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Fault Tolerant Model Predictive Control applied on the Barcelona Sewer Network

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Abstract— **Fault-tolerant model predictive control strategies of sewer networks are investigated in this article. A general linear model of sewer networks is presented and the corresponding expression into MPC formalism are discussed. From FTC and MPC, two strategies are proposed and compared: Passive Fault Tolerant MPC, that takes advantage of natural tolerance (robustness) of MPC and Active Fault Tolerant MPC, that uses active fault tolerance techniques in combination with MPC. Finally, some results are obtained applying these strategies over a portion of the Barcelona sewage network.**

I. Introduction

UNDER the city of Barcelona there is a complex sewer system, almost 1500 Km in length, which is characterized for being unitary, that is, its collectors carry together rain water and residual water. Having a unitary system, it is specially important that the sewer network is capable of complying with its role during severe rains as well as in non-threatening weather, see [1]. The yearly rainfall in Barcelona is not very high (600 mm/year), but it includes heavy storms typical of the Mediterranean climate that cause a lot of flooding problems and *Combined Sewer Overflows* (CSO) into the streets and to the receiving waters. This has motivated investigation of global control strategies of the sewer network, capable of minimizing flooding inside the city limits and the amount of untreated sewage going into the environment (pollution). The actuators present in a sewer system are retention and redirection gates as well as pump stations. Detention tanks are built to increase the ability of the sewer network to respond to rain storms.

An interesting approach to global control on the sewer networks includes the use of an operative model of the system dynamics and the network constrains to calculate, for a future horizon, optimal strategies to handle actuators taking into account the actual system state information (given by sensors connected to a SCADA) and a adequate rain prediction [1] as well as system restrictions. One of the most natural ways to calculate optimal control set points to be applied at the actuators is based on model predictive control.

Model Predictive Control (MPC) is one of the few advanced control methodologies which has had a significant impact on industrial control engineering. Due to its ability to handle multivariable problems with constraints, it is one of the most successful control strategies in process control (see [2], [3]). The successful application of MPC on sewer systems has been reported in [4], [5], among others.

Fault tolerant control is a relatively new concept which aims at improving control performance by treating failures and exemptions in the operation of technical processes in a systematic way, see [6]. The robustness of feedback control systems gives rise to an implicit fault tolerance. Faults that occur under closed loop control are often compensated by the control action. The same applies when MPC is used. It has been furthermore demonstrated that even when knowledge of the fault is not available, when the estimation of external disturbances affecting the loop is performed in a special way and the input levels have hard constraints, the MPC controller automatically takes advantage of actuator redundancy when available, see [7]. As all states are assumed measurable in the application of current article, this fault tolerance property does not apply.

On the other hand, it is possible, when using the MPC formalism, to increase fault tolerance, if knowledge of faults is available by modifying parameters of the optimization problem which is solved in each sample. Faults that affect the internal model or system constraints can in this way be incorporated into a MPC controller in a natural way. Furthermore, due to the flexibility that control objectives can be expressed within the MPC formalism, when faults cause control objectives to become unattainable, they can be dropped from the optimization problem or degraded in priority, for example, by changing hard constraints to soft ones.

The control strategy which results when information about faults is used to modify the optimization problem of the model predictive controller will be referred to as Active Fault Tolerant Model Predictive Control (AFTMPC). To distinguish between cases, the control strategy when no knowledge is assumed available to modify the optimization problem will be referred to as Passive Fault Tolerant Model Predictive Control (PFTMPC). The exact characteristics of the two strategies will be discussed in a later section.

The aim with the current paper is to compare the performance of the two strategies for realistic rain and fault scenarios. Performance improvements when using AFTMPC motivate investment in diagnosis systems to monitor the health of actuators when rain storms occur. Another aim is to investigate the importance of objective prioritization and reconfiguration for sewer network control.

The paper structure is the following: in Section II, the modelling approach of sewer network is introduced along

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with the formulation of the MPC problem. In Section III, fault tolerant is discussed. In Section IV, the assumptions made for the comparison are presented and in Section V the results are presented. Finally, conclusions and further work are discussed in Section VI.

II. MPC ON SEWER NETWORKS

A. System Modeling

Several modelling techniques have been presented in the literature that deal with sewer networks, see [5], [8]. The modelling approach used in this paper is based on the approach presented in [4]. There, the sewer system was divided into connected subgroups of sewers and treated as interconnected *virtual tanks*. At any given time, the stored volumes represent the amount of water inside the mains associated with the tank and are calculated on the basis of area rainfall and flow exchanges between the interconnected virtual tanks. The volume is calculated through the mass balance of the stored volume, the inflows and the outflow of the tank and the input rain, as:

$$x_i(k+1) \quad = \quad x_i(k) + \varphi SP_i(k) + \Delta t(q_i^{in}(k) - q_i^{out}(k))$$

where φ is the ground absorption coefficient of the *i-th* tank, S is the area of the *i-th* tank catchment, P is the precipitation intensity in Δt of the *i-th* tank catchment and Δt is the time interval between measurements. $q_i^{in}(k)$ and $q_i^{out}(k)$ are the sum of inflows and outflows, respectively. *Real detention tanks* are modelled in the same way but without the precipitation term.

The tanks are connected with flow paths or links which represents the main sewage pipes between the tanks. The manipulated variables of the system, denoted as u_i, are related to the outflows from the tanks. The outflows are assumed to be proportional to the tank level, that is, $q_i^{out}(k) = \beta_i x_i(k)$. In the case of a real tank, a *retention gate* is present to control the outflow. This means that this outflow can be closed completely in the fault free case. Virtual tank outflows can not be closed but can be redirected with *redirection gates*. The redirection gates divert the flow from a nominal flow path which the flow follows if the redirection gate is closed. This nominal flow is denoted as Q_i in the equation below, which expresses mass conversation at the redirection gate.

$$q_i^{out}(k) = Q_i(k) + \sum_j u_i^j(k) \tag{1}$$

j is an index over all manipulated flows coming from the redirection gate. The flow path which Q_i represents is assumed to have a certain capacity and when this capacity reaches its limits, an overflow situation occurs. This flow limit will be denoted $\overline{Q}_i$. When Q_i reaches its capacity, two cases are considered: first, the water starts to flow on the streets, causing an overflow situation and secondly, it exits the sewer network and is considered lost to the environment. In the first case, the overflow water either follows the nominal flow path and ends up in the same tank as Q_i or it is diverted to an other virtual tank. Flow to the environment physically represents the situation when the sewage water ends up in a river or, in the case of the Barcelona situation, in the Mediterranean sea.

Model constraints: When using this modelling approach where the inherent nonlinearities of the sewer network are simplified by assuming that only flow rates are manipulated, physical restrictions need to be included as constraints on system variables. For example, variables u_i^j that determinate outflow from a tank should never be larger than the outflow from the tank. This is expressed with the following inequality

$$\sum_j u_i^j(k) \leq q_i^{out}(k) = \beta_i x_i(k) \tag{2}$$

Usually the range of actuation is also limited so that the manipulated variable has to fulfill $\underline{u}_i^j \leq u_i^j(k) \leq \overline{u}_i^j$, where $\underline{u}_i$ denotes the lower limit of manipulated flow and $\overline{u}_i$ denotes its upper limit. When $\underline{u}_i^j$ equals zero, this constraint is convex but if the lower bound is larger than zero, constraint in Eq. (2) has to be included in the range limitation. This leads to the following non-convex inequality:

$$\min(\underline{u}_i^j, q_i^{out}(k) - \sum_{w \neq j} u_i^w(k)) \leq u_i^j(k) \leq \overline{u}_i^j \tag{3}$$

The sum in the expression is calculated for all outflows related to tank i except j. A further complication is that if the control signal is a inflow to a real tank that has hard constraints on its capacity, then the situation can occur that this lower limit is also limited by this maximum capacity and the outflow from the real tank.

The limit on the range of real tanks is expressed as

$$0 \leq x_i(k) \leq \overline{x}_i \tag{4}$$

As this constraint is physical, it is impossible to send more water to a real tank than it can hold. The virtual tanks do not have a physical limit on their capacity. When they rise above a decided level an overflow situation occurs. This represents the case when the level in the sewers has reached a limit so that an overflow situation can occur in the streets. Notice that in practice, the difference between tank and link CSO is often small.

Using the modelling formalism presented, a difference equation for the tank volumes in the sewer network can be written as

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \mathbf{B_p}\mathbf{d}(k) \tag{5}$$

where $\mathbf{x}(k)$ is a vector with the tank volumes, $\mathbf{u}(k)$ represents manipulated flows, vector $\mathbf{d}(k)$ corresponds to rain perturbations and constant matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{B_p}$ are the system matrices. When the lower limit on $\mathbf{u}(k)$ is zero, the model constraints can be written as $\mathbf{Ex} + \mathbf{Hu} \leq \mathbf{b}$.

Remark: Sewer networks are inherently highly nonlinear systems. Many of the parameters that in the current article are assumed to be constant are in fact dependent on various variables. On the other hand, as the focus of this study is on differences between control strategies and not absolute values, it is assumed that the linear models used approximate the real behavior sufficiently well for the conclusions drawn to be valid.

B. MPC Problem Formulation

As the model is linear and constraints are in most cases linear as well, the MPC controller presented in this paper could be designed using text book formalisms such as presented in [3]. Using Eq. (5), the states are expressed as affine functions of the changes in the control signal, $\Delta\mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$ for a prediction horizon H_p. The control signal is, on the other hand, only allowed to change over the control horizon, H_u. The cost function and constraints are expressed using this affine function and the optimization is solved using $\Delta\mathbf{u}$ as optimization variables. The manner in which peculiarities of the problem such as how to model overflow and how to deal with the non convex constraint given by Eq. (3) will be discussed later.

1) Control objectives: The sewer system control problem has multiple objectives with varying priority. The main objectives are listed below in order of decreasing priority.

1) minimize CSO in streets (virtual tank overflow)
2) minimize CSO in links
3) minimize losses to the environment (pollution)
4) maximize sewage treatment

The purpose of the last objective is to reduce the levels in the tanks to anticipate future rainstorms.

2) Constraints included in the optimization problem: The physical constraints of the system that were presented in Eqs. (2), (3) and, in the case of real tanks, Eq. (4), are added as constraints in the optimization problem.

The preferable way to deal with the control objectives is to include them as well as constraints in the MPC optimization problem. This would mean including constraints on the virtual tanks as expressed in Eq. (4) for control objective 1 while in the case of link overflow, the constraint to add would be $0 \le Q_i(k) \le \overline{Q}_i$.

The problem with including these control objectives as constraints is that the optimization problem can become infeasible. It is easy to see that constraints on the level in the virtual tank can be violated if the precipitation, $d(k)$ is very high. Eq. (1) can be violated if $\beta_i x_i$ is larger than the sum of the upper limit of the variables on the right side of the equation. Infeasibility is therefore related to the balance of mass in the system. The optimization problem can, on the other hand, be made feasible if the constraints related to control objectives are relaxed and changed into soft constraints. On-line, this can be done by manual intervention or by an automatic procedure.

3) Soft constraints: When the optimization problem becomes infeasible, the constraints related to control objectives are included in a "soft" way into the cost function, see [3]. This includes adding a new variable ϵ_i to relax each constraint. In the case of the tank constraint, this would be

$$0 \le x_i(k) - \overline{x}_i \le \quad \epsilon_i^x(k) \tag{6}$$

while the link overflow constraint would be expressed as

$$0 \le Q_j(k) - \overline{Q}_j \le \quad \epsilon_j^Q(k) \tag{7}$$

The auxiliary variables $\epsilon_i^x(k)$ related to the tank constraint are gathered into a vector $\epsilon^x(k) = [\epsilon_1^x(k)\ \epsilon_2^x(k) \ldots \epsilon_M^x(k)]$, where M is the number of tank constraints relaxed and the auxiliary variables related to link overflow into a vector $\epsilon^Q(k)$. The vectors $\epsilon^x(k)$ and $\epsilon^Q(k)$ are then added with an appropriate weight to the cost function. Notice that vector ϵ^Q could contain elements related to link overflow or flow lost to the environment.

4) The cost function: The cost function has the form

$$\begin{aligned} V(k) = & \sum_{i=0}^{H_p-1} \|\epsilon^x(k+i)\|^2_{\Gamma_x} + \left\|\epsilon^Q(k+i)\right\|^2_{\Gamma_Q} \\ & + \sum_{i=0}^{H_p-1} \left\|\overline{Q}_i - Q_i(k+i)\right\|^2_{\Gamma_{\overline{Q}}} + \sum_{i=0}^{H_u-1} \|\Delta\mathbf{u}(k+i)\|^2_{\Gamma_{\Delta u}} \end{aligned}$$

where Γ corresponds to weight matrix associated with each term through its subscript. The first term corresponds to control objective 1. The second term includes both control objectives 2 and 3. The flow links in the third term are the links connected to waste water treatment plants. This term therefore corresponds to the 4 objective. Finally, the fourth term is added to obtain smooth changes in the control signal over the horizon. Although the presented cost function is of quadratic nature, linear terms can be included in a similar manner.

As the cost function is of quadratic nature, efficient quadratic programming algorithms can be used in the case when the constraints are convex ($\underline{u}_i^j = 0$). In the example presented in the current article, the function `quadprog` in the MATLAB Optimization Toolbox was used. When the lower limits of u_i^j were different than zero, the more general `fmincon` was used. This function permits introducing constraints as in Eq. (3).

III. Fault Tolerant MPC

In Fig. 1, a schematic diagram of an architecture for Fault Tolerant MPC (FTMPC) is shown. Into the plant block enter disturbances $d(k)$ and the control signal $u(k)$ while the state $x(k)$ exits. The MPC block contains the optimizer that uses the measured states and disturbances, the plant model and model constraints as well as the control objectives to calculate the minimum of the cost function.

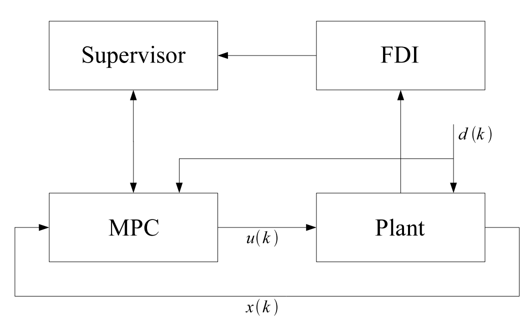

Fig. 1. Schematic diagram of FTMPC.

If the control loop has an active fault tolerance strategy, the FDI block detects and diagnoses the faults and their magnitude and sends the information to the supervisor. The supervisor changes the parameters of the optimization problem for the optimizer based on the FDI information. Thus, the difference between active FTMPC and passive FTMPC is the existence of the supervisor and FDI block in the active case. Notice that, in this article, the FDI block is assumed to work optimally. It detects and isolates the faulty components and returns the fault magnitude. The fault information is assumed readily available and is used to modify the optimization problem.

Only actuator faults are considered in this article. Actuation faults manifest themselves as limitations to the range of the manipulated flow rates, that is, when a fault occurs, the constraint in Eq. (3) is replaced with new range limits, $\underline{u}_{i,f}^{j}$ and $\bar{u}_{i,f}^{j}$. These limits are used in the AFTMPC strategy but not in the PFTMPC strategy.

The flow range can be limited from below due to the inability to close a gate and it can be limited from above due to the inability to open a gate sufficiently or reduction in pump capacity. A stuck actuator means the range is limited to a point or very narrow interval.

IV. Comparison of fault tolerant MPC strategies

The purpose of this section is to compare AFTMPC and PFTMPC for realistic scenarios of rain storms and actuator faults for a portion of the Barcelona sewer network. The assumptions made for the comparison will be presented and their validity discussed before the results are given.

A. Sewer network example

The example represents a portion of the Barcelona sewer network that includes two virtual tanks, one real detention tank (Escola Industrial Tank 35.000 m^3) and three actuator gates (two redirection gates and one retention gate). The real tank is designed to avoid flooding downstream and help minimize pollution. In Fig. 2 the system is shown. The CSO flow elements are not depicted in the figure but, as explained with Eq. (1), their origin and end follow Q_i. An exemption is Q_3, where overflow pollutes the sea. In what follows, the comparison of control strategies will concentrate on comparing CSO in links (objective 2) and losses to the environment (objective 3) as the variables associated with these objectives have the same unites making the comparison easier. Control objective 1, CSO in virtual tanks was therefore not considered in the comparison.

The considered system is a small part of the Barcelona sewer network that includes 50 virtual tanks and 5 real tanks. 15 more real tanks are being built for the purpose of avoiding flooding and undesired releases of sewage to the environment. The system matrices are

$$\mathbf{A} = \begin{pmatrix} 1-\Delta t\beta_1 & 0 & 0 \\ 0 & 1 & 0 \\ \Delta t\beta_1 & 0 & 1-\Delta t\beta_3 \end{pmatrix}$$

$$\mathbf{B} = \Delta t \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} \quad \mathbf{B_p} = \Delta t \begin{pmatrix} 0 & \alpha_2 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & \alpha_3 \end{pmatrix}$$

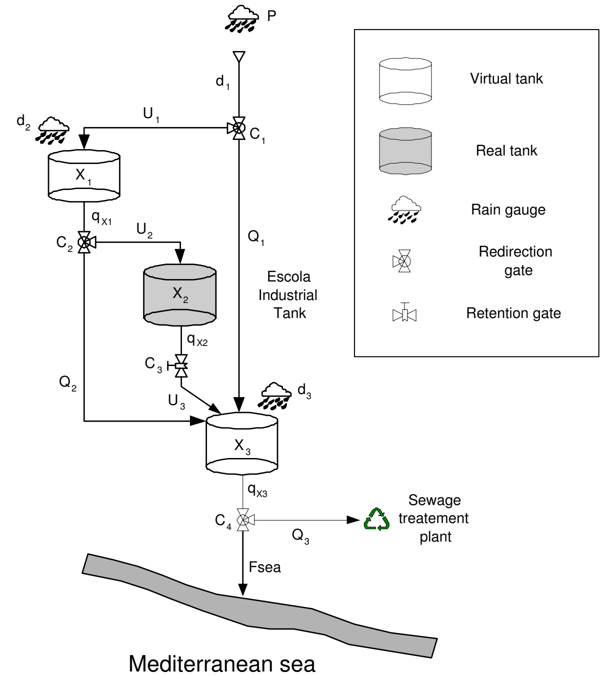

Fig. 2. Example of Barcelona test catchment.

where the sampled period is $\Delta t = 300s$. The numerical values of the system parameters are $\alpha_2 = 0.5715$, $\alpha_3 = 0.0783$, $\beta_1 = 5.8 \times 10^{-4}$ and $\beta_3 = 1.0 \times 10^{-3}$. Associated range limitations are in m^3/s (superscript j is not used as there is only one manipulated output from each tank)

$$u_1(k) \in [0, 11], \quad u_2(k) \in [0, 25], \quad u_3(k) \in [0, 7]$$

The flow d_1 into the redirection gate at the top of the figure is the outflow from a virtual tank not considered in the example since there are no control signals that affect its volume. When $u_1(k)$ is limited from below, constraint Eq. (3) is replaced by

$$\min(d_1(k), \underline{u}_1(k)) \leq u_1(k) \leq \overline{u}_1(k) \tag{8}$$

The flow to the sewage treatment plant is given by the link flow Q_3. The capacity of the links were $\overline{Q}_1 = 9.14$, $\overline{Q}_2 = 3.4$ and $\overline{Q}_3 = 9$.

B. Rain scenarios

The rain scenarios considered were based on real rain gauge data obtained within the city of Barcelona on the given dates. In Fig. 3 a scenario is shown which occurred the 14 of September 1999. The rain is shown in cubic meters per second. To get these units, the area of the virtual tank S was multiplied with the reading of the rain gauge and with ground absorbtion factor φ to obtain the total rain that entered the virtual tank per time unit. To get the final volume of water that entered the tank in each sample, Δt had to be taken into account. The rain storm shown in the figure caused severe flooding in the area that the example represents. Rain storms with this amount of precipitation have a 4-year average return period. Rain storms with the same maximum intensity have on the other hand a 12-year return period.

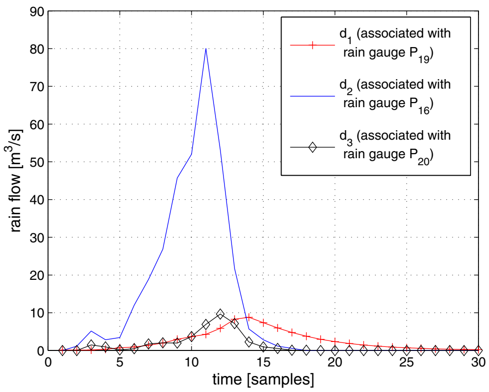

Fig. 3. Heavy rain scenario that occurred 14 September, 1999.

The fault tolerant control strategies were compared for a number of rain scenarios. The scenario for which the results will be presented has a 0.7-year average return period with regard to total amount and 10-year return period with regard to maximum intensity. It occurred 17 October, 1999.

C. Fault scenarios

The fault scenarios consist of limiting the range of the actuators in three ways:

1) Limit range from below (range is 50-100%), denoted as $f\underline{u}_i$.
2) Limit range from above (range is 0-50%), denoted as $f\overline{u}_i$.
3) Limit from below and above, simulating stuck actuator (50-51%) and denoted as $f\overline{\underline{u}}_i$.

In scenario, $f\underline{u}_2$ the lower limit of actuator 2, $\underline{u}_2$ was put equal to the upper limit of actuator 3, $\overline{u}_3$. The reason for this arrangement is that the optimization problem is infeasible if tank 2 is full and $\underline{u}_2 > \overline{u}_3$.

D. Simulation of scenarios

The control strategies were compared by simulating the closed loop system for various fault and rain scenarios. The model used for simulation was the same as was used for the model predictive controller but in the PFTMPC case, the actuator ranges were limited in the plant.

The duration of the simulations was selected as 48 samples or 4 hours as the rain storm generally had peaks of duration around 10 samples or 50 minutes. The tanks were empty in the beginning of the scenarios. The rain storm peaks generally occurred in the first 20 samples. To compare strategies, total flooding and pollution released was added over the whole scenario.

The prediction horizon and control horizon were selected as 6 samples or 30 minutes. This selection was based on the reaction time of the system to disturbances. An other reason for this selection is that rain prediction becomes less reliable for larger horizons.

Two cases were considered with regard to the prediction of rain $d(k)$ over the control horizon. In the first case, $d(k)$ was assumed to be equal to the last measurement over the whole horizon. The other case considered was when perfect knowledge of rain was available over the horizon. The difference between the two cases was found to be small, confirming similar results reported in [4]. Real rain predictions based on radars would be somewhere between these two cases, better than assuming rain constant over the horizon but worse than perfect prediction. In what follows, results are reported only for the second case, that is when rain is known over the prediction horizon.

E. Nominal control tuning

When hard constraints have to be relaxed, the soft constraints that enter the cost function have to be weighted in some way to reflect the control objectives and their priority. This involves selecting the weighting matrixes Γ_x, Γ_Q in the cost function. The units of the variables in the cost functions have to be taken into account when the weights are selected.

The variables related to objective 2 (Q_1 and Q_2) were weighted equally in the cost function while the weight of the variable related to environmental losses (Q_3) was weighted less. Various ratios between the weights of the two objectives were considered. For many scenarios, increasing weights on objective 2 improved CSO link overflow marginally while causing a large increase in losses to the environment. This kind of problems could be solved by lexicographic prioritization as presented in [9]. Denoting the ratio between the weights as γ, results will be presented for $\gamma = 100$. The weight associated with objective 4 was selected small enough to not interfere with the other objectives. Weights for $\Delta\mathbf{u}$ were selected small as well.

V. Results

Generally, CSO flooding in streets was reduced when AFTMPC was used compared to PFTMPC. The biggest improvements were obtained when precipitation was large enough so that actuators needed to operate close to the upper limit of their range, that is when the precipitation brought the sewer network close to its capacity.

AFTMPC did not yield great improvements when the heavy rain scenario presented in Fig. 3 was simulated. The reduction in CSO that could be archived was about 0-10%. The reason for this is that the considered portion of the Barcelona sewer network does not have the capacity to handle rain storms with that intensity even in the fault free case. Therefore it did not matter if actuation limits were known to the controller or not.

When very common rain scenarios with little precipitation were studied the same thing occurred, that is, AFTMPC did not give a great improvement. The reason for this is that in those scenarios the constraints are usually not reached in terms of actuation and thus, faults in actuators rarely affect performance.

Results are shown in Table I for a rain storm that occurred 17. October 1999. For each actuator of the system and for each fault type, the total CSO volume that flooded to the streets during the simulation of the scenario and the total volume of sewage lost to the environment are shown as pairs (V_{str}, V_{env}) with the unit of cubic meters (m^3). Even though results are shown for one specific scenario, the conclusions presented were based on simulation of various scenarios.

TABLE I

RESULTS FOR RAIN STORM 17. OCTOBER, 1999

FAULT SCENARIO		PFTMPC	AFTMPC	
ACTUATOR	TYPE		HIGHER PRIORITY OBJECTIVE 2	HIGHER PRIORITY OBJECTIVE 3
u_1	$f\underline{u}_1$	(**2200**, 3400)	(**80**, 500)	(1100, **620**)
	$f\overline{u}_1$	(**0**, 1000)	(**0**, 2400)	(0, **0**)
	$f\overline{\underline{u}}_1$	(**900**, 1600)	(**0**, 360)	(970, **650**)
u_2	$f\underline{u}_2$	(**190**, 4700)	(**0**, 6000)	(300, **0**)
	$f\overline{u}_2$	(**21400**, 9000)	(**11300**, 15200)	(23700, **6400**)
	$f\overline{\underline{u}}_2$	(**20600**, 8800	(**10900**, 18600)	(21700, **5900**)
u_3	$f\underline{u}_3$	(**0**, 3400)	(**0**, 5500)	(1200, **550**)
	$f\overline{u}_3$	(**4700**, 3900)	(**1400**, 5500)	(6000, **0**)
	$f\overline{\underline{u}}_3$	(**4200**, 3700)	(**0**, 3400)	(6000, **160**)
No fault		—	(**0**, 5000)	(0, **0**)

To show the consequences of the objective prioritization, the last column in the tables shows the variables when the priorities of the objectives were reversed, that is, when objective 3 was considered of higher priority then objective 2 and the weights were reversed accordingly in the cost function. The bold number in each pair in the table represents the variable with the higher priority.

When comparing these numbers, a reference to consider is the total CSO flooding when the heavy rain scenario presented in Fig. 3 was simulated in the fault free case. The total CSO to the streets in that scenario was 20000 m^3. As was said before, that rain storm caused severe flooding in the catchments present in the example.

In Table I, it can be seen that in three scenarios, flooding to the streets would have been prevented by using the AFTMPC strategy. These are scenarios $f\overline{\underline{u}}_1$, $f\underline{u}_2$ and $f\overline{\underline{u}}_3$. In the first two scenarios, the flooding was not of a large magnitude but in case $f\overline{\underline{u}}_3$, the PFTMPC strategy resulted in a street flooding of 4200 m^3.

The largest flooding reduction was obtained for scenarios $f\overline{u}_2$ and $f\overline{\underline{u}}_2$. There the flooding was reduced from roughly 20000 to around 11000 which is a large reduction when the reference value from the heavy rain scenario mentioned before is kept in mind. In Fig. 4, the CSO flooding reduction (upper bars) using AFTMPC compared to PFTMPC is shown in percentages. As seen, for most cases the reduction was quite large. Notice that to keep the comparison simple, hard constraints were not included in the MPC controllers in the study. Constraint management for sewer networks is a challenging problem as the number of variables and actuators is very large. Also notice that in case $f\overline{u}_1$, the volume of the sewage lost to the environment is 2400 when objective 2 has higher priority. When priorities are switched, sewage lost to the environment is reduced to zero while CSO to the streets does not increase. The same thing occurs in the case when no fault is present. With a lower weight ratio γ between objectives, these number did not improve. This shows that hard constraints should be used to achieve control objectives when possible.

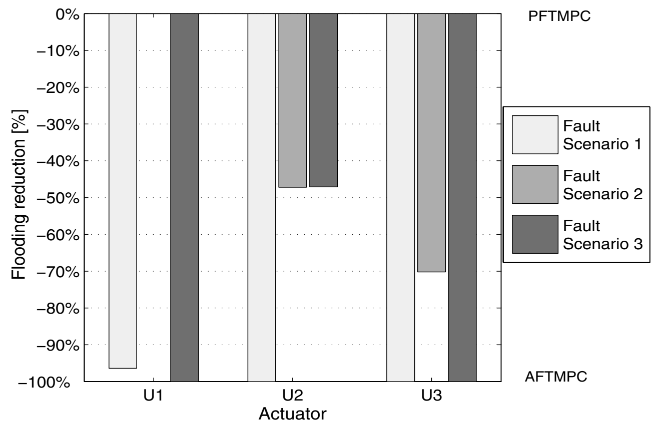

Fig. 4. Flooding reduction using AFTMPC

VI. CONCLUSIONS AND FURTHER WORK

The article presented a comparison between AFTMPC and PFTMPC applied to sewer networks under realistic rain and fault scenarios. The result showed that AFTMPC reduces CSO flooding in all cases. Furthermore, rain scenarios where the sewer network reached its design limit, using AFTMPC could prevented flooding or reduce it considerably.

It was shown that it is preferable to include control objectives as hard constraints if possible because secondary objective might suffer needlessly if soft constraints with different weights are used.

The study presented motivates the use of FDI algorithms to diagnose actuator faults in sewer networks. The diagnosis algorithm should provide the limits on the actuator range to be useful for fault tolerant control.

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