Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Energy-based nonlinear control of hydraulically actuated mechanical systems

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Abstract— **This contribution is devoted to a nonlinear energy based controller design for plants consisting of a mechanical 1-DOF load system coupled to a general hydraulic actuator. A systematic controller design which maintains the Port Hamiltonian structure of the plant is demonstrated for the case of a double-ended piston with only one servovalve. The developed controller shows good robustness properties and is able to inject additional damping into the load system without velocity measurement. Finally, the performance of the control law is demonstrated by application to a special industrial plant.**

I. Introduction

In typical industrial applications hydraulic actuators are connected to a mechanical load system of usually one mechanical degree of freedom (1-DOF). In this contribution an energy-based control concept is presented which is applicable to a broad class of mechanical 1-DOF plants with a general hydraulic actuator, i.e., single and double-ended – with one or two servovalves – and even rotatory piston actuators. Here, only the case of a double-ended piston actuator with one single 3-land-4-way servovalve, it is the most complex one, is treated. The proposed controller is able to inject damping into poorly or even undamped mechanical systems without velocity or acceleration measurements.

As soon as a fast and reliable set-point control is required for the whole operating range of a hydro-mechanical assembly nonlinear methods become indispensable. Towards a nonlinear controller design, one can find different approaches ranging from exact linearization techniques [4] or some flatness based concepts [2] to backstepping designs [1] and passivity based concepts [8]. The first family of nonlinear control concepts usually requires an accurate velocity signal. [8] and the approach of [5] based on a special variant of the input-to-outptut linearization, e.g., presents controllers for a linear load system which do not rely on the measurement of velocity. The energetic interpretation and the embedding of hydraulic actuator systems into the Port Hamiltonian (PH) context, see e.g. [11], has been established in [3] for the single-ended and in [6] for the double-ended actuator. A backstepping based approach with disturbance observer under the assumption of velocity measurement is given in [7].

This contribution is organized as follows. It will start with a brief summary of the PH description of hydraulically actuated mechanical systems. Afterwards, the nonlinear energy-based control concept is developed modularly. The control scheme for a general mechanical load system is applied to an industrial plant, namely a so-called wrapper assembly. Finally, the performance of the controller design is demonstrated by simulation results.

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II. Hydraulically actuated mechanical systems

Let us consider a simple 1-DOF mechanical system with a Hamiltonian H_l of the following form.

$$H_l = \frac{1}{2} m(q)^{-1} p^2 + V(q) \tag{1}$$

Let q, p denote the generalized coordinate respectively the generalized momentum. The generalized mass is denoted by $m(q)$ and $V(q)$ indicates the potential energy. The damping force is assumed to be linear in the generalized velocity, i.e.,

$$F_{ld} = d(q)\, m(q)^{-1}\, p \ , \quad d(q) > 0 \ .$$

The corresponding Hamiltonian equations with state $x_l = (q, p)$ and the hydraulic force F_h as fictitious input read as

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t} q &= m(q)^{-1} p \\ \tfrac{\mathrm{d}}{\mathrm{d}t} p &= \frac{1}{2} \frac{\partial_q m(q)}{m(q)^2} p^2 - \partial_q V - d(q)\, m(q)^{-1} p + \\ &+ \partial_q x_h(q)\, F_h \ . \end{aligned} \tag{2}$$

Here and in the following, ∂_q indicates partial differentiation with respect to q. The mechanical load system (2) is rigidly connected to a general hydraulic actuator system which produces the force F_h, see Fig. 1. The term $x_h(q)$ indicates the

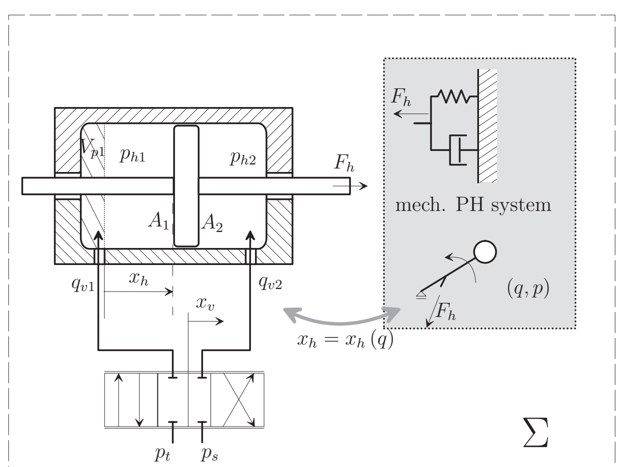

Fig. 1. Hydraulically actuated mechanical systems.

piston position and the term $\partial_q x_h(q) F_h$ is the generalized force on the mechanical system due to the hydraulic chamber pressures. This follows directly from the Lagrange formalism considering the virtual displacements. It is assumed that

$$0 < k_{x_h,\min} \leq |\partial_q x_h(q)| \leq k_{x_h,\max} < \infty$$

is met, what expresses the fact that the generalized force exerted on the load system should be finite and non-zero. The PH description of the system (2) is given by

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t} x_l &= (J_l - R_l)\, \partial_{x_l} H_l^T + G_l F_h \\ y_l &= G_l^T \partial_{x_l} H_l^T \end{aligned}$$

with the Jacobian $\partial_{x_l} H_l$ of H_l. J_l denotes the canonical skew symmetric structure matrix, R_l is the positive semidefinite dissipation matrix and G_l the input vector such that

$$J_l - R_l = \begin{bmatrix} 0 & 1 \\ -1 & -d(q) \end{bmatrix}, \quad G_l = \begin{bmatrix} 0 \\ \partial_q x_h(q) \end{bmatrix}.$$

Since the dynamics of the plant are usually much slower than those of the servo valve, the valve dynamics will be neglected and the spool position x_v of the *critically centered three-land-four-way spool valve*, see Fig. 1, will be considered as the plant input. The valve is assumed to be rigidly connected to the hydraulic actuator, to an accumulator with pressure pump and to the tank at supply and tank pressures p_s respectively p_t, $p_s > p_t$. In this case, the turbulent volumetric flows q_{vi} into chamber $i = 1, 2$ at pressure p_{hi} are given by

$$-q_{vi} = \begin{cases} (-1)^i x_v k_v \sqrt{p_s - p_{hi}}\,, & (-1)^i x_v \leq 0 \\ (-1)^i x_v k_v \sqrt{p_{hi} - p_t}\,, & (-1)^i x_v > 0 \end{cases} \tag{3}$$

with the valve coefficient k_v, see, e.g. [9]. For short, $q_{vi} = \Xi_i x_v$ will be used. The neglect of the usually small leakages, which can be compensated if known, leads to the standard equations

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t} p_{h1} &= \frac{E}{V_{h1}} \left(-A_1 v_h + q_{v1}(x_v)\right) \\ \tfrac{\mathrm{d}}{\mathrm{d}t} p_{h2} &= \frac{E}{V_{h2}} \left(A_2 v_h + q_{v2}(x_v)\right) \end{aligned} \tag{4}$$

for the chamber pressures with hydraulic bulk modulus E, effective piston areas A_i, chamber volumes $V_{hi} = V_{pi} - (-1)^i A_i x_h$ and piston velocity $v_h = \partial_q x_h m^{-1} p$. The V_{pi}'s denote the chamber offset and pipe volumes. The overall plant dynamics Σ are given by (2), (4) with $F_h = A_1 p_{h1} - A_2 p_{h2}$. The state of Σ is $x_{sen} = (q, p, p_{h1}, p_{h2}) \in \chi = (q_-, q_+) \times \mathbb{R} \times (p_t, p_s) \times (p_t, p_s)$. Here, (q_-, q_+) denote the mechanical end-positions due to a finite actuator length. The set (q_-, q_+) is therefore invariant. A possibility how to incorporate these constraints into V is presented in [8]. Apart from that, the construction of the hydraulic chambers guarantees $V_{h1}(q_-) > 0$ and $V_{h2}(q_+) > 0$.

For the purpose of controller design, the PH structure of Σ will be given in so-called *canonical coordinates* $\xi = (q, p, z_{h1}, z_{h2})$, where p denotes the generalized momentum and z_{h1} and z_{h2} are special coordinates due to the hydraulic system part. In fact, they are functions of the fluid masses in the cylinder chambers. z_{h1} and z_{h2} are used instead of the fluid masses because in these coordinates the zero dynamics subsystem (z_{h2}) for the output q is clearly identifiable. The special coordinates are given by

$$\begin{aligned} z_{h1} &= F_h + \textstyle\sum_i (-1)^{i-1} E A_i \ln\left(V_{hi} V_{pi}^{-1}\right) \\ z_{h2} &= p_{h2} A_2 + E A_2 \ln\left(V_{h2} V_{p2}^{-1}\right)\,. \end{aligned} \tag{5}$$

The PH description of the system Σ is given by

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t} \xi &= (J - R)\, \partial_\xi H^T + G u_v \\ y &= G^T \partial_\xi H^T\,. \end{aligned}$$

J denotes the canonical skew symmetric 4×4 structure matrix and R is the dissipation matrix such that

$$J - R = \begin{bmatrix} J_l - R_l & 0 \\ 0 & 0 \end{bmatrix}.$$

Therefore, z_{h1} and z_{h2} are special invariants, so-called Casimir functions, of the PH structure. The Hamiltonian is given by the sum of the potential energy stored in the hydraulic chambers and the energy of the mechanical subsystem (1)

$$\begin{aligned} H = H_h + H_l\,,\; H_h &= \textstyle\sum_i E V_{hi} \left(\ln\left(V_{hi} V_{pi}^{-1}\right) - 1\right) + \\ &+ V_{p1}(p_{h0} + E)\, \mathrm{e}^{\frac{z_{h1}+z_{h2}}{A_1 E} - \frac{p_{h0}}{E}} + \\ &+ V_{p2}(p_{h0} + E)\, \mathrm{e}^{\frac{z_{h2}}{A_2 E} - \frac{p_{h0}}{E}} + \\ &- z_{h1} x_h - (z_{h1} + z_{h2}) \tfrac{V_{p1}}{A_1} - z_{h2} \tfrac{V_{p2}}{A_2}\,. \end{aligned} \tag{6}$$

p_{h0} denotes some offset pressure. The input vector G for the transformed input u_v yields

$$\begin{aligned} G^T &= \begin{bmatrix} 0 & 0 & 1 & g_4 \end{bmatrix}, \quad g_4 = \frac{\frac{EA_2}{V_{h2}} \Xi_2}{\frac{EA_1}{V_{h1}} \Xi_1 - \frac{EA_2}{V_{h2}} \Xi_2} \\ u_v &= \left(\frac{EA_1}{V_{h1}} \Xi_1 - \frac{EA_2}{V_{h2}} \Xi_2\right) x_v\,. \end{aligned}$$

It is straight forward to check that $-1 < g_4 < 0$ since the Ξ_i's have opposite sign. The time change of the Hamiltonian (6) along a trajectory is given by

$$\tfrac{\mathrm{d}}{\mathrm{d}t} H = -\left(\partial_\xi H\right) R \left(\partial_\xi H\right)^T + \langle y, u \rangle \tag{7}$$

with the canonical product of the input u and the collocated output y, and thus the plant is *passive* with storage function H. For convenience, the system Σ is partitioned into a part Σ_1 with state $\xi_1 = (q, p, z_{h1})$ and corresponding (J_1, R_1, G_1) which represents the load system and the hydraulically generated force F_h and an independent pure hydraulic part Σ_2 with $\xi_2 = (z_{h2})$ which represents, e.g., the second chamber pressure. Σ_2 is the zero-dynamics subsystem for the output q.

III. Nonlinear Controller Design

The key idea of a PH controller design is to choose an augmented Hamiltonian H_a for the controlled plant, such that H_a has a strict minimum at the desired locus. Stability of the closed loop follows immediately from the passivity property (7). Nevertheless, one has to ensure that the closed loop plant has again PH structure. Towards this end, new structure and dissipation matrices J_a and R_a can be assigned in order to accomplish more freedom in controller design. Once (J_a, R_a, H_a) are assigned appropriately, the control law is uniquely determined.

The equilibria $\breve{x}_{sen}$ of the plant Σ specified by the desired position $\breve{q}$ are given by the set

$$\left\{ \breve{x}_{sen} \in \chi | \, \breve{p} = 0 \ , \ A_1 \breve{p}_{h1} - A_2 \breve{p}_{h2} = \partial_q V \partial_q x_h^{-1} \big|_{q=\breve{q}} \right\} \ .$$

Further, $x_v = 0$ has to be met. The equilibrium in canonical coordinates is $\breve{\xi} = (\breve{q}, 0, z_{h1}(\breve{q}), \breve{z}_{h2})$ with a suitable $\breve{z}_{h2}$. The goal is to design a controller which stabilizes the closed loop plant at $\breve{q}$ exploiting the decoupling of the state ξ. This energy based control law will be developed in two steps. First, a static controller is designed under the assumption that $\partial_q V|_{q=\breve{q}}$, i.e., the equilibrium force, is perfectly known and that the damping in the mechanical system is sufficient. Afterwards, the control law is extended by a disturbance observer for $\partial_q V|_{q=\breve{q}}$. This observer provides an estimate of the velocity signal, which is used to inject additional dissipation into the plant. The section finishes with a discussion of the damping properties of the dynamic controller and some considerations concerning additional acceleration feedback. In the following, relative coordinates $\bar{\xi} = \xi - \breve{\xi}$ will be used.

Static controller part. Since the system Σ consists of Σ_1 and the zero-dynamics subsystem Σ_2, the closed loop PH system is chosen such that this structure is preserved. Therefore, an augmented Hamiltonian $H_a = H_{a1}(\bar{\xi}_1) + H_{a2}(\bar{\xi}_2)$ for Σ with a strict minimum at $\breve{\xi}$ will be assigned. The mechanical part of H_a is obtained by the special PH structure preserving transformation of the mechanical load system to relative coordinates $\bar{\xi}_1$

$$H_l|_{\xi_1 = \breve{\xi}_1 + \bar{\xi}_1} - V(\breve{q}) - \frac{\partial_q V(q)|_{q=\breve{q}}}{\partial_q x_h(q)|_{q=\breve{q}}} (x_h - \breve{x}_h) \ .$$

The same kind of transformation is applied to the potential stemming from the hydraulic fluid. H_a is finally chosen to

$$\begin{aligned} H_a = H_{a1}(\bar{\xi}_1) + H_{a2}(\bar{\xi}_2) = H_l|_{\xi_1 = \breve{\xi}_1 + \bar{\xi}_1} + \\ - V(\breve{q}) - \underbrace{\frac{\partial_q V(q)|_{q=\breve{q}}}{\partial_q x_h(q)|_{q=\breve{q}}}}_{\breve{F}_h} (x_h - \breve{x}_h) + \\ \Gamma_1 \frac{\bar{z}_{h1}^2}{2} + E \int_{x_h(\breve{q})}^{x_h(\breve{q}+\bar{q})} \left(A_1 \ln\left(\frac{V_{h1}(\breve{x}_h + \tau)}{\breve{V}_{h1}} \right) + \right. \\ \left. A_2 \ln\left(\frac{\breve{V}_{h2}}{V_{h2}(\breve{x}_h + \tau)} \right) \right) \mathrm{d}\tau + \Gamma_2 \frac{\bar{z}_{h2}^2}{2} \end{aligned} \tag{8}$$

with any constants $\Gamma_i > 0$ and $\breve{x}_h = x_h(\breve{q})$. The necessary condition for a minimum of H_{a1} at $\breve{\xi}_1$ is clearly satisfied. Since the integral term can be considered as the potential of the hydraulic spring relative to $\breve{q}$ with large E this term usually dominates and leads to a strict global minimum of H_{a1} at $\breve{\xi}_1$ and further to one of H_a at $\breve{\xi}$, what is assumed in the following. For the controller design the focus is laid on the subsystem Σ_1 since it is not influenced by the zero-dynamics Σ_2. This can be done in the PH context since H_a is the sum of the two subsystem Hamiltonians $H_{a1}(\bar{\xi}_1) + H_{a2}(\bar{\xi}_2)$. Finally, it will be shown that the overall closed loop system is stable. In order to maintain the PH structure of Σ_1 the restrictions (9)

$$G_1^{\perp} \left((J_{a1} - R_{a1}) \partial_{\bar{\xi}_1} H_{s1}^T + (J_{s1} - R_{s1}) \partial_{\bar{\xi}_1} H_1^T \right) = 0 \tag{9}$$

have to be fulfilled, with $H_{a1} = H_1 + H_{s1}$, $J_{a1} = J_1 + J_{s1}$, $R_{a1} = R_1 + R_{s1}$ and the annihilator $G_1^{\perp}$ of G_1. Therefore, the augmented subsystem matrices are chosen such that

$$J_{a1} - R_{a1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -d(q) & 2\beta \\ 0 & 0 & -\frac{k_p}{\Gamma_1} \end{bmatrix}$$

$$\beta = \frac{\partial_q x_h(q)|_{q=\breve{q}+\bar{q}}}{2\Gamma_1} \ .$$

Then, $\breve{\xi}_1$ is a stable equilibrium of the closed loop system $\mathring{\Sigma}_1$ for a controller gain $k_p > 0$ as long as

$$k_p d(q) - \frac{\left(\partial_q x_h(q)|_{q=\breve{q}+\bar{q}} \right)^2}{4\Gamma_1} \geq 0 \ , \tag{10}$$

i.e., $R_{a1} \geq 0$. This condition is not really restrictive since the free parameter $\Gamma_1 > 0$ of H_a can be chosen arbitrary large since it does not appear in the static control law

$$u_v = -k_p \bar{z}_{h1} \ , \tag{11}$$

which follows from

$$\begin{aligned} \left(G_1^T G_1 \right) u_v = \\ G_1^T \left((J_{a1} - R_{a1}) \partial_{\bar{\xi}_1} H_{s1}^T + (J_{s1} - R_{s1}) \partial_{\bar{\xi}_1} H^T \right) \ . \end{aligned} \tag{12}$$

Further, it follows with $\Gamma_1 \to \infty$ that the natural damping of the load system can be quite small without violating (10). $H_{a1}(q, \cdot)$ is radially unbounded in $\mathbb{R}^2$ and due to La'Salle's invariance principle the system $\mathring{\Sigma}_1$ (Σ_1 and (11)) with state ξ_1 and state space $(q_-, q_+) \times \mathbb{R}^2$ has an asymptotically stable equilibrium at $\breve{\xi}_1$. Linearizing $\mathring{\Sigma}_1$ at $\breve{\xi}_1$ shows that the equilibrium is locally exponentially stable. The coupling of $\mathring{\Sigma}_1$ to $\mathring{\Sigma}_2$ due to

$$\tfrac{\mathrm{d}}{\mathrm{d}t} \bar{z}_{h2} = g_4(\bar{\xi}_1, \bar{z}_{h2}) \, u_v(\bar{\xi}_1)$$

satisfies the linear growth condition in $\bar{z}_{h2}$ since $0 < |g_4| < 1$ and $u_v|_{\breve{\xi}_1} = 0$, see [10], and thus the overall closed loop system $\mathring{\Sigma}$ is stable and bounded with state space $(q_-, q_+) \times \mathbb{R}^3$. Nevertheless, the restriction of the pressures p_{hi} is not considered. The valve characteristics (3) are only valid if the p_{hi}'s do not increase beyond respectively decrease below the finite pressures p_s and p_t. For technical plants this is usually fulfilled. (8) can be used to look for a suitable region of attraction, which depends of course on the special plant. For the reason of steady-state accuracy $\breve{F}_h$ has to be known due to (5). $\breve{F}_h$ is not known if, e.g., a constant but unknown external load acts on the plant. This partial drawback will be dealt with in the next paragraph.

Dynamical extension. Again, the PH controller is designed with focus on the subsystem Σ_1. From here on, it is assumed for the reason of simplicity that the generalized coordinate q is chosen such that the generalized mass is constant, i.e. $\partial_q m(q) = 0$. This is *no restriction of generality* since, e.g., $\acute{m} = 1$ can be always achieved by the transformation

$$\acute{q} = \int_{\breve{q}}^{q} m(\tau)^{\frac{1}{2}}\, \mathrm{d}\tau \ , \quad \acute{p} = pm(q)^{-\frac{1}{2}} \ .$$

For notational convenience, the original symbols will be used instead of the primed ones. Hence, the transformed load system (2) is state-affine in the unmeasured state p. The fact that the damping term is linear in p is not changed by the preceding transformation. It will be assumed that the uncertainty in V occurs affine in form of the unknown but constant load force F_l

$$V = -F_l q + \tilde{V}(q) \ ,$$

i.e., F_l is collocated to the generalized velocity $\dot{q}$. For the observer design, one investigates (2), which has now the form

$$\begin{aligned}
\tfrac{\mathrm{d}}{\mathrm{d}t} q &= \frac{p}{m} \\
\tfrac{\mathrm{d}}{\mathrm{d}t} p &= -d(q)\frac{p}{m} - F_l + \underbrace{\left(-\partial_q \tilde{V}\right) + \partial_q x_h(q) F_h}_{=u_{obs}} \ .
\end{aligned}$$

The transformation u_{obs} will simplify the observer design. In fact, $\left(-\partial_q \tilde{V}\right)$ will be canceled. Now, one can build a nonlinear second order disturbance observer for (2) and $\dot{F}_l = 0$ with the state

$$\hat{x}_{obs} = \begin{bmatrix} \hat{p} \\ \hat{F}_l \end{bmatrix} + \begin{bmatrix} (\lambda_1 + \lambda_2) m \\ \lambda_1 \lambda_2 m \end{bmatrix} \bar{q}$$

and the λ_i's to be fixed. The nonlinear observer dynamics read as

$$\begin{aligned}
\tfrac{\mathrm{d}}{\mathrm{d}t}\hat{x}_{obs} &= \begin{bmatrix} \lambda_1+\lambda_2 & -1 \\ \lambda_1\lambda_2 & 0 \end{bmatrix} \hat{x}_{obs} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_{obs} + \\
&- \begin{bmatrix} d(q) m^{-1} \\ 0 \end{bmatrix} \hat{x}_{obs,1} + \begin{bmatrix} (\lambda_1+\lambda_2) d(q) \\ 0 \end{bmatrix} \bar{q} + \\
&\begin{bmatrix} -\left((\lambda_1+\lambda_2)^2 - \lambda_1\lambda_2\right) m \\ -\lambda_1\lambda_2(\lambda_1+\lambda_2) m \end{bmatrix} \bar{q} \ , \ \mathring{\hat{F}}_h = \frac{\hat{F}_l + \partial_q \tilde{V}}{\partial_q x_h(q)} \ .
\end{aligned} \tag{13}$$

Here, λ_1, λ_2 denote the eigenvalues of the linear part of the observer error dynamics in the left half plane. In order to incorporate the observer into an overall PH structure for the final controller design one studies the observer error dynamics Σ_e with $e_1 = \hat{p} - p$ and $e_2 = \hat{F}_l - F_l$

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1+\lambda_2 & -1 \\ \lambda_1\lambda_2 & 0 \end{bmatrix}}_{A_{obs}} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} - \begin{bmatrix} d(q(t)) m^{-1} \\ 0 \end{bmatrix} e_1 \ . \tag{14}$$

Since A_{obs} is Hurwitz there exists a positive definite solution P^{-1} of the Lyapunov-type equation

$$A_{obs}^T P + P A_{obs} + 2\tilde{Q} = 0 \tag{15}$$

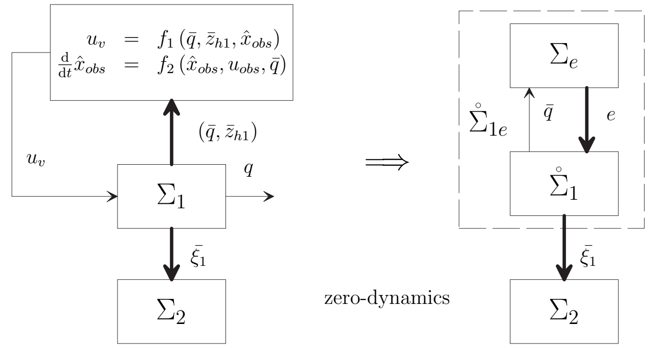

Fig. 2. The structure of the plant with the dynamic controller and the corresponding PH systems.

for any positive definite $\tilde{Q}$. With the abbreviation $Q = P^{-1}\tilde{Q}P^{-1}$, the error dynamics (14) have PH structure (J_o, R_o, H_o) with the decomposition $A_{obs} = (J_o - Q) P$, $J_o = (A_{obs} + QP) P^{-1}$, $H_o = \frac{1}{2} e^T P e$, and

$$R_o = Q + \underbrace{\begin{bmatrix} d(q(t)) m^{-1} P_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix}}_{R_{o2} \geq 0} \ .$$

The equilibrium $(0,0)$ of (J_o, R_o, H_o) is *globally exponentially stable* due to

$$\tfrac{\mathrm{d}}{\mathrm{d}t} H_o = -e^T P R_o P e < 0 \ .$$

Now, the observer PH structure can be combined with that of $\mathring{\Sigma}_1$ ((J_{a1}, R_{a1}, H_{a1})) to the extended closed loop system $\mathring{\Sigma}_{1e}$, see Fig. 2.

The overall desired PH structure for $\mathring{\Sigma}_{1e}$ is given by (J_{ae}, R_{ae}, H_{ae}), $H_{ae} = H_{a1} + \frac{1}{2} e^T P e$ for $\bar{\xi}_{1e} = (\bar{q}, p, \bar{z}_{h1}, e_1, e_2)$ with

$$J_{ae} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & \beta+\gamma & 0 \\ 0 & -\beta-\gamma & 0 & -\delta^T \\ 0 & 0 & \delta & J_o \end{bmatrix} \ , \quad \tilde{\delta} = \begin{bmatrix} \tilde{\delta}_1 \\ \tilde{\delta}_2 \end{bmatrix}$$

$$R_{ae} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & d(q) & -\beta+\gamma & 0 \\ 0 & -\beta+\gamma & \frac{k_p}{\Gamma_1} & \delta^T \\ 0 & 0 & \delta & R_o \end{bmatrix} \ , \quad \delta = P^{-1}\tilde{\delta}$$

and

$$\begin{aligned}
\beta &= \frac{\partial_q x_h(q)|_{q=\breve{q}+\bar{q}}}{2\Gamma_1} \ , \quad \gamma = \frac{k_d}{2} f_d(\bar{q}) \\
\tilde{\delta}_1 &= -\frac{k_d}{2}\frac{f_d(\bar{q})}{m} \ , \quad \tilde{\delta}_2 = -\frac{k_p}{2\,\partial_q x_h(q)|_{q=\breve{q}+\bar{q}}} \ .
\end{aligned}$$

The term γ is responsible for the injected additional damping and δ reflects the coupling of the error dynamics (14) to $\mathring{\Sigma}_1$ due to

$$\begin{aligned}
\tfrac{\mathrm{d}}{\mathrm{d}t}\bar{z}_{h1} &= -k_p \bar{z}_{h1} + k_p \frac{e_2}{\partial_q x_h(q)|_{q=\breve{q}+\bar{q}}} + \\
&- k_d f_d(\bar{q}) \frac{p}{m} + k_d f_d(\bar{q}) \frac{e_1}{m} \ .
\end{aligned}$$

The positive semidefiniteness of R_{ae} follows from that of

$$\begin{bmatrix} I_{3\times 3} & 0 \\ 0 & P \end{bmatrix} R_{ae} \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & P \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \tilde{Q} + P R_{o2} P \end{bmatrix}$$

by decomposition into a sum of positive semidefinite matrices if $k_p, k_d > 0$, the tunable nonlinearity $f_d(\bar{q})$ such that $\operatorname{sign}(f_d(\bar{q})) = \operatorname{sign}(\partial_q x_h(q)) = \text{const}$, and

$$\frac{k_p d(\breve{q} + \bar{q})}{\Gamma_1} - \left(\frac{\partial_q x_h(q)|_{q=\breve{q}+\bar{q}}}{2\Gamma_1} - \frac{k_d}{2} f_d(\bar{q}) \right)^2 \geq 0 \ ,$$

since $\tilde{Q}$ can be chosen arbitrarily in (15). Here, $\Gamma_1 > 0$ has to be selected in order to fulfill the inequality above. Therefore, this choice of (J_{ae}, R_{ae}, H_{ae}) guarantees local asymptotic stability of the dynamically extended closed loop system $\mathring{\Sigma}_{1e}$ due to

$$\tfrac{\mathrm{d}}{\mathrm{d}t} H_{ae} = -\partial_{\bar{\xi}_{1e}} H_{ae} R_{ae} \partial_{\bar{\xi}_{1e}} H_{ae}^T \leq 0$$

and La Salle's invariance principle. An argumentation similar to that at the end of the static controller part shows that the closed loop subsystem is asymptotically stable in $(q_-, q_+) \times \mathbb{R}^4$ and the overall system is stabilized and bounded in $(q_-, q_+) \times \mathbb{R}^5$. $f_d(\bar{q})$ can be used to shape the damping behaviour of the system. For the special choice of $f_d(\bar{q})$ and Γ_1 such that $\gamma = \beta$ it directly follows that the original load system (2) is even not required to have any damping.

The final dynamic control law is given by

$$\begin{aligned} u_{v,dyn} &= -k_p \left(z_{h1} - \hat{\mathring{F}}_h \right) - k_d f_d(\bar{q}) \frac{\hat{p}}{m} \\ &\quad - k_p \sum_i (-1)^{i-1} E A_i \ln \left(V_{hi} V_{pi}^{-1} \right) \Big|_{q=\breve{q}} \\ x_{v,dyn} &= \left(\frac{EA_1}{V_{h1}} \Xi_1 - \frac{EA_2}{V_{h2}} \Xi_2 \right)^{-1} u_{v,dyn} \end{aligned}$$

combined with the observer equations from (13) and the transformation (5). The term $u_{v,dyn}$ follows similar to (12). The only tuning parameters of the controller besides $f_d(\bar{q})$, which can be set, e.g., to the constant $\operatorname{sign}(\partial_q x_h(q))$, are the proportional gain k_p, the damping gain k_d and the λ_i's of the observer. Following the same idea as for the damping part of the controller, k_p can be replaced by $k_p f_p(\bar{q}) > 0$ in order to influence the proportional controller part. This can be used, e.g., to reduce the plant input signal for large position errors significantly without loosing the stabilizing properties of the controller.

It is possible to interpret the additional dissipation added by the dynamic feedback in terms of a mechanical damper. Investigating

$$\varepsilon_1 \tfrac{\mathrm{d}}{\mathrm{d}t} \bar{z}_{h1} = -\varepsilon_2 \bar{z}_{h1} - f_d(\bar{q}) \frac{p}{m}$$

with $\varepsilon_1 = k_d^{-1}$ and $\varepsilon_2 = k_p k_d^{-1}$ gives in the limit

$$\begin{aligned} \lim_{\varepsilon_1 \to 0} \varepsilon_1 \tfrac{\mathrm{d}}{\mathrm{d}t} \bar{z}_{h1} &= -\varepsilon_2 \bar{z}_h - f_d(\bar{q}) \frac{p}{m} \\ \bar{z}_h &= -\frac{1}{\varepsilon_2} f_d(\bar{q}) \frac{p}{m} \end{aligned}$$

and by singular perturbation arguments – compare to (2) –

$$\tfrac{\mathrm{d}}{\mathrm{d}t} p = \ldots - \partial_q x_h(q) f_d(\bar{q}) \frac{1}{\varepsilon_2} \frac{p}{m} + \partial_q x_h(q)(\ldots) \ .$$

By choosing $f_d(\bar{q}) = \partial_q x_h(q)^{-1}$ one obtains in the limiting case a linear damper in $\dot{q}$ with gain $\varepsilon_2^{-1} = \dfrac{k_d}{k_p}$. This gives already some hints how to tune the controller. On the other hand, if one only feeds back the velocity, i.e. $k_p = 0$, one changes only the mechanical stiffness of $\mathring{\Sigma}_1$ due to the integrator $\frac{\mathrm{d}}{\mathrm{d}t}\bar{z}_{h1}$.

Following this idea, there is an alternative possibility in order to inject damping into the plant if the acceleration is available by measurement. Then, the control law can be extended by an acceleration feedback term.

$$u_{v,acc} = -k_{d,acc} \operatorname{sign}(\partial_q x_h(q)) \dot{v}$$

$u_{v,acc}$ mimics then exactly a mechanical damper due to the integrator $\frac{\mathrm{d}}{\mathrm{d}t}\bar{z}_{h1}$.

IV. Industrial application

A possible application is a hydraulically actuated wrapper assembly , i.e., a lever mechanism commonly used in rolling mills, with a double-ended piston connected to only one single servovalve, see Fig. 3. The constant nominal moment of inertia of the assembly is about $m = 8000 \ \mathrm{Nm}^2$ – the moment of inertia and the mass of the actuator are neglected –, the nominal bulk modulus is $E = 1.6e9 \ \mathrm{Nm}^{-2}$ and hydraulic forces occur up to some kN. The energy-based controller can be easily adapted to this plant by identifying the wrapper angle α as generalized position. Therefore, the plants mathematical model is given by (2), (4) with

$$\begin{aligned} x_h &= \sqrt{l_1^2 + l_2^2 - 2 l_1 l_2 \cos(\varepsilon - \alpha)} - l_{h0} \\ V &= m_l g l_g \sin(\alpha + \kappa) \ , \end{aligned}$$

$l_{h0} = l_h|_{x_h=0}$, the constant of gravity g, the mass of the wrapper $m_l = 7000$ kg, a damping constant $d = 1e5 \ \mathrm{Nsrad}^{-1}$, piston areas $A_1 = A_2 = 0.025 \ \mathrm{m}^2$, offset volumes $V_{p1} = V_{p2} = 0.05 A_1$ m. l_1, l_2, l_g, ε, κ are some geometrical constants of the construction. The observer eigenvalues are $\lambda_1 = \lambda_2 = -100$ and $k_p = 5000$, $k_d = 0.2E$. The performance of the energy-based dynamic controller with damping compared to the pure static controller (11) is demonstrated by the simulations in Fig. 4. At the

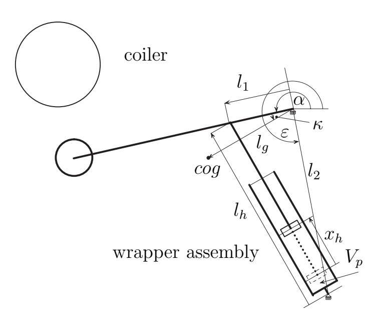

Fig. 3. The wrapper assembly with center of gravity (cog).

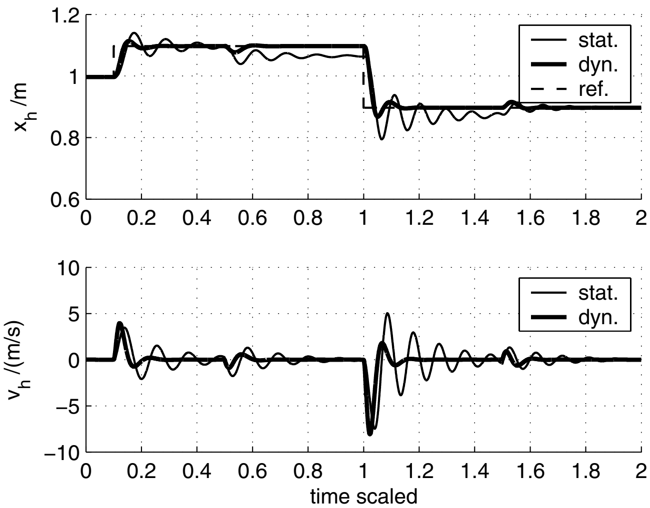

Fig. 4. Piston position (x_h) and velocity (v_h) responses with static and dynamic controller.

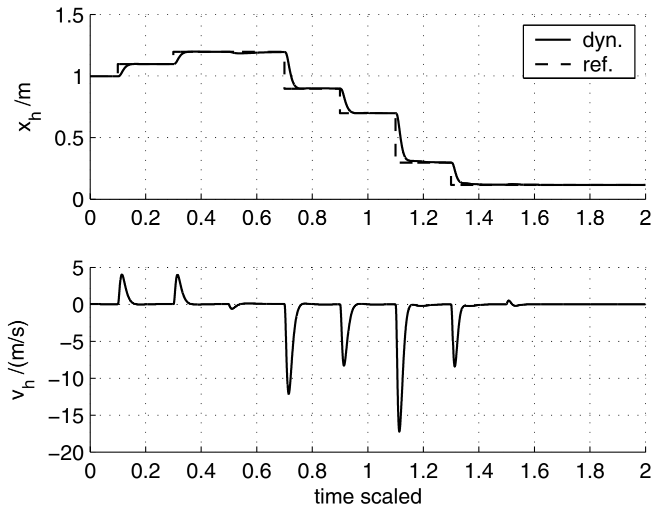

Fig. 5. Piston position (x_h) and velocity (v_h) responses with dynamic controller over a large position range.

scaled timevalues 0.1 and 1 position changes are induced and at 0.5 a disturbance load torque is applied, which is removed at 1.5. The pure static controller is not able to eliminate the steady-state error due to the external disturbance and further it causes oscillations due to the poorly damped plant. In the case that the parameters, e.g., k_v, E, m, d, differ from their nominal values one can proof the stability of the closed loop using Lyapunov theory and the Hamiltonian (8) with the modified parameters as a Lyapunov function. The quality of the control law with parameters differing from their nominal values and a finite sampling time has been studied in extensive simulations of the plant and the dynamic controller. E.g., even in the case that the hydraulic bulk modulus is decreased to half of its nominal value and the mass is twice the nominal value one can proof stability and the responses hardly defer from those of Fig. 4. Therefore, these responses are not depicted. Further, the performance of the dynamic controller over a large position range is given in Fig. 5.

V. Concluding remarks

The nonlinear energy-based PH controller proposed in this contribution stabilizes the desired equilibrium of a 1-DOF hydro-mechanical plant asymptotically independent of its natural damping. Further, the control law can mimic a mechanical damper without velocity measurement. The dependence on the knowledge of some physical parameters, e.g., on the hydraulic bulk modulus E, the moment of inertia m, etc. has been studied in simulations.

Additionally, one can modify the frequency dependence of the damping by tuning the observer as a filter of possibly higher order. In the presence of acceleration measurement it is shown how this can be directly incorporated into the controller. The behaviour of the controlled actuator at different operating points is essential for a good performance of the overall system. This is one of the major differences of the proposed controller design compared to conventional linear ones. The presented concept is easy to commission at industrial plants since the few tuning parameters are well interpretable in a physical sense.

VI. Acknowledgments

We gratefully acknowledge the financial support from the *Voest Alpine Industrieanlagenbau* (VAI) within the scope of the *Christian Doppler Laboratory (CDL) for Automatic Control of Mechatronic Systems in Steel Industries*.

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