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Robust exact pole placement via an LMI-based algorithm

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Abstract— **This paper deals with the robust exact pole placement problem in connection with the solvability of a Sylvester equation. The main issue is to compute a well-conditioned solution to this equation. The best candidate solution must possess the minimal condition number. This problem is cast as a global optimization under LMI constraints for which a numerical convergent algorithm is provided and compared with the most attractive methods in the literature.**

Keywords: Pole placement, Sylvester equation, condition number, robustness, global optimization, Linear Matrix Inequality.

I. INTRODUCTION

The classical pole placement problem consists of determining a state feedback such that the eigenvalues of the closed-loop system are at a desired location in the complex plan. This problem is deeply treated in the seminal paper [25] (see also [26]). Since then, the pole placement technique for designing suitable dynamic response of a linear plant, becomes among the most important tools in control theory.

It is well-known that the pole placement for the single-input case has a unique solution which can be easily computed by using Ackermann formula [1]. In contrast, the multi-input case of this problem may have an infinite number of possible solutions. Moreover, it is not easy to come up with a numerical solution to the multi-input case problem. For example, the function *place* (from MATLAB control toolbox) cannot assign poles with multiplicity greater than the number of the inputs of the system. In this paper, we provide an algorithm which seems to be numerically efficient and can locate any desired poles (distinct, multiple or overlapping with the open-loop spectra).

In real world problems, the dynamic of the system is not exactly known and may be subject to possible perturbations. Hence, the best pole placement strategy must take into account the sensitivity of the located poles to possible errors in the model of the plant or to external perturbations.

A numerical study of the conditioning of the pole placement problem in terms of the perturbations in the data of the system, can be found in [13], [14]: it is shown that the sensitivity of the located poles depends on the condition number of the closed-loop matrix, the norm of the feedback and the distance to uncontrollability. An earlier result concerns the Bauer-Fike theorem [9]: assume that the matrix $A + BK$ is diagonalizable and X is the associated eigenvector matrix. If λ_{pert} is a pole of the perturbed matrix $A + BK + \Delta$, then there exists a pole λ of $A + BK$ such that

$$|\lambda - \lambda_{pert}| \leq C(X) \parallel \Delta \parallel,$$

where $C(X)$ is the condition number of the eigenvector matrix X.

According to the above fact it is clear that one has to achieve small condition number of the eigenvector matrix to guarantee small variation of the assigned poles against possible perturbations. This is what is called : *Robust Pole Placement problem* (RPP). Many computational methods are devoted to this problem: early work appeared in [10], [18] which was followed by other approaches based on the solvability of a Sylvester equation [2], [5]. The most attractive computational methods seems to be [12], [4], [19], [23]. A numerical treatment of RPP problem with state constraints can be found in [6].

This paper treats the robust exact pole placement problem in connection with the solvability of a Sylvester equation. The problem of computing a well-conditioned solution to this equation is addressed. Some equivalent formulations to this problem are given. Especially, the robust pole placement problem is formulated as a global optimization under LMI constraints. Two LMI-based convergent algorithms are provided. The conception of theses algorithm is based on the same idea of Frank and Wolf algorithm [8], which originally was designed for quadratic optimization under linear constraints. A related LMI-based algorithm [7] has been used for the solution to the static output feedback stabilization problem. This algorithm has a great success and seems to work efficiently for others non convex control problems.

In this paper, some benchmark examples presented in [4] are numerically treated. These examples come from many industrial applications and are known to be ill-conditioned. Numerical comparisons with some famous approaches [4], [12], [19], [23] show the effectiveness of our proposed algorithm.

The remainder of the paper is organized as follows. In section 2, some preliminary results are given and the RPP problem is settled in terms of an optimization problem. In section 3, a special and important case of the RPP problem is treated. Also, a numerical convergent algorithm is designed for this specific case. In section 4, we consider the general case of the RPP problem in terms of a global optimization under LMI constraints. A numerical convergent algorithm is given. Finally, section 5 presents a numerical comparison of the proposed LMI-based approach with others numerical algorithms.

II. RPP problem and Preliminaries

Consider the following time-invariant system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ are given, (A, B) is controllable.

If a state feedback control law $u = Kx$ is used to locate a desired poles $\lambda_1, \cdots, \lambda_n$, then the closed loop matrix $A+BK$ has the following real Jordan decomposition

$$A + BK = X\,\Sigma X^{-1}, \tag{2}$$

where Σ is the *real Jordan matrix* associated to the desired poles and X is the corresponding eigenvector matrix.

It is well-known that a measure of the sensitivity of the eigenvalues of the closed-loop system is the condition number:

$$C_F(X) \equiv \| X \|_F \| X^{-1} \|_F,$$

where the norm $\| \,.\, \|_F$ is the Frobenius norm:

$$\| X \|_F^2 = \sum_{i,j=1}^{n} | X_{ij} |^2 = \mathbf{Tr}(XX^T).$$

Note that an arbitrary set of eigenvalues can be assigned by solving equation (2) as follows. After constructing the *real Jordan matrix* Σ in function of the desired poles, we make the change of variable $Y = KX$ in order to have the following equivalent equation to (2)

$$AX + BY = X\Sigma\,. \tag{3}$$

So that the pole placement problem is solved by looking for an invertible matrix X solution to the equation (3) and consequently the feedback gain $K = Y X^{-1}$ is solution to (2).

It is important to derive the solution to the pole placement problem from an equation which does not involve the matrix gain of the state feedback controller. This formulation is numerically important since the number of variables is reduced. Moreover, it will be shown that the gain matrix K can be computed with a higher accuracy.

In the sequel, we derive an equivalent formulation to (3) which involves less variables. For this purpose, we use the following well-known Lemma.

Lemma 2.1: The matrix system $M\,S = N$ has a solution in the variable S if and only if

$$(I - M\,M^+)N = 0.$$

Moreover, all the solutions are

$$S = M^+N + (I - M^+M)Z,$$

where Z is an arbitrary matrix.

An immediate consequence of the above Lemma is the following result.

Theorem 2.2: Let Σ be the real Jordan matrix associated to a desired poles $\{\lambda_1, ..., \lambda_n\}$. Then there exists a state feedback gain K such that $\sigma(A + BK) = \{\lambda_1, ..., \lambda_n\}$ if and only if there exists a non singular matrix X satisfying

$$(I - BB^+)(X\Sigma - AX) = 0. \tag{4}$$

Moreover, the state feedback gain K is given by

$$K = B^+(X\,\Sigma\,X^{-1} - A)\,. \tag{5}$$

Remark 2.3: In addition to the robustness interpretation of the condition number of the matrix X solution to (4), we can have an accurate numerical computation of the gain matrix K by looking for a well-conditioned X satisfying (4). This fact was also pointed out in [12].

Remark 2.4: Theorem 2.2 has an important consequence: if the condition number of the matrix X is small, then we may also have a controller with a small gain. Effectively, the formula (5) leads to

$$\| K \|_F \leq \| B^+ \|_F \| \Sigma \|_F \, C_F(X) + \| B^+A \|_F \tag{6}$$

Remark 2.5: Throughout this paper the matrix Σ and the variable matrix X are implicitly considered as associated to the Jordan real form of $A + BK$ for some state feedback gain K which, as we have seen can be computed by using formula (5).

According to the preceding considerations we are now able to formulate the robust pole placement problem as the following global optimization problem.

Find X optimal solution to:

$$(P_1)\left\{\begin{array}{l} \min \mathbf{Tr}(XX^T)\mathbf{Tr}(X^{-1}X^{-T}) \\ \text{subject to :} \\ \ (I - BB^+)(AX - X\Sigma) = 0. \end{array}\right. \tag{7}$$

As matter of fact the optimization problem (P_1) is difficult to solve numerically, since the objective function $\mathbf{Tr}(XX^T)\mathbf{Tr}(X^{-1}X^{-T})$ is highly nonlinear and nonconvex in the variable X.

In the sequel, we provide other equivalent formulations to (P_1) in order to design our LMI-based algorithm for the numerical treatment of RPP problem.

III. Ideal RPP problem

In this section, we treat an important special case of the robust pole placement problem. It is shown that this problem can be cast as a global optimization under LMI constraints.

With regard to the preceding analysis we want to minimize the condition number of the matrix X associated to the real Jordan matrix of $A + BK$. If X is orthogonal (i.e: $XX^T = I$), then the condition number achieves the smallest value n (the size of the system). Consequently, we can guarantee small variations of the spectrum of the closed loop system against possible perturbations. The problem of finding an orthogonal eigenvector matrix is shown to be equivalently formulated by the following optimization problem.

Find X optimal solution to:

$$(P_2)\left\{\begin{array}{l}\min -\mathbf{Tr}(XX^T)\\ \text{subject to:}\\ \left\{\begin{array}{l}(I-BB^+)(AX-X\Sigma)=0,\\ \left[\begin{array}{cc} I & X\\ X^T & I\end{array}\right]\geq 0.\end{array}\right.\end{array}\right. \tag{8}$$

We have following result.

Theorem 3.1: The optimal value of the optimization problem (P_2) is exactly $-n$ if and only if there exists a state feedback gain K such that

$$A+BK=X\Sigma X^{-1},\ and\, XX^T=I. \tag{9}$$

Proof: Assume that the global optimum of problem (P_2) is n and achieved by X^*. By Schur lemma the following LMI

$$\left[\begin{array}{cc} I & X^*\\ X^{*T} & I\end{array}\right]\geq 0,$$

is equivalent to $X^*X^{*T}\leq I$. since $\mathbf{Tr}(X^*X^{*T})=n$ we must necessarily have $X^*X^{*T}=I$. By using the result of Theorem 2.2, we see that condition (9) is satisfied. The rest of the proof follows the same line of argument. ∎

In order to treat numerically the above optimization problem (P_2), we introduce the following algorithm.

Algorithm 3.2:

Step 0: Set $X_o=I$

Step 1: $X_{i+1}=\arg\min -\mathbf{Tr}(X_i^TX)$

subject to :

$$\left\{\begin{array}{l}(I-BB^+)(AX-\Sigma X)=0\\ \left[\begin{array}{cc} I & X\\ X^T & I\end{array}\right]\geq 0\end{array}\right.$$

Step 2: If the sequence $\mathbf{Tr}(X_i^TX_i)$ is stationnary stop, else go to step 1.

Here it is shown that the algorithm 3.2 generates a sequence (X_i) such that $\mathbf{Tr}(X_iX_i^T)$ is increasing and converges to a stationary value. In the case when the stationary value is equal to n the problem (P_2) is solved.

Let us now prove our claim. The following trivial identity holds for any i.

$$\begin{array}{l}\mathbf{Tr}((X_i-X_{i+1})(X_i-X_{i+1})^T)-\mathbf{Tr}(X_{i+1}X_{i+1}^T)\\ \quad=\ -2\mathbf{Tr}(X_iX_{i+1}^T)+\mathbf{Tr}(X_iX_i^T),\end{array}$$

since $\mathbf{Tr}((X_i-X_{i+1})(X_i-X_{i+1})^T)\geq 0$, this implies

$$-\mathbf{Tr}(X_{i+1}X_{i+1}^T)\leq -2\mathbf{Tr}(X_iX_{i+1}^T)+\mathbf{Tr}(X_iX_i^T). \tag{10}$$

At each step of the algorithm, the objective function $-\mathbf{Tr}(X_iX^T)$ is minimized and since X_{i+1} is the optimal solution, we must have

$$\mathbf{Tr}(X_iX_i^T)\leq \mathbf{Tr}(X_iX_{i+1}^T).$$

Combining the above inequality with the inequality(10) it follows

$$\mathbf{Tr}(X_iX_i^T)\leq \mathbf{Tr}(X_{i+1}X_{i+1}^T).$$

We conclude that the sequence $\mathbf{Tr}(X_iX_i^T)$ is increasing and bounded by n (because of $X_iX_i^T\leq I$). Then the sequence necessarily converges to a stationary value.

IV. General RPP problem

We have shown in section 2 that the robust pole placement problem can be expressed equivalently as

Find X optimal solution to:

$$(P_1)\left\{\begin{array}{l}\min \mathbf{Tr}(XX^T)\mathbf{Tr}(X^{-1}X^{-T})\\ \text{subject to :}\\ \ (I-BB^+)(AX-X\Sigma)=0.\end{array}\right. \tag{11}$$

The following result provides a simple and equivalent expression of problem (P_1).

Lemma 4.1: Consider the following optimization problem

$$(P_3)\left\{\begin{array}{l}\min \mathbf{Tr}(X^{-1}X^{-T})\\ \text{subject to:}\\ \left\{\begin{array}{l}(I-BB^+)(AX-X\Sigma)=0,\\ \mathbf{Tr}(XX^T)\leq 1.\end{array}\right.\end{array}\right. \tag{12}$$

Then the optimal values of (P_1) and (P_3) are equal. Moreover, any optimal solution to (P_3) is also optimal for (P_1).

Proof: Denote by X_3 any optimal solution of (P_3). We show first that $\mathbf{Tr}(X_3X_3^T)=1$. Effectively, if $\mathbf{Tr}(X_3X_3^T)<1$, this will contradicts the fact that X_3 is an optimum. To see this, let $\alpha=\mathbf{Tr}(X_3X_3^T)^{-1}>1$ and define $X^*=\alpha^{1/2}X_3$. Then X^* is feasible for (P_3) and satisfies $\mathbf{Tr}(X^{*-1}X^{*-T})<\mathbf{Tr}(X_3^{-1}X_3^{-T})$, which is impossible.

Now, denote by X_1 any optimal solution of (P_1). Since X_3 is feasible for (P_1) and $\mathbf{Tr}(X_3X_3^T)=1$, we have necessarily

$$\mathbf{Tr}(X_3^{-1}X_3^{-T})\geq \mathbf{Tr}(X_1X_1^T)\mathbf{Tr}(X_1^{-1}X_1^{-T}).$$

Also, since $(\mathbf{Tr}(X_1X_1^T))^{-1/2}X_1$ is feasible for (P_3) we also have

$$\mathbf{Tr}(X_3^{-1}X_3^{-T})\leq \mathbf{Tr}(X_1X_1^T)\mathbf{Tr}(X_1^{-1}X_1^{-T}),$$

and the proof is complete. ∎

At this stage, we are in position to establish the following.

Theorem 4.2: The solution to the robust pole placement problem is given by the following optimization problem

$$(P_4)\left\{\begin{array}{l}\min \mathbf{Tr}(X^{-1}X^{-T})\\ \text{subject to:}\\ \left\{\begin{array}{l}(I-BB^+)(AX-X\Sigma)=0,\\ \mathbf{Tr}(P)=1,\\ \left[\begin{array}{cc} P & X\\ X^T & I\end{array}\right]\geq 0.\end{array}\right.\end{array}\right. \tag{13}$$

Proof: It suffices to show that the constraint $\mathbf{Tr}(XX^T) \leq 1$ is equivalent to the LMIs

$$\mathbf{Tr}(P) = 1, \quad \begin{bmatrix} P & X \\ X^T & I \end{bmatrix} \geq 0,$$

then by using Lemma 4.1 the proof is straightforward. ■

A. LMI-based algorithm

To address numerically problem (P_4), we consider the following algorithm:

Algorithm 4.3:

Step1: $X_1 = \arg\min -\mathbf{Tr}(X)$ subject to constraints (13).

Step 2: Let $X = \arg\min -\mathbf{Tr}(X_i^{-1}X_i^{-T}X_i^{-1}X)$ subject to constraints (13).
Compute $X_{i+1} = \arg\min C_F(\alpha X_i + (1-\alpha)X)$, for $\alpha \in [0,1]$

Step 3: If the sequence $C_F(X_i)$ is stationary, exit. Else go to step 2.

In Algorithm 4.3 we minimize a linearization of the function $\mathbf{Tr}(X^{-1}X^{-T})$. Effectively, one can show that the derivative of $\mathbf{Tr}(X^{-1}X^{-T})$ at X_i is exactly $-2\mathbf{Tr}(X_i^{-1}X_i^{-T}X_i^{-1}X)$.

Algorithm 4.3 is based on the same idea of Frank and Wolf algorithm [8], which originally was designed for quadratic optimization under linear constraints. A related LMI-based algorithm [7] has been used for the solution to the static output feedback stabilization problem. This algorithm has a great success and seems to work efficiently for others non convex control problems.

B. Convergence and behavior of Algorithm 4.3

By conception Algorithm 4.3 provides a sequence with decreasing condition numbers and then converges to a stationary value.

Extensive numerical tests showed that this algorithm exhibits a fast convergence and provides a big decrease of the condition number. Its numerical effectiveness is compared with many existing algorithms (see below the numerical comparisons).

One of the limitation of some pole assignment algorithms is that an arbitrary set of poles cannot be assigned. For example, the function *place* (from MATLAB control toolbox) cannot assign poles with multiplicity greater than the number of the inputs of the system (rank(B)). Note that *place* is based on the algorithm of [12]. This limitation also applies to the improved algorithm in [19]. As mentioned in [24], another possible limitation is when the closed and open-loop spectra overlap.

Choosing any desired poles (distinct, multiple or overlapping with the open-loop spectra), the proposed Algorithm 4.3 has a nice and same numerical behavior. By extensive numerical tests we have found that the first iteration always provides a nonsingular solution and since the condition number of the iterates progressively decreases these iterates remain nonsingular.

We stress out the proposed LMI formulation is very flexible, for example, it can take into account other constraints such as bound on the norm of the state feedback gain or decoupling modes.

V. Numerical results

For the numerical experiments we have used Matlab 6.0. Our Algorithm 4.3 is implemented by using the Semidefinite Programming code SP [21] interfaced with Matlab.

We have treated 11 benchmark examples presented in [4]. These examples come from different industrial applications and are known to be ill-conditioned. Our numerical results are compared with the following Algorithms: *place* from the Matlab Control Toolbox based on the algorithm of [12], *robpole* based on the algorithm proposed in [19], *sylvpalce* proposed in [23] and the results of the algorithm proposed in [4].

Our algorithm performs largely better than *place* for all the examples. For the others algorithm, TABLE I, TABLE II and TABLE III illustrate the fact that our algorithm performs better for the minimization of the condition number C_F. Moreover, even if we are not minimizing the norm of the gain K of the controller our algorithm performs almost identically or even better for some examples.

The behaviour of our Algorithm 4.3 is exhibited in the figures given below. Note that for all 11 examples, the algorithm has a fast convergence and stabilizes in very few iteration.

Examples	place			robpole	
	$C_2(X)$	$C_F(X)$	$\| K \|_2$	$C_2(X)$	$\| K \|_2$
1	3.43	6.56	1.45	4.27	1.28
2	40.06	53.142	219.85	39.85	225.5
3	37.47	53.425	59.40	39.29	49.1
4	10.78	13.428	9.83	10.77	9.44
5	91.62	146.17	4.53	88.56	5.14
6	3.619	5.997	21.32	3.63	19.41
7	4.803	12.16	329.115	4.65	235.2
8	26.038	36.986	19.231	3.61	19.84
9	18.464	23.954	844.07	18.44	820.5
10	1.05	4.0029	1.33	1.00	1.41
11	12526	14618	6692	12443	6580

TABLE I

Examples	sylvplace		Byers-Nash		
	$C_2(X)$	$\| K \|_2$	$C_2(X)$	$C_F(X)$	$\| K \|_2$
1	3.39	1.45	3.35	6.40	1.46
2	37.68	354.8	33.07	39.11	354.85
3	35.48	77.25	33.85	41.23	77.15
4	10.77	9.44	10.77	11.87	9.44
5	89.05	4.22	85.37	137.50	4.22
6	3.58	23.0	3.55	5.84	23.00
7	4.38	270.3	4.74	11.91	305.50
8	3.61	19.25	3.61	5.89	28.25
9	18.42	829.2	18.59	21.24	807.57
10	1.00	1.41	1.00	4.00	1.42
11	12443	6580	1.2E4	1.2E4	6.6E3

TABLE II

Examples	LMI-based approach		
	$C_2(X)$	$C_F(X)$	$\| K \|_2$
1	3.347	6.40	1.434
2	33.069	39.106	354.9
3	33.83	41.22	77.37
4	10.773	11.866	9.44
5	85.367	137.498	4.224
6	3.940	6.263	12.65
7	4.544	11.539	346.6
8	3.61	5.887	21.13
9	18.075	20.728	829.18
10	1.000	4.000	1.417
11	11670	11749	6582

TABLE III

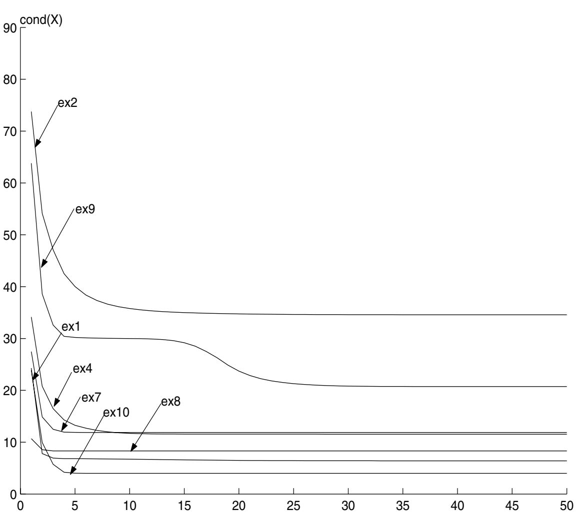

figure1: illustrates the evolution of the $C_F(X)$

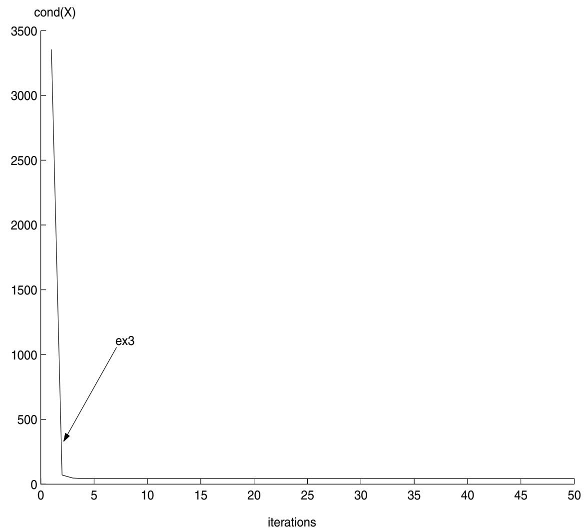

figure2: illustrates the evolution of the $C_F(X)$

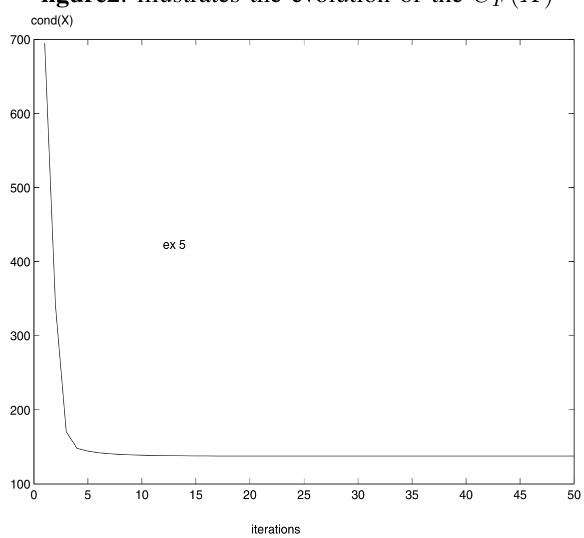

figure3: illustrates the evolution of the $C_F(X)$

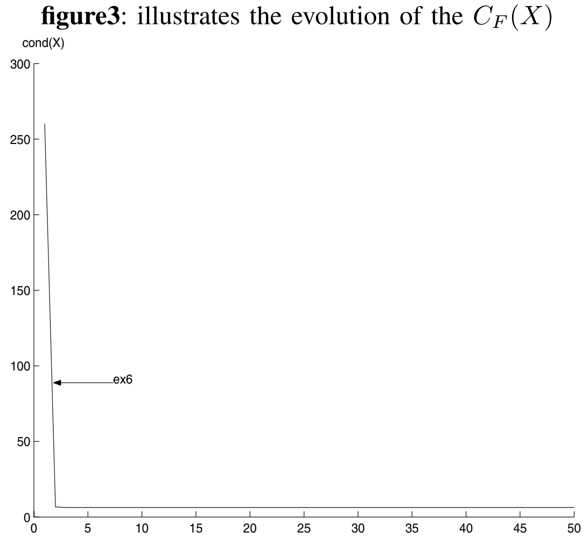

figure4: illustrates the evolution of the $C_F(X)$

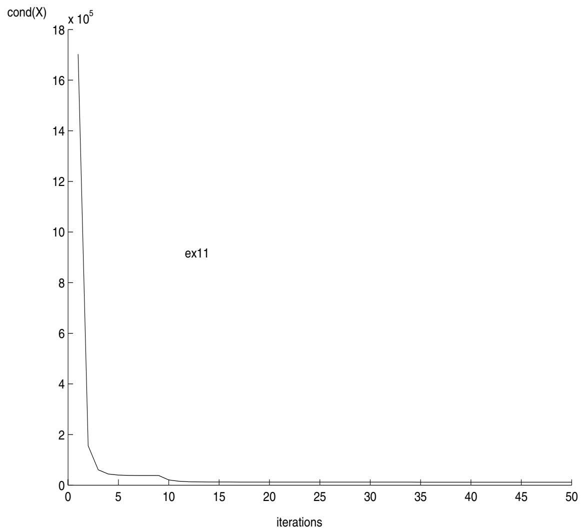

figure5: illustrates the evolution of the $C_F(X)$

VI. CONCLUSION

This paper treated the robust exact pole placement problem in connection with the solvability of a Sylvester equation. We have addressed the problem of computing a well-conditioned solution to this equation. In other words, we have seen that the problem reduces to the minimization of the condition number. Some equivalent formulations to this problem are given. Especially, the robust pole placement problem is formulated as a global optimization under LMI constraints. Two LMI-based convergent algorithms are provided. Numerical comparison with other approaches in the literature, shows the effectiveness and the good behavior of the proposed algorithm.

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