***Simulation results***

Stick-slip phenomena of TDC and TDCIM are simulated with the plant in Fig. 6 under the regulation control with initial error of 0.0057deg. Its results are showed in Fig. 10.

Through the results, it can be easily made out that stick-slip of TDCIM has smaller amplitude and chatters more frequently than that of TDC.

From now, the reasons are analyzed with the division of stick state and state change from stick to slip.

- ***Analysis for Stick state***

In stick state with regulating error, input torque increases if the controller has any integrator. TDC has 1st order integrator, therefore, the control input increases linearly. But, in TDCIM case with 2nd order integrator and high gain include L^{-2}, the control input increases in proportion to time square. It's easily understood that the process of TDCIM is much faster and escapes stick state more rapidly than that of TDC. It's the

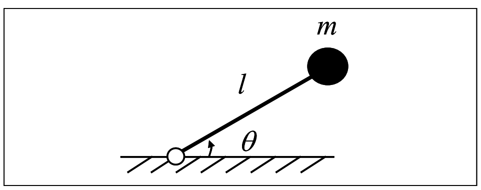

Fig. 6. 1 DOF manipulator. $l = 1.0(m)$, $m = 1.0(kg)$

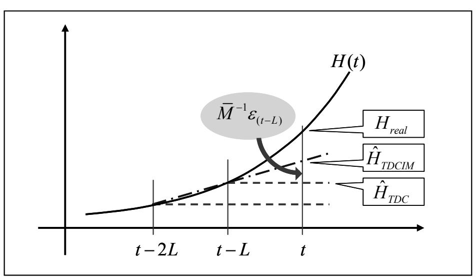

Fig. 7. Estimation of slow **H** in TDC, TDCIM

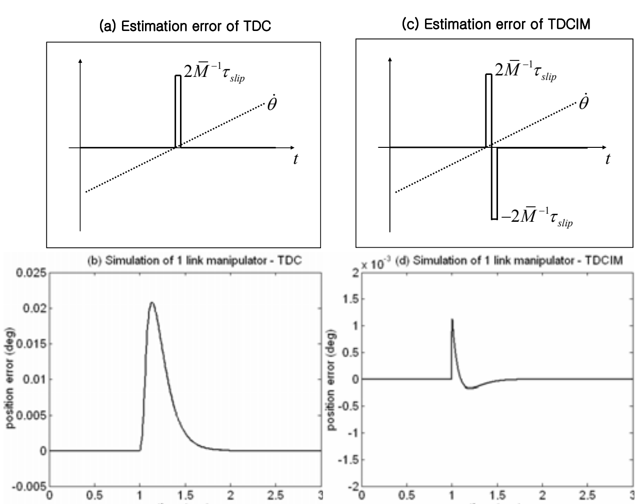

Fig. 8. Effect of Coulomb friction in TDC and TDCIM. Where, τ_{slip}=5Nm, L=0.001sec.

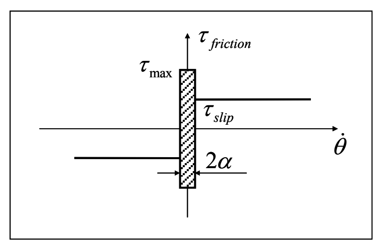

Fig. 9. Karnopp's friction model. Where, τ_{max} denotes max. static friction.

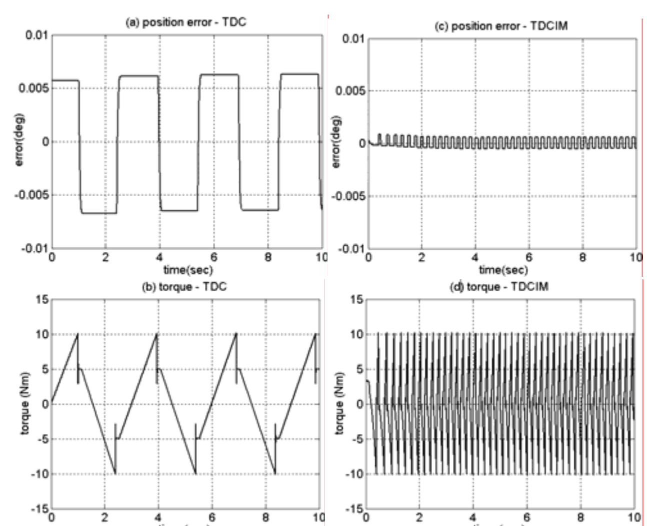

Fig. 10. Simulation results of Stick-Slip in TDC and TDCIM. Where, τ_{max}=10Nm, τ_{slip}=5Nm, α=10^{-5}rad/sec.

reason why stick-slip of TDCIM chatters more frequently than that of TDC.

▪ ***Analysis for the state change from Stick to Slip***

In changing state from stick to slip, the friction value of the system jumps from static friction to moving friction. So, TDE error becomes large just like the case of coulomb friction and TDC has large tracking error. In TDCIM case, on the other hand, the tracking error reduces considerably by compensating TDE error. Therefore, the amplitude of Stick-Slip in TDCIM is smaller than that of TDC.

V. EXPERIMENTS

In this section, we confirm the robustness of TDCIM though the experiments with 2 DOF SCARA type robot, and compare with that of TDC.

The SCARA type robot used in the experiments is showed in Fig. 11. The first link length is 0.35m and the second is 0.2m.

A. Experimental set up

$\overline{M} = diag(\alpha_1, \alpha_2)$ of TDC and TDCIM described in TABLE II are best tuned to minimize tracking error. And, PD gains are set to k_{Di}=20, k_{Pi}=100 for both controllers to that make natural frequencies ω_n=10rad/sec and damping ratios ξ=1 of the error dyanmics eq. (10) and (24).

Fig. 11. SCARA type robot system

TABLE II
CONTROL GAINS($\overline{M} = diag(\alpha_1, \alpha_2)$) FOR SCARA ROBOT

	TDC	TDCIM
α_1	0.1	0.03
α_2	0.025	0.0058

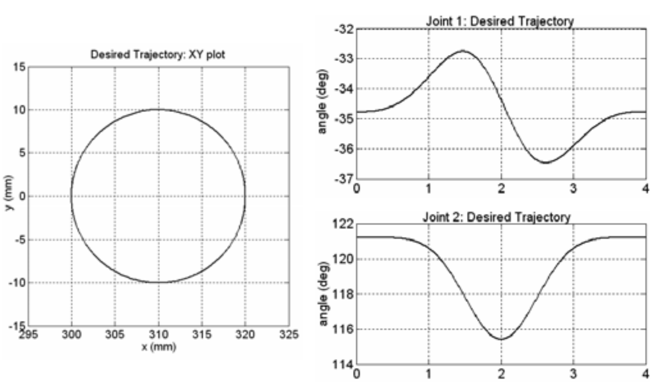

Fig. 12. Desired trajectory

Fig. 12 shows the reference trajectory, which let the end effecter make a circle with 10mm radius during 4 seconds.

Both the sampling time and the time delay for TDE are set to L=0.001sec.

B. Experimental results

Experimental results are showed in Fig. 13 and Fig. 14. Fig. 13 represents tracking error and torque of each joint. And, Fig. 14 shows XY position responses of the end effecter. Maximum tracking errors are arranged in TABLE III.

From the results in Fig.13 (a), (c) compared to references in Fig. 12, we can observe that the plant controlled by TDC has large tracking error due to coulomb friction when the plant passes by zero velocity. In TDCIM case, tracking error is reduced decently, and that confirms the compensator works well. Fig. 12 (b) and (d) show that TDC and TDCIM have simlier torque profiles, because input torque is decided to compensate the plant dynamics and to make plant track the

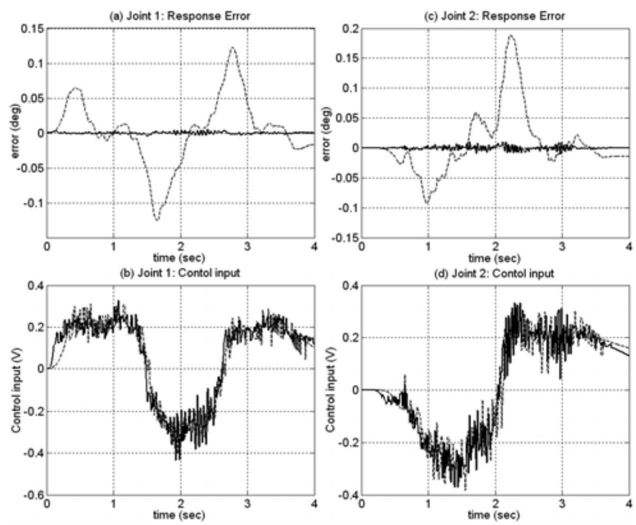

Fig. 13. Experimental results(solid: TDCIM, dashed: TDC).

TABLE III
MAXIMUM TRACKING ERRORS IN TDC AND TDCIM

Symbol	① TDC	② TDCIM	②/①*100
Joint 1	0.1257 deg	0.0049 deg	3.9%
Joint 2	0.1883 deg	0.0107 deg	5.7%

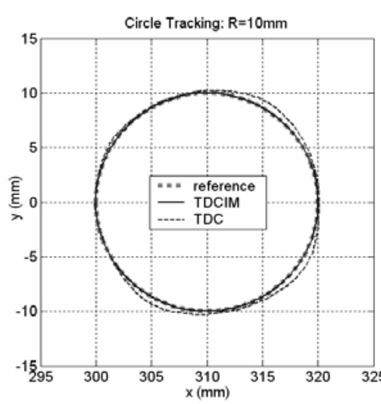

Fig. 14. Experimental results – XY plot

refereces. But, you can find that input torque of TDCIM changes faster than that of TDC to reduce tracking error, when TDC has large tracking error due to Coulomb friction.

Through the experimental results above, it is certificated that TDCIM is applicable to multi DOF system and improves the robustness of TDC against the friction.

VI. CONCLUSION

This paper proposes an enhanced controller using TDE and internal model concept. It is the controller that improves the tracking performance of TDC by compensating TDE error using IMC scheme. It has outstanding robustness against the friction effects in the plants, and, moreover, has simple structure which is easily applicable without whole model computation of the plants. Through the analysis and experiments, its robustness is showed when TDE error becomes large by coulomb friction or stribeck effect.

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