Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Active Vibration Suppression of a Flexible Beam via Sliding Mode and H∞ Control

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Abstract—**In this study, sliding mode and H_∞ control techniques are applied to a flexible beam in order to suppress some of the vibration modes. The beam is a clamped-free flexible structure having piezoelectric (PZT) patches as actuators and a laser displacement sensor for measuring the tip point deflection. The beam is modeled in two different ways for each control algorithm. To implement sliding mode control (SMC), Euler-Bernoulli beam model is used and a finite dimensional LTI model is formed by using assumed mode method. As the SMC requires state measurement, an observer is designed to estimate the states from the measured tip deflection. In order to implement H_∞ control algorithm, the model of the flexible beam, which is an approximate transfer function, is constructed by using system identification technique. The experimental results of designed SMC and H_∞ control algorithms are presented.**

I. INTRODUCTION

VIBRATION control of flexible structures is of great interest as light structures in all engineering applications are getting much more important. Various control strategies have been suggested and applied to different flexible systems in order to suppress vibration of flexible structures. Some of these studies are SMC [1, 2, 3], LQG control [4], QFT Control [5], and H_∞ control [6, 7, 8]. In these studies, infinite dimensional flexible structures are, in generally, modeled as finite dimensional linear systems by taking some of the vibration modes into account by means of the assumed mode method and Finite Element Model [1, 4, 5, 8, 9, 10].

SMC is a particular type of the so-called Variable Structure Control (VSC) that changes the control direction(s) to drive the system to a specified manifold in the state space and then keep the system within a neighborhood of this manifold. Hence, SMC design for a system consists of two stages; design of the manifold such that the so-called reduced-order system has stable dynamics and the design of high frequency discontinuous control so that the system trajectories are directed to the manifold for all initial conditions [11, 12]. The main feature of SMC is its insensitivity to some class of uncertainties which makes it attractive in the control applications for uncertain systems. Another approach to control of uncertain systems is H_∞ control which uses an optimization problem with an operator norm, called H_∞ norm [13, 14].

In this study, SMC and H_∞ control techniques are designed and applied to a clamped-free flexible beam in order to suppress the first two vibration modes of the flexible structure. The partial differential equation (PDE) of the beam obtained from Euler-Bernoulli beam equation is transformed to finite dimensional ordinary differential equations (ODEs) by means of the assumed mode method in the SMC design. As the SMC uses states of the system, which are unachievable, these are obtained by means of observer design. With the observer design, the output (tip deflection) is used and the structure of SMC becomes very similar to that of H_∞ control, i.e., the output of the system is measured and used in the creation of control law. On the other hand, experimental system identification method based on the work in [15] is used to obtain system model (transfer function) and then H_∞ control is applied to this model.

The organization of the paper is as follows; in Section 2, the system modeling is described for each control algorithms. Section 3 gives the controller design. In section 4, experimental setup and results are given. Finally, conclusions are given in Section 5.

II. SYSTEM MODELING

The control algorithms studied here require different system modeling in order to implement the designed control laws. For instance, SMC law is based on finite dimensional state space model of the system, whereas H_∞ control law needs the transfer function. Therefore, in this study, two models of the flexible beam are formed and used for each control algorithms. For the SMC law, Euler-Bernoulli beam is used and PDE of the system is transformed into a finite dimensional ODEs and the state space form of the finite dimensional system is used. For the H_∞ control law, by using experimental results, an approximate transfer function is obtained.

A. *Euler-Bernoulli Beam Model*

The model of clamped-free flexible beam studied here is given in Fig. 1.

Manuscript received March 7, 2005. This work was supported in part by the State Planning Organization of Turkey.

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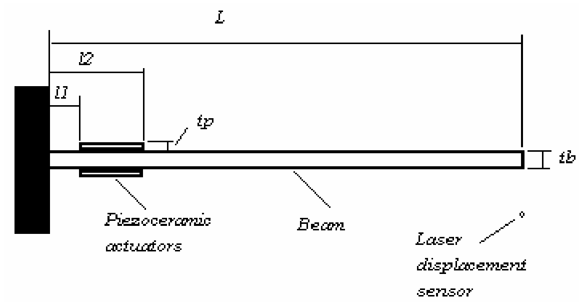

Fig. 1, Flexible beam model

Two piezoelectric patches are bonded to the flexible beam as actuators near the fixed end and laser displacement sensor is used to measure the tip point displacement. By using the Euler-Bernoulli beam equation, the infinite dimensional mathematical expression of the beam can be written as follows;

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y(x,t)}{\partial x^2}\right]+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}=C_a\frac{\partial^2 V_a(x,t)}{\partial t^2} \quad (1)$$

where $y(x,t)$ is the deflection along the x–axis, E is the Young's modulus, I is the moment of inertia, A is the cross-sectional area, and ρ is the density of the uniform beam. $V_a(x,t)$ is the applied control voltage to piezoelectric actuators. Since piezoelectric actuators are uniform along their lengths, the control voltage $V_a(x,t)$ can be replaced by $V_a(t)$. The PDE given by (1) can be solved by using the so-called assumed mode approach which yields finite dimensional ordinary differential equation set. In that case, we assume;

$$y(x,t)=\sum_{i=1}^{\infty} q_i(t)\phi_i(x) \qquad \text{and}$$

$$\ddot{q}_i(t)+2\xi_i w_i \dot{q}_i(t)+w_i^2 q_i(t)=\frac{C_a}{\rho A L^3}[\phi_i'(l_1)-\phi_i'(l_2)]V_a(t)$$

$$\text{for } i=1,2,\ldots n \quad (2)$$

In (2), $q_i(t)$, $\phi_i(x)$ $\phi_i'(x)$ are the i^{th} modal coordinate, mode shape function, and modal slope respectively. On the other hand, w_i and ξ_i are the natural frequency and the damping ratio of the i^{th} mode respectively. In this study, first two modes of the beam are controlled ($i= 1,2$). The mode shape function for clamped-free beam is given by

$$\phi_i(x)=L(\cosh(\lambda_i x)-\cos(\lambda_i x)-k_i(\sinh(\lambda_i x)-\sin(\lambda_i x)) \quad (3)$$

where $k_i=\dfrac{\cosh(\lambda_i L)+\cos(\lambda_i L)}{\sinh(\lambda_i L)+\sin(\lambda_i L)}$.

The quantities λ_i are the real roots of the following equation

$$\cos(\lambda_i L)\cosh(\lambda_i L)=-1 \quad (4)$$

For the first two modes of the beam, the quantities are calculated as $\lambda_1=3.7955$ and $\lambda_2=9.5020$. The natural frequencies can be calculated from,

$$w_i=\sqrt{\frac{EI}{\rho A}}\lambda_i^2 \quad (5)$$

where, $I=bt_b^3/12$, $A=bt_b$. For the beam studied here, the natural frequencies are found to be w_1=6.5720 and w_2=41.1890 Hz. Damping ratios of the first two modes of the flexible beam are obtained as ξ_1=0.07 and ξ_2=0.02 [1]. The constant C_a in (2) can be calculated from

$$C_a=E_p d_{31} b(t_b+t_p) \quad (6)$$

where E_p is the Young's modulus, t_p is the thickness, and d_{31} is the electric charge constant of the piezoelectric patches. b and t_b are the width and thickness of the beam respectively. The beam model for the first two modes can be written in the state space form as follows;

$$\dot{x}(t)=\begin{bmatrix}0 & 1 & 0 & 0\\ -w_1^2 & -2\xi_1 w_1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -w_2^2 & -2\xi_2 w_2\end{bmatrix}x(t)+\frac{C_a}{\rho A L^3}\begin{bmatrix}0\\ \phi_1'(l_1)-\phi_1'(l_2)\\ 0\\ \phi_2'(l_1)-\phi_2'(l_2)\end{bmatrix}u(t) \quad (7)$$

For the SMC implementation state space modeling of the beam given in (7) will be used.

B. *System Identification of The Beam*

In order to implement H_∞ control law, transfer function of the system is necessary. In this study, the transfer function of the system is approximated by using experimental system identification technique which is based on the works [6] and [15]. Experimental identification of the beam includes two approaches; white-box and black-box modeling. White-box modeling is theoretical and black-box modeling is the experimental part of the system identification. The method having two of them is called gray-box modeling [15]. Parametric and nonparametric techniques can be used to construct system model in gray-box modeling. In parametric system identification, system model is considered with parameter vector to be determined. The important thing in the determination of parameters of the system is the adjustment of frequency responses of the system with the experimental frequency responses.

In this study, nonparametric and parametric system identification methods are used. In the nonparametric part, a sinusoidal signal with variable frequency is applied to the beam and response of the beam is obtained. Windowing and Welch average techniques are used to obtain smooth response of the beam since smooth response is important for parametric identification. After nonparametric identification, the frequency response model is obtained by using least square curve fitting method. The system model obtained by nonparametric method can be expressed as follows;

$$g(z)=\frac{\sum_{j=1}^{p} n_j z^j}{z^p+\sum_{j=1}^{p-1} d_j z^j} \tag{8}$$

where p shows the order of the equation. The parameters to be found are the coefficients of the numerator and denominator of the transfer function. The order of the equation (p) is selected by doing the experimental part of the system identification (black-box system modeling). The system is excited with a sinusoidal chirp signal of frequency up to selected mode frequencies, and then the response of the system is stored to be used in the white-box system modeling. The order of the equation is determined by curve fitting (order of the curve) so that estimated transfer function response satisfies the experimental part. After some mathematical operations [6], (8) can be considered as a "least square method" problem of the following form;

$$A\hat{x}=\hat{b}+\hat{r}\ . \tag{9}$$

where,

$$A=\begin{bmatrix} g(z_1) & g(z_1)z_1 & \cdots & g(z_1)z_1^{p-1} & -1 & -z_1 & \cdots & -z_1^p \\ g(z_2) & g(z_2)z_2 & \cdots & g(z_2)z_2^{p-1} & -1 & -z_2 & \cdots & -z_2^p \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g(z_m) & g(z_m)z_m & \cdots & g(z_m)z_m^{p-1} & -1 & -z_m & \cdots & -z_m^p \end{bmatrix}$$

and

$$\hat{x}=\begin{bmatrix}\hat{d}\\ \hat{n}\end{bmatrix}\ ,\quad \hat{b}=-\begin{bmatrix} g(z_1){z_1}^p \\ g(z_2){z_2}^p \\ \vdots \\ g(z_m){z_m}^p \end{bmatrix}$$

where m is the number of frequency point, r is the minimum norm vector which satisfies (9).

After some mathematical and experimental operations, the transfer function of the flexible beam is approximated as follows [6];

$$G(s)=\frac{-0.00024\mathrm{s}^8-0.01185\mathrm{s}^7-128.9\mathrm{s}^6+3552\mathrm{s}^5}{\mathrm{s}^8+41.43\mathrm{s}^7+5.648\times10^5\mathrm{s}^6+1.637\times10^7\mathrm{s}^5}$$

$$\frac{-2.294\times10^7\mathrm{s}^4-9.158\times10^8\mathrm{s}^3+3.117\times10^{11}\mathrm{s}^2+3552\mathrm{s}^5}{+3.492\times10^{10}\mathrm{s}^4+6.383\times10^{11}\mathrm{s}^3+1.262\times10^{14}\mathrm{s}^2} \tag{10}$$

$$\frac{+6.433\times10^{12}s+7.224\times10^{14}}{+1.134\times10^{15}\mathrm{s}+1.1\times10^{17}}$$

By using the transfer function of the beam obtained experimentally, the frequency response of the system is plotted in Fig.2. As it can be seen from Fig.2, the estimated transfer function obtained by (10) can represent the system well enough since the error between the estimated and fitted (experimental) transfer functions is quite small.

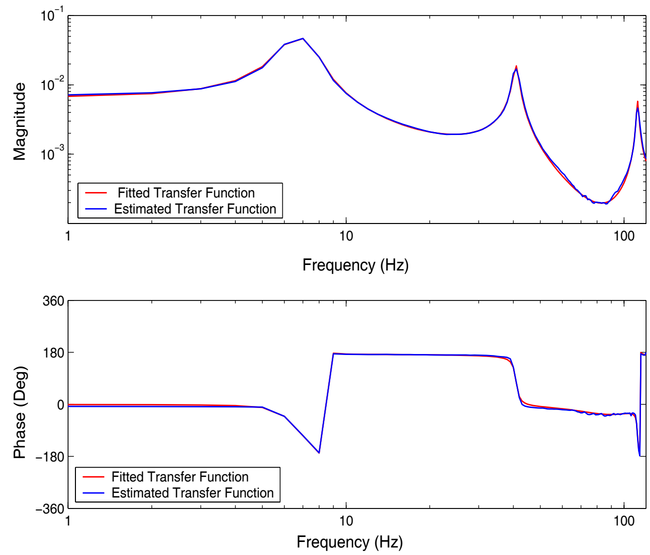

Fig. 2, Frequency response of the flexible beam.

III. CONTROLLER DESIGN

In this section, two control algorithms for the active vibration suppression of the flexible beam are introduced.

A. Sliding Mode Controller

SMC design for flexible beam comprises two stages: the design of a manifold so that sliding motion satisfies a stable dynamics and synthesis of a control law such that the trajectories of the closed loop motion are directed towards the surface. Since the states of the flexible beam system, which are necessary to construct the controller, can not be measured directly, an observer is designed which estimates the system states by measuring the tip point displacement of the beam.

Controller Design: The flexible beam model given by (7) is an LTI system and it can be represented by

$$\begin{aligned}\dot{x}(t)&=Ax(t)+Bu(t)\\ y(t)&=Cx(t)\end{aligned} \tag{11}$$

where the triple *(A, B, C)* is controllable and observable. The standard linear sliding surface design procedure can be applied to (10), which starts with the following sliding surface equation [11, 12];

$$\sigma(x,t)=Sx(t) \tag{12}$$

The controller which directs the system states to the surface and keeps it on the surface has two parts; equivalent control and high frequency discontinuous control. The equivalent control is given by $u_{eq}(t) = -(\widetilde{S}\widetilde{B})^{-1}\widetilde{S}\widetilde{A}\,x(t)$ and it is continuous one which can be implemented quite reasonably with high performance microprocessors. The high frequency control input, on the other hand, is given by

$$u_n(t) = -K\,sign(\sigma(x)) \quad (13)$$

where K is a positive constant. Note that $u_n(t)$ is a high frequency discontinuous control and PZT patches in the system can produce high frequency control signals. Now the total control input becomes

$$u(t) = u_{eq}(t) + u_n(t) \quad (14)$$

Observer Design: The modal coordinates and modal velocities, which are the states of the flexible beam model, can not be measured directly. Therefore we design an observer in order to estimate the system states. The observer design is based on [11, 12]. The estimates have,

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y - C\hat{x}(t)) \quad (15)$$

where $\hat{x} \in R^n$ is the n-dimensional estimated states which gives the error between real and estimated states as $e = x - \hat{x}$. The error dynamics of the observed system becomes $\dot{e}(t) = (A - LC)e(t)$ which implies that $(A - LC)$ must have negative eigenvalues to obtain a stable observer. The observer gain matrix, L, should be chosen carefully so that the estimated states should converge the system states as quick as possible. Improper choice of observer gain matrix may lead to system instability.

B. H_∞ Controller Design

In this section, H_∞ controller is designed to increase the damping ratio of the flexible beam in the working frequency interval and stability of the system under disturbances.

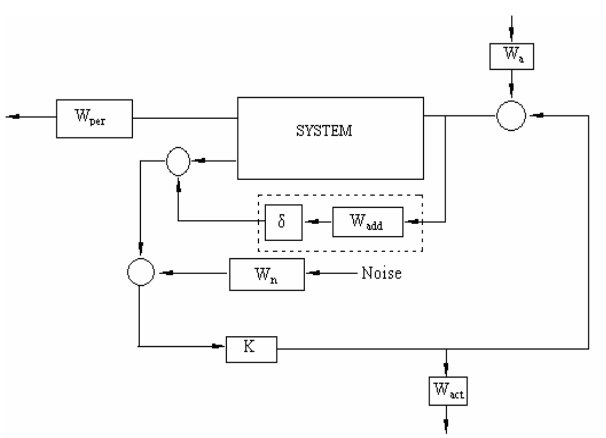

Fig.3, Block diagram of the H_∞ Controller

Schematic diagram of the H_∞ controller is given in Fig.3. In the figure, system represents the model of the beam obtained from system identification. The uncertainties are represented as W_{add} at the block diagram. W_{per} is the expected performance specifications of the system, W_{act} is saturation of the actuators, W_d is for external disturbances and W_n is for noises at the measuring device in the system. Details of designing H_∞ controller for this flexible beam can be found in [6].

IV. EXPERIMENTAL SETUP AND RESULTS

The beam used in this study is shown in Fig.4. In the experimental setup 507x51x2 mm Aluminum beam and 20x25x0.61 mm dimensional 8 pieces BM500 type piezoelectric (PZT) patches were used. Parameters of the flexible beam and PZT patches are given in Table 1.

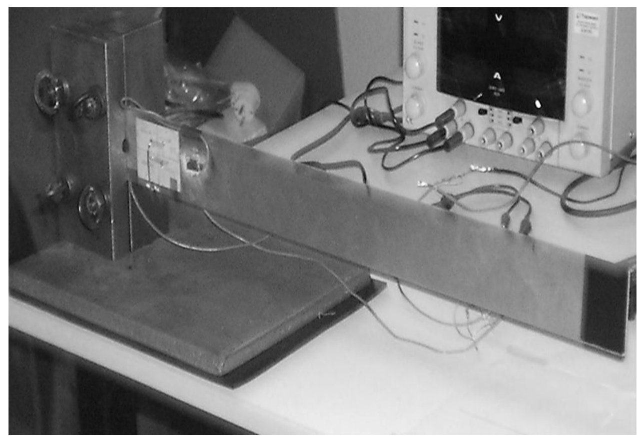

Fig.4, The flexible beam system

Table1. Parameters of flexible beam

Beam length, L	0.507 m
Beam width, b	0.051 m
Beam thickness, t_b	0.002 m
Beam density, ρ	2480 kg/m^3
Beam Young's modulus, E	70x10^9 N/m^2
PZT position, l_1	0.026 m
PZT position, l_2	0.076 m
Charge constant, d_{31}	-200x10^{-12} m/V
PZT Young's modulus, E_p	60x10^9 N/m^2
PZT width, w	0.051 m
PZT thickness, t_p	6.1x10^{-4} m

In order to implement the controllers designed in section 3, Sensortech SS10 type four-channel programmable controller and data acquisition system was used. This programmable controller system is controlled by a personal computer which runs with Linux operating system. The control signal was send to Sensortech SA10 high voltage power amplifier in order to apply to PZT patches. The controller system can send signal between -10V and +10V, then this signal was amplified 15 times by high voltage power amplifier. This means that control signal is bounded at +150V and –150V. Tip point displacement of the beam was measured by Keyence laser displacement sensor which has the maximum sampling frequency of 1024 μs. The measured signal was fed back to controller system. The experimental setup of the flexible beam system is given in Fig.5.

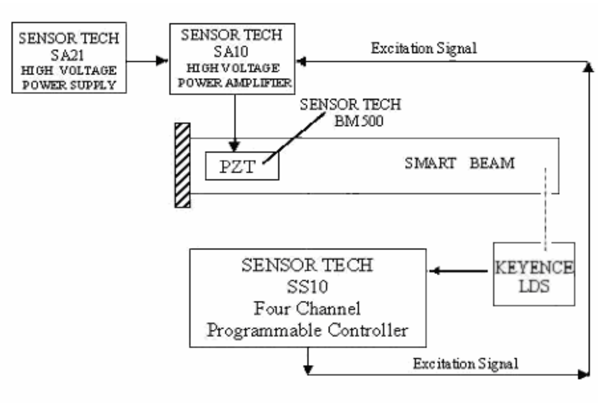

Fig.5, Experimental setup of the flexible beam system

The open loop time response of the beam for 0.04 m initial tip point displacement and zero velocity is given in Fig. 6.

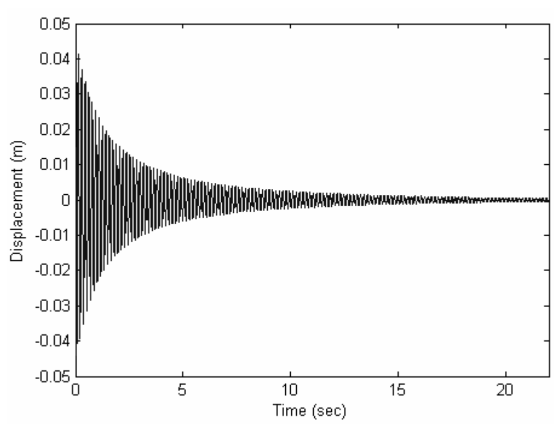

Fig.6, Free response of the beam for 0.04 m tip point displacement.

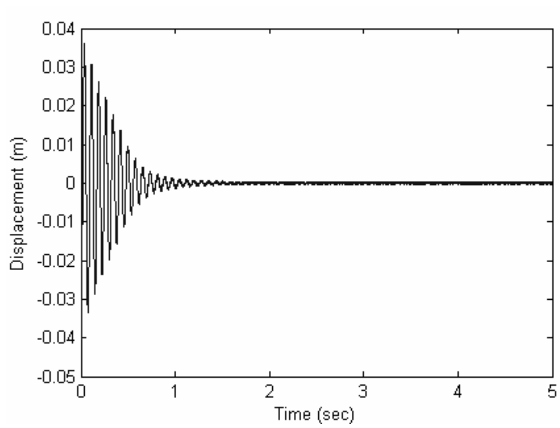

Fig.7, Closed loop response of the beam for SMC.

Time response of the beam for SMC and applied control input to PZT patches are given in Fig.7 and Fig.8 respectively. In the experimental studies, the positive constant *K,* in (13) is selected to be 0.5. The output is produced by D/A National Instruments PCI-6713 card and the maximum sampling rate for producing the output signal is 2048 sample/s. The SMC algorithm suppresses the vibration of the beam in less then 1 sec. As expected, because of the discontinuous part of SMC (and because of vibration), the control signal changes its direction and a high frequency control signal is required. Because of bound in the control signal, SMC control signal is between -150 and +150 V.

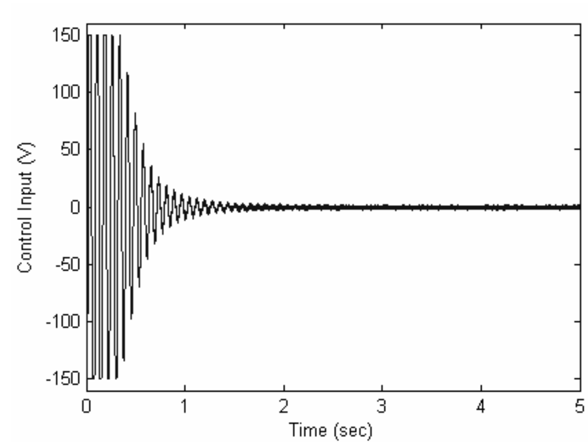

Fig.8, Applied control voltage to PZT patches for SMC.

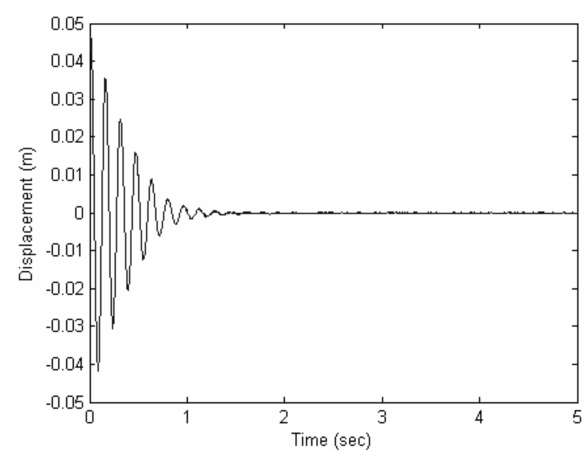

Fig.9, Closed loop response of the beam for H_∞ control.

Time response of the beam for H_∞ control and applied control input to PZT patches are given in Fig.9 and Fig.10 respectively. The designed H_∞ control algorithm also suppresses the vibration in a very short time and the applied voltage is again discontinuous. However, H_∞ control algorithm presents a more smooth control signal. On the other hand, because of high order transfer function modeling, applied control voltage takes longer time compared to SMC.

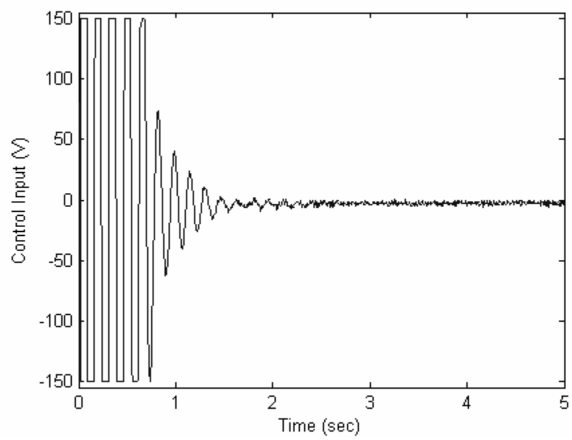

Fig.10, Applied control voltage to PZT patches for H_∞ control.

V. Conclusions

In this work, two different control algorithms, SMC and H_∞ control were designed and implemented in order to suppress some of the vibration modes of a flexible beam. The SMC design is based on finite dimensional model of the Euler-Bernoulli beam and the states of the model are obtained by using an observer. The observer uses the output information, which is the tip deflection of the beam, and generates the system states. By this configuration, the SMC structure looks like the structure of the H_∞ controller, i.e., the output information of the system is utilized. On the other hand, H_∞ controller was designed by constructing system model from system identification. By performing experimental identification method, an estimated transfer function which represents the system is formed. Two different control strategies were applied to the same system and the experimental results showed the success of the control approaches.

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