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New Approaches for the Stabilization of Discrete Takagi-Sugeno Fuzzy Models

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Abstract—**The work presented deals with the stability and the stabilization of discrete Takagi-Sugeno fuzzy models. Several results show that stability and stabilization conditions using a non quadratic function outperform those obtained with a quadratic one. In this work we will use some new Lyapunov's functions and show that the results obtained include all the other approaches. To show the effectiveness of the method, several examples are given.**

I. Introduction

Takagi-Sugeno (TS) models [1] were hardly investigated in the last years. Most of the papers use the direct Lyapunov method. This method needs the use of a candidate function which is chosen very often as a quadratic in the continuous case, see for example [2] and the references therein. Some works exist with a piecewise Lyapunov function [3] and [4]. In the discrete case another class of Lyapunov function can be used [5]–[7]. This class remains in quadratic functions blended together by the membership functions of the TS model. It is proved that the quadratic results are included in the non quadratic ones and the set of solutions is considerably enhanced [5]–[7]. In these papers the conditions are always derived from the variation of the Lyapunov function in one sample. This paper deals with a new approach to derive sufficient conditions of stability and stabilization which are less conservative.

This paper is organized as it follows: in the first part the notations and the material are given. Especially the matrix properties and the Lyapunov function that will be used. The next part is devoted to the stability of the discrete TS model problem. Firstly the extension of the quadratic case is treated and secondly the one of the non quadratic case. The third part treats the stabilization case. Several Examples are also given to illustrate the effectiveness of the approach. At last, a discussion is made to show the advantages and the drawbacks of the proposed approach.

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II. Notations and Materials

Notation: For any positive scalar function $h_i(z(\cdot)) \geq 0$, and matrices A_i $i \in \{1,\dots,r\}$ we define $A_{z(\cdot)} = \sum_{i=1}^{r} h_i(z(\cdot))A_i$. Let us also define for $a,b \in \mathbb{N}$, $a \leq b$ $\prod_{i=a}^{b} Y_i = Y_a Y_{a+1} \dots Y_b$. At last a $(*)$ indicates a transpose quantity. For example $A_z^T P_{z+}(*) - P_z < 0$ stands for $A_z^T P_{z+} A_z - P_z < 0$ and $\begin{bmatrix} P_z^{-1} & (*) \\ A_z & P_{z+}^{-1} \end{bmatrix} > 0$ for $\begin{bmatrix} P_z^{-1} & A_z^T \\ A_z & P_{z+}^{-1} \end{bmatrix} > 0$.

Lemma 1 [8] [5]: Let $P > 0$, Γ, Φ and Ψ matrices of appropriate dimension, the two following statements are equivalent:

Find $P > 0$ such that $\Phi^T P \Phi - \Gamma < 0$ (1)

Find $P > 0$ and Ψ such that $\begin{bmatrix} -\Gamma & (*) \\ \Psi\Phi & -\Psi - \Psi^T + P \end{bmatrix} < 0$ (2)

Proof: $(\Leftarrow)$ Using the congruence with $\begin{bmatrix} I & -\Phi^T \end{bmatrix}$ gives directly the result. $(\Rightarrow)$ Choose $\Psi = P$ then using Schur's complement on (2) gives directly the result.

Remark 1: The equivalence is not more true if Ψ is under constraint such that it cannot be equal to P.

In the following, r rules discrete Takagi-Sugeno models (TS) are considered:

$$x(t+1) = A_{z(t)} x(t) \tag{3}$$

Remind that TS models are a set of linear models blended with the nonlinear functions $h_i(z(t)) \geq 0$. Any control affine nonlinear model $x(t+1) = f(x(t)) + g(x(t))u(t)$ can be represented exactly by a TS model in a compact set of the state space [2]. The so-called nonlinearity sector approach is a systematic way to put a nonlinear model in a TS form [2]. The TS representation is not unique for a given nonlinear model [2] [9].

Lemma 2: Considering a Lyapunov function $V(x(t))$, the model (3) is globally asymptotically stable if $\Delta_k V(x(t)) = V(x(t+k)) - V(x(t)) < 0$ for a given positive integer $k \in \mathbb{N}^*$ and for all $t \in \mathbb{N}$ and for all initial conditions.

Proof: By choosing the following Lyapunov function:

$$V_2(x(t)) = \sum_{l=0}^{k-1} V(x(t+l)) \tag{4}$$

The model (3) is globally asymptotically stable if:

$$\Delta V_2(x(t)) = V_2(x(t+1)) - V_2(x(t)) < 0 \tag{5}$$

$$\Delta V_2(x(t)) = \sum_{l=0}^{k-1} V(x(t+l+1)) - \sum_{l=0}^{k-1} V(x(t+l)) \tag{6}$$

$$\Delta V_2(x(t)) = V(x(t+k)) - V(x(t)) = \Delta_k V(x(t)) \tag{7}$$

And if $\Delta_k V(x(t)) < 0$ so $\Delta V_2(x(t))$ is too.

Remark 2: The function $V_2(x(t))$ is a Lyapunov function because it is a sum of Lyapunov functions.

III. STABILITY

First consider the classical quadratic Lyapunov function:

$$V(x(t)) = x(t)^T P x(t), \; P > 0 \tag{8}$$

Quadratic stability is ensured if there exists a common matrix $P > 0$ such that [10]:

$$A_i^T P A_i - P < 0 \tag{9}$$

Considering $\Delta_k V(x(t))$ using $V(x(t))$ defined in (8) we can write:

$$\Delta_k V(x(t)) = x(t+k)^T P x(t+k) - x(t)^T P x(t) \tag{10}$$

then with (3):

$$\Delta_k V(x(t)) = x(t)^T \left(\prod_{l=0}^{k-1} A_{z(t+l)}^T P (*) - P \right) x(t) \tag{11}$$

Finally we get the following result.

Theorem 1: Considering the open-loop TS model (3), if there exists a common matrix $P > 0$ such that $A_{i_0}^T A_{i_1}^T \dots A_{i_{k-1}}^T P A_{i_{k-1}} \dots A_{i_1} A_{i_0} - P < 0$ for all combinations of $i_l \in \{1,2,\dots,r\}$, $l \in \{0,1,\dots,k-1\}$ then model (3) is globally asymptotically stable.

Proof: The result is straightforward from (11).

For non quadratic Lyapunov function we use the following Lyapunov function [5]:

$$V(x(t)) = x(t)^T \sum_{i=1}^{r} h_i(z(t)) P_i x(t) = x(t)^T P_{z(t)} x(t) \tag{12}$$

with matrices $P_i > 0$, $i \in \{1,\dots,r\}$. Then:

$$\Delta_k V(x(t)) = x(t)^T \left(\left(\prod_{l=0}^{k-1} A_{z(t+l)}^T \right) P_{z(t+k)} (*) - P_{z(t)} \right) x(t) \tag{13}$$

Using lemma 1 leads to:

$$\begin{bmatrix} P_{z(t)} & \left(\prod_{l=0}^{k-1} A_{z(t+l)}^T \right) \Psi_{z(t+k)} \\ (*) & \Psi_{z(t+k)} + \Psi_{z(t+k)}^T - P_{z(t+k)} \end{bmatrix} > 0 \tag{14}$$

Theorem 2: Considering the open-loop TS model (3), if there exists matrices Ψ_i, $P_i > 0$, $i \in \{1,\dots,r\}$, such that $\begin{bmatrix} P_i & A_{i_0}^T A_{i_1}^T \dots A_{i_{k-1}}^T \Psi_j \\ (*) & \Psi_j + \Psi_j^T - P_j \end{bmatrix} > 0$ for all combinations of $i_l \in \{1,2,\dots,r\}$, $l \in \{0,1,\dots,k-1\}$ then model (3) is globally asymptotically stable.

Remark 3: The classical results of stability [5] with the Lyapunov function (12) are recovered considering $k = 1$ for the conditions of theorem 2. Note also that a simplified version can be obtained with: $\Psi_i = P_i$.

To show the interest of the proposed approach we consider the following example:

$$A_1 = \begin{bmatrix} a & -1 \\ 0 & -0.9 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0.9 & 0 \\ -0.6 & b \end{bmatrix} \tag{15}$$

It is obvious that for any pair (a,b), $a,b \notin]-1 \;\; 1[$ the two linear models are unstable. Classical quadratic stability conditions (9) cannot prove the stability of this model whatever the pair (a,b) is. The next figure presents the result for non quadratic stabilization in the plane (a,b). The region in the centre (in black) is the region where solutions exist using the classical non quadratic stabilization conditions [5], see remark 3. The other region uses theorem 2 conditions with $k = 4$. The real improvement of the results for that particularly simple example shows the interest of this approach.

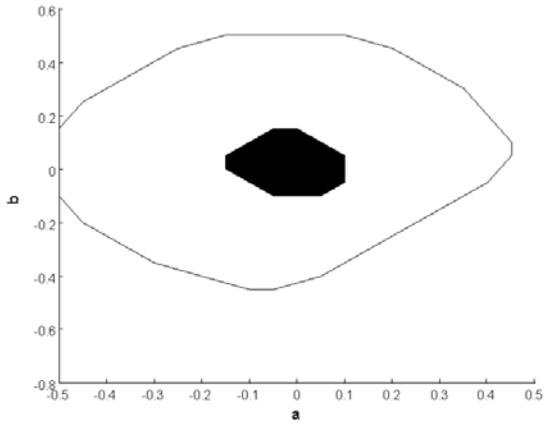

Fig. 1. Stability domains using non quadratic conditions (black center $k=1$, white region $k=4$)

Next sections are interested in the stabilization of TS discrete models following the same approach.

IV. STABILIZATION

Consider the model:

$$\begin{cases} x(t+1) = A_{z(t)}x(t) + B_{z(t)}u(t) \\ y(t) = C_{z(t)}x(t) \end{cases} \tag{16}$$

together the control law (non PDC) [5], we will suppose that $G^{-1}_{z(t)}$ exists for all $z(t)$, this point will be discussed after theorem 3 conditions:

$$u(t) = -L_{z(t)}G^{-1}_{z(t)}x(t) \tag{17}$$

The closed loop can be written as:

$$x(t+1) = \left(A_{z(t)} - B_{z(t)}L_{z(t)}G^{-1}_{z(t)}\right)x(t) = \hat{A}_{zz(t)}x(t) \tag{18}$$

Consider the candidate Lyapunov function [5]:

$$V(x(t)) = x(t)^T\, G^{-T}_{z(t)}P_{z(t)}G^{-1}_{z(t)}x(t) \tag{19}$$

$$\Delta_k V(x) = x^T(t+k)\,G^{-1}_{z(t+k)}P_{z(t+k)}(*) - x^T(t)\,G^{-1}_{z(t)}P_{z(t)}(*) \tag{20}$$

Along the trajectories of (18):

$$\Delta_k V = x(t)^T \left\{ \prod_{l=0}^{k-1} \hat{A}^T_{zz(t+l)} G^{-T}_{z(t+k)} P_{z(t+k)}(*) - G^{-T}_{z(t)}P_{z(t)}G^{-1}_{z(t)} \right\} x(t) \tag{21}$$

$\Delta_k V(x) < 0$ is ensured if:

$$G^T_{z(t)} \prod_{l=0}^{k-1} \hat{A}^T_{zz(t+l)} G^{-T}_{z(t+k)} P_{z(t+k)}(*) - P_{z(t)} < 0 \tag{22}$$

Note that (22) is obtained after congruence with the full rank matrix $G_{z(t)}$. Let us define:

$$A_{zz(t)} = \hat{A}_{zz(t)}G_{z(t)} = A_{z(t)}G_{z(t)} - B_{z(t)}L_{z(t)}$$

then:

$$G^T_{z(t)}\left(\prod_{l=0}^{k-1}\hat{A}^T_{zz(t+l)}\right)G^{-T}_{z(t+k)} = \prod_{l=0}^{k-1}\left(A^T_{zz(t+l)}G^{-T}_{z(t+l+1)}\right) \tag{23}$$

Applying lemma 1 one time will give:

$$\begin{bmatrix} -P_{z(t)} & (*) \\ A_{zz(t)} & -G_{z(t+1)} - G^T_{z(t+1)} + \prod_{l=1}^{k-1}\left(A^T_{zz(t+l)}G^{-T}_{z(t+l+1)}\right)P_{z(t+k)}(*) \end{bmatrix} < 0 \tag{24}$$

Using it a second time gives:

$$\begin{bmatrix} -P_{z(t)} & (*) & 0 \\ A_{zz(t)} & -G_{z(t+1)} - G^T_{z(t+1)} & (*) \\ 0 & A_{zz(t+1)} & \begin{pmatrix} -G_{z(t+2)} - G^T_{z(t+2)} + \\ \prod_{l=2}^{k-1}\left(A^T_{zz(t+l)}G^{-T}_{z(t+l+1)}\right)P_{z(t+k)}(*) \end{pmatrix} \end{bmatrix} < 0 \tag{25}$$

Applying it k times leads to condition (26) given at the bottom of the page. Then it corresponds to the multiple sum of (27) with the $\Upsilon^l_{i_0 i_1 \cdots i_{k-1}, j_0 j_1 \cdots j_{k-1}}$ defined in (28) next page.

$$\begin{bmatrix} -P_z & (*) & \cdots & 0 & 0 \\ A_zG_z - B_zL_z & -G_{z(t+1)} - G^T_{z(t+1)} & \ddots & 0 & 0 \\ 0 & A_{z(t+1)}G_{z(t+1)} - B_{z(t+1)}L_{z(t+1)} & \ddots & (*) & \vdots \\ \vdots & \vdots & \ddots & -G_{z(t+k-1)} - G^T_{z(t+k-1)} & (*) \\ 0 & 0 & \cdots & A_{z(t+k-1)}G_{z(t+k-1)} - B_{z(t+k-1)}L_{z(t+k-1)} & -G_{z(t+k)} - G^T_{z(t+k)} + P_{z(t+k)} \end{bmatrix} < 0 \tag{26}$$

$$\sum_{l=1}^{r}\sum_{i_0=1}^{r}\sum_{j_0=1}^{r}\cdots\sum_{i_{k-1}=1}^{r}\sum_{j_{k-1}=1}^{r} h_{i_0}(z(t))\,h_{j_0}(z(t))\cdots h_{i_{k-1}}(z(t+k-1))\,h_{j_{k-1}}(z(t+k-1))\,h_l(z(t+k))\,\Upsilon^l_{i_0 i_1 \cdots i_{k-1}, j_0 j_1 \cdots j_{k-1}} < 0 \tag{27}$$

$$\Upsilon^{l}_{i_0 i_1 \cdots i_{k-1}, j_0 j_1 \cdots j_{k-1}} = \begin{bmatrix} -P_{i_0} & (*) & \cdots & 0 & 0 \\ A_{i_0} G_{j_0} - B_{i_0} L_{j_0} & -G_{j_1} - G_{j_1}^T & \ddots & 0 & 0 \\ 0 & A_{i_1} G_{j_1} - B_{i_1} L_{j_1} & \ddots & (*) & \vdots \\ \vdots & \vdots & \ddots & -G_{j_{k-1}} - G_{j_{k-1}}^T & (*) \\ 0 & 0 & \cdots & A_{i_{k-1}} G_{j_{k-1}} - B_{i_{k-1}} L_{j_{k-1}} & -G_l - G_l^T + P_l \end{bmatrix} \quad (28)$$

Let us note that it is straightforward to derive a LMI problem from (26) with respect to the variables $P_i > 0$, G_i and L_i $i \in \{1,\ldots,r\}$. For example basic results of [2] can be written as $i_0, i_1, \cdots, i_{k-1}, j_0, \cdots, j_{k-1}, l \in \{1,2,\cdots,r\}$:

$$\Upsilon^{l}_{i_0 i_1 \cdots i_{k-1}, i_0 i_1 \cdots i_{k-1}} < 0 \quad (29)$$

$$\Upsilon^{l}_{i_0 i_1 \cdots i_{k-1}, j_0 j_1 \cdots j_{k-1}} + \Upsilon^{l}_{j_0 j_1 \cdots j_{k-1}, i_0 i_1 \cdots i_{k-1}} < 0$$

$$j_0, \cdots, j_{k-1} \neq i_0, i_1, \cdots, i_{k-1} \quad (30)$$

Note that any existing relaxation can be also used [11], [12]. Nevertheless those involving additional variable matrices, [12] for example, are to be avoided due to the actual solver possibilities.

Theorem 3: Considering the model (16) together the control law (17), if there exist matrices $P_i > 0$, G_i and L_i $i \in \{1,\ldots,r\}$ such that (29) and (30) are satisfied then the closed loop is globally asymptotically stable.

Proof: The fact that (12) and (19) are Lyapunov functions is demonstrated in [5]. It is enough to check that $G_{z(t)}^{-1}$ exists for all $z(t)$.

Considering $-G_{z(t+k)} - G_{z(t+k)}^T + P_{z(t+k)}$ the last term of (26), then $G_{z(t+k)} + G_{z(t+k)}^T > P_{z(t+k)} > 0$ which ensures the property.

Remark 4: The set of solution for the classical non quadratic conditions [5] are obtained with $k = 1$.

To show the interest of these new conditions of stabilization, this second example is built in a way that there is no stabilization condition with $k = 1$ [5] (and of course no quadratic ones). Consider the following TS model:

$$A_1 = \begin{bmatrix} -1.9 & -1.44 \\ 2 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.08 & 0.8 \\ 0.58 & 0.69 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.26 \\ 0.92 \end{bmatrix},\ B_2 = \begin{bmatrix} -0.86 \\ 0.21 \end{bmatrix}.$$

The solution found with $k = 2$ is:

$$P_1 = \begin{bmatrix} 8.5269 & -6.6734 \\ -6.6734 & 7.0602 \end{bmatrix},\ P_2 = \begin{bmatrix} 5.6446 & -4.3773 \\ -4.3773 & 5.7374 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 8.9624 & -7.1013 \\ -7.597 & 7.8830 \end{bmatrix},\ G_2 = \begin{bmatrix} 9.2349 & -6.9426 \\ -6.3173 & 6.9310 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 5.5642 & -5.6420 \end{bmatrix},\ F_2 = \begin{bmatrix} 5.1814 & -5.6202 \end{bmatrix}$$

For the simulation purpose, the following membership functions satisfying the convex sum property are chosen: $h_1(z(t)) = 1 - h_2(z(t)) = (1 - \cos(x_1))/2$.

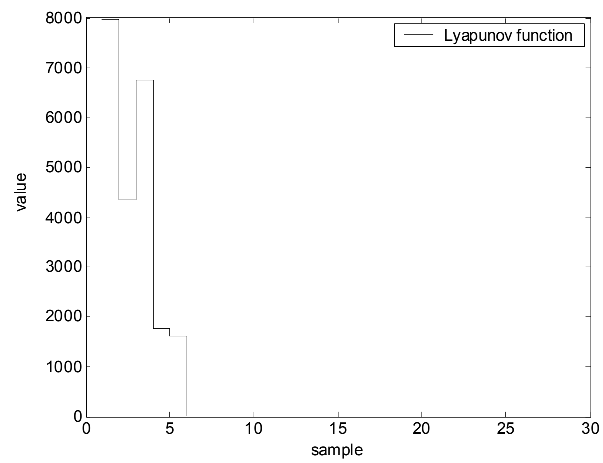

Fig. 2. Evolution of the Lyapunov function

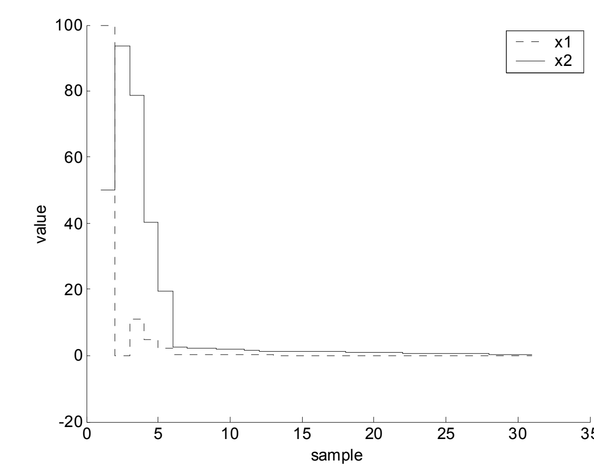

Fig. 3. Evolution of state variables with $x(0) = [100 \quad 50]^T$

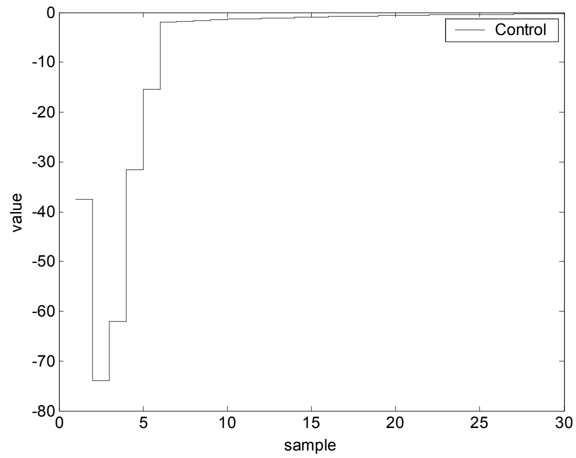

Fig. 4. Control signal evolution

Starting with the initial conditions $x(0)=[100 \quad 50]^T$, figure 2 presents the evolution of the Lyapunov function, figure 3 the evolution of the state variables through time and figure 4 the signal for the control law defined in (17).

Figure 2 shows clearly that locally the Lyapunov function $V(x(t))$ can grow, i.e. see for example sample 2 and 3 where $V(x(3))>V(x(2))$ but as $k=2$ we recover as expected: $V(x(3))<V(x(1))$.

I. Discussion of the results

The examples proposed in stability and stabilization show that this way of dealing with TS discrete fuzzy models can be helpful. This last part will exhibit the different advantages and drawbacks. First of all, the theorem 4 will give a result in terms of inclusion of the different conditions according to the Lyapunov function variation sample k considered.

Theorem 4: Consider the Lyapunov function candidate $V(x(t))=x^T(t)G_z^{-T}P_zG_z^{-1}x(t)$ and two numbers $p,k\in\mathbb{N}^*$. If the conditions (29) and (30) are fulfilled considering the Lyapunov function $\Delta_k V(x(t))$, then the conditions (29) and (30) coming from the Lyapunov function $\Delta_{p\times k}V(x(t))$ are also fulfilled.

Proof:

First of all, if $\Delta_k V(x(t))<0$ then $\Delta_k V(x(t+s\times k))<0$ for $0\le s\le p-1$, thus:

$\Delta_{p\times k}V(x(t))=\sum_{s=0}^{p-1}\Delta_k V(x(t+s\times k))<0$. Now consider the case $p=2$, the general proof follows the same path.

Let us note:

$\bar{A}_{i_m j_m}=A_{i_m}G_{j_m}-B_{i_m}L_{j_m}+A_{j_m}G_{i_m}-B_{j_m}L_{i_m}$ and

$\bar{G}_{i_m j_m}=G_{i_m}+G_{i_m}^T+G_{j_m}+G_{j_m}^T$

$$\Upsilon^l_{i_0\cdots i_{2k-1},j_0\cdots j_{2k-1}}+\Upsilon^l_{j_0\cdots j_{2k-1},i_0\cdots i_{2k-1}}=\begin{bmatrix} -P_{i_0}-P_{j_0} & (*) & \cdots & 0 & 0 & \cdots & 0 \\ \bar{A}_{i_0j_0} & -\bar{G}_{i_1j_1} & \ddots & 0 & 0 & & \\ 0 & \bar{A}_{i_1j_1} & \ddots & (*) & \vdots & & \vdots \\ & \vdots & \ddots & -\bar{G}_{i_{k-1}j_{k-1}} & (*) & \ddots & \\ \vdots & 0 & \cdots & \bar{A}_{i_{k-1}j_{k-1}} & -\bar{G}_{i_kj_k} & \ddots & 0 \\ & & & & \ddots & \ddots & (*) \\ 0 & & \cdots & & 0 & \bar{A}_{i_{2k-1}j_{2k-1}} & -2G_l-2G_l^T+2P_l \end{bmatrix}<0 \tag{31}$$

$$\begin{bmatrix} -P_{i_0}-P_{j_0} & (*) & \cdots & 0 & 0 \\ \bar{A}_{i_0j_0} & -\bar{G}_{i_1j_1} & \ddots & 0 & \vdots \\ 0 & \bar{A}_{i_1j_1} & \ddots & (*) & 0 \\ \vdots & 0 & \ddots & -\bar{G}_{i_{k-1}j_{k-1}} & (*) \\ 0 & \cdots & 0 & \bar{A}_{i_{k-1}j_{k-1}} & -\bar{G}_{i_kj_k}-\begin{bmatrix}\bar{A}_{i_kj_k}\\0\\ \\0\end{bmatrix}^T\begin{bmatrix}-\bar{G}_{i_{k+1}j_{k+1}} & (*) & & 0 \\ \bar{A}_{i_{k+1}j_{k+1}} & \ddots & & \\ 0 & & -\bar{G}_{i_{2k-1}j_{2k-1}} & (*) \\ & & \bar{A}_{i_{2k-1}j_{2k-1}} & -2G_l-2G_l^T+2P_l\end{bmatrix}\begin{bmatrix}\bar{A}_{i_kj_k}\\0\\ \\0\end{bmatrix} \end{bmatrix}<0 \tag{32}$$

The conditions (29) and (30) for $\Delta_{2k}V\left(x(t)\right)$ are given with (31). Then, using a Schur's complement (31) leads to (32). As $\Delta_k V\left(x(t+k)\right)<0$ (33) holds:

$$\begin{bmatrix} -P_{i_k}-P_{j_k} & (*) & \cdots & 0 & 0 \\ \bar{A}_{i_k j_k} & -\bar{G}_{i_{k+1} j_{k+1}} & \ddots & 0 & 0 \\ 0 & \bar{A}_{i_{k+1} j_{k+1}} & \ddots & (*) & \vdots \\ & \vdots & \ddots & -\bar{G}_{i_{2k-1} j_{2k-1}} & (*) \\ \vdots & 0 & \cdots & \bar{A}_{i_{2k-1} j_{2k-1}} & -2G_l-2G_l^T+2P_l \end{bmatrix}<0 \tag{33}$$

then using the Schur's complement on (33)leads to:

$$-\begin{bmatrix} \bar{A}_{i_k j_k} \\ 0 \\ \\ 0 \end{bmatrix}^T \begin{bmatrix} -\bar{G}_{i_{k+1} j_{k+1}} & (*) & & 0 \\ \bar{A}_{i_{k+1} j_{k+1}} & \ddots & & \\ 0 & & -\bar{G}_{i_{2k-1} j_{2k-1}} & (*) \\ & & \bar{A}_{i_{2k-1} j_{2k-1}} & -2G_l-2G_l^T+2P_l \end{bmatrix} \begin{bmatrix} \bar{A}_{i_k j_k} \\ 0 \\ \\ 0 \end{bmatrix}<P_{i_k j_k}+P_{j_k i_k} \tag{34}$$

Then using property (34) on (32) gives immediately the result as i_k in (34) plays the same role as l for (32) applied with $\Delta_k V\left(x(t)\right)$. For $p>2$, just repeat the same operation $p-1$ times.

Note also that an inclusion relation exists between two numbers k_1 and k_2 only if one is the divisor of the other. It can also be seen that the more k increases the larger the solution domain is, i.e. the number of TS discrete fuzzy models possibly stabilized increases. At last, let us give some words about the feasibility. Note first that no additional variables are required using this approach when k increases. However, the number of LMI involved according to the Lyapunov function variation sample k grows quickly as their number does: $r\left(\frac{r(r+1)}{2}\right)^k$. Of course it maybe inconsistent with actual LMI solvers. Using our own experience, after numerous trials using a PC Pentium 4, 3GHz with 512 Mo RAM, MATLAB LMI Toolbox and two linear models, the stability problem resolution is available with reasonable computation time, until $k=12$ and the stabilization until $k=4$. Note that we consider in these trials only TS models with no solution for $k=1$. It shows that even if the approach seems promising we are, for now, still faced with numerical resolution problems.

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